

8F8all 2018 COT 3100 Section 2 Final Exam - Free Response

Last Name: _____, **First Name :** _____

Please show your work and put a box around your final answer for each question.

1) (8 pts) Jaylon is 7 miles south of a stream that flows due east. He is also 12 miles west and 21 miles north of his apartment. He wishes to go to the stream to collect some water and then return to his apartment. What's the minimum distance he can travel to accomplish this task? (Hint: the result is an integer number of miles.)

2) (8 pts) Find the ordered pair (x,y) that satisfy the pair of equations shown below:

$$\log_2 x^3 + \log_4 y^2 = 6$$

$$\log_4 x^4 + \log_2 y^6 = 20$$

3) (12 pts) Recall that $\frac{d}{dx}\sin(x) = \cos(x)$ and $\frac{d}{dx}\cos(x) = -\sin(x)$. We denote the n^{th} derivative of a function as $\frac{d^n y}{dx^n}$. Let $y = \sin(2x)$. Conjecture a guess for the function $\frac{d^{4n}y}{dx^{4n}}$, using the function y given, for all non-negative integers n . (Thus, your formula will be a formula for the 0^{th} , 4^{th} , 8^{th} , 12^{th} , etc. derivatives of $\sin(2x)$.) Prove your guess via mathematical induction on n .

4) (15 pts) A number of electronic coins have recently gained value. In particular, a bytecoin is worth 10 cents, a megacoin is worth one dollar, an opticoins is worth one dollar and finally a knightcoin is also worth one dollar. How many different combinations of bytecoins, megacoins, opticoins and knightcoins are worth exactly 20 dollars? Please leave your answer in powers, combinations, factorials, etc. and carefully explain what each expression in your answer and work represent.

5) (10 pts) What is the probability that a randomly chosen positive divisor of 15^{79} is an integer multiple of 45^{30} ? Please express your answer as a fraction in lowest terms.

6) (10 pts) Let $f(x) = \frac{3x-2}{x+5}$, with a domain of all reals except $x = -5$. Determine $f^{-1}(x)$ as well as the domain and range of $f^{-1}(x)$.

7) (12 pts) Define a relation R over the set of rational numbers as follows: $R = \{ (p, q) \mid p - q \text{ has a denominator less than } 200 \text{ when reduced to lowest terms} \}$. Determine, with proof, whether or not R is (a) reflexive, (b) irreflexive, (c) symmetric, (d) anti-symmetric and (e) transitive.

Scratch Page - Please carefully mark any work on this page you would like graded.