COT 3100 Fall 2017 Homework #1 Solutions

1) Write out a truth table for the following logical expressions:

a) $[p \wedge (p \to q)] \to q$ b) $[(p \to q) \wedge (q \to r)] \to (p \to r)$

c) $p \wedge \neg(q \wedge (\neg p \vee r))$

Solution

a)

p	q	p→q	p ∧ (p→q)	[p ∧ (p→q)] →q
0	0	1	0	1
0	1	1	0	1
1	0	0	0	1
1	1	1	1	1

b)

p	q	r	p→q	q→r	[(p→q) ∧ (q→r)]	p→r	[(p→q) ∧ (q→r)] →(p→r)
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	0	0	1	1
0	1	1	1	1	1	1	1
1	0	0	0	1	0	0	1
1	0	1	0	1	0	1	1
1	1	0	1	0	0	0	1
1	1	1	1	1	1	1	1

c)

p	q	r	¬p∨r	q∧(¬p∨r)	¬(q∧(¬p∨r))	p ∧ ¬(q∧(¬p∨r))
0	0	0	1	0	1	0
0	0	1	1	0	1	0
0	1	0	1	1	0	0
0	1	1	1	1	0	0
1	0	0	0	0	1	1
1	0	1	1	0	1	1
1	1	0	0	0	1	1
1	1	1	1	1	0	0

2) Determine all truth value assignments for the primitive statements p, q, r, s, t that make the following expression false: $[(p \wedge q) \wedge r] \rightarrow (s \vee t)$.

Solution

In order for an implication to be false, the first part must be true while the second false. This means that $p \wedge q \wedge r$ is true while $s \vee t$ is false. The first clause could only be true if all three of p, q, and r are true, while the second clause can only be false if both s and t are false. So, there is one truth assignment that makes the entire statement false. It's

p=q=r=true, s=t=false

3) Negate the following boolean expression and simplify the result as much as possible. Please show each step and name the rule you are using at each step:

$$p \vee q \vee (\neg p \wedge \neg q \wedge r)$$

Solution

$\neg[p \vee q \vee (\neg p \wedge \neg q \wedge r)]$	$\Leftrightarrow$
$\neg(p \vee q) \wedge \neg(\neg p \wedge \neg q \wedge r)$	$\Leftrightarrow$ (DeMorgan's)
$\neg(p \vee q) \wedge \neg(\neg(p \vee q) \wedge r)$	$\Leftrightarrow$ (DeMorgan's)
$\neg(p \vee q) \wedge (\neg\neg(p \vee q) \vee \neg r)$	$\Leftrightarrow$ (DeMorgan's)
$\neg(p \vee q) \wedge ((p \vee q) \vee \neg r)$	$\Leftrightarrow$ (Double Negation)
$[\neg(p \vee q) \wedge (p \vee q)] \vee [\neg(p \vee q) \wedge \neg r)]$	$\Leftrightarrow$ (Distributive)
$F \vee [\neg(p \vee q) \wedge \neg r)]$	$\Leftrightarrow$ (Inverse)
$[\neg(p \vee q) \wedge \neg r)]$	$\Leftrightarrow$ (Identity)
$\neg p \wedge \neg q \wedge \neg r$	$\Leftrightarrow$ (DeMorgan's)

4) Show that the following two logical expressions are equivalent using the laws of logic:

$$(p \to q) \wedge [\neg q \wedge (r \vee \neg q)] \quad \text{and} \quad \neg(q \vee p)$$

Solution

$(p \to q) \wedge [\neg q \wedge (r \vee \neg q)]$ $\Leftrightarrow$
$(\neg p \vee q) \wedge [\neg q \wedge (r \vee \neg q)]$ $\Leftrightarrow$ (Implication Identity)
$(\neg p \vee q) \wedge [(\neg q \wedge r) \vee (\neg q \wedge \neg q)]$ $\Leftrightarrow$ (Distributive)
$(\neg p \vee q) \wedge [(\neg q \wedge r) \vee \neg q]$ $\Leftrightarrow$ (Idempotent)
$(\neg p \vee q) \wedge [\neg q \vee (\neg q \wedge r)]$ $\Leftrightarrow$ (Commutative)
$(\neg p \vee q) \wedge \neg q$ $\Leftrightarrow$ (Absorption)
$\neg q \wedge (\neg p \vee q)$ $\Leftrightarrow$ (Commutative)
$(\neg q \wedge \neg p) \vee (\neg q \wedge q)$ $\Leftrightarrow$ (Distributive)
$(\neg q \wedge \neg p) \vee F$ $\Leftrightarrow$ (Inverse)
$\neg q \wedge \neg p$ $\Leftrightarrow$ (Identity)
$\neg(q \vee p)$ (Demorgan's)

5) Prove the following logical argument using the rules of implication. Please show each step and state which rule you use.

$(\neg p \vee q) \to r$
$r \to (s \vee t)$
$\neg s \wedge \neg u$
$\neg u \to \neg t$

$\therefore\ p$

Solution

1) $\neg s \wedge \neg u$	Premise
2) $\neg u$	Rule of Conjunctive Simplification (Step 1)
3) $\neg u \to \neg t$	Premise
4) $\neg t$	Modus Ponens (Steps 2 and 3)
5) $\neg s$	Rule of Conjunctive Simplification (Step 4)
6) $\neg s \wedge \neg t$	Rule of Conjunction (Steps 4 and 5)
7) $r \to (s \vee t)$	Premise
8) $\neg(s \vee t) \to \neg r$	Contrapositive (Step 7)
9) $(\neg s \wedge \neg t) \to \neg r$	De Morgan's Law (Step 8)
10) $\neg r$	Modus Ponens (Steps 6 and 9)
11) $(\neg p \vee q) \to r$	Premise
12) $\neg r \to \neg(\neg p \vee q)$	Contrapositive (Step 11)
13) $\neg r \to (p \wedge \neg q)$	De Morgan's Law and Double Negation (Step 12)
14) $p \wedge \neg q$	Modus Ponens (Steps 10 and 13)
15) $\therefore\ p$	Rule of Conjunctive Simplification

6) Create simple statements for p, q, r, s, t and u for problem number 5 above that make reasonable sense in real life.

Solution
Here is a set of assignments for the variables:

p = "my friends are in town"	s = "I will watch TV"
q = "I am out of cash"	t = "I will play guitar hero"
r = "I will stay home"	u = "I did my homework"

Given these assignments, the argument goes as follows: If my friends are out of town or I am out of cash, then I will stay home. If I stay home, then either I will watch TV or play guitar hero. I didn't watch TV and I didn't do my homework. If I don't do my homework, then I don't play guitar hero. Thus, my friends are in town.

7) Let ? be an unknown boolean logical operator. The logical statement $[(\neg p \wedge q) \vee r] \Rightarrow (q \; ? \; r)$ is equivalent to $(p \vee \neg q \vee r)$. Given this information, there are 2 possible truth tables for the boolean logical operator ?. List, with proof, both of these truth tables.

Solution

p	q	r	$(\neg p \wedge q) \vee r$	(q ? r)	$[(\neg p \wedge q) \vee r] \Rightarrow$ (q ? r)	$(p \vee \neg q \vee r)$
0	0	0	0	?	1	1
0	0	1	1	1	1	1
0	1	0	1	0	0	0
0	1	1	1	1	1	1
1	0	0	0	?	1	1
1	0	1	1	1	1	1
1	1	0	0	?	1	1
1	1	1	1	1	1	1

Here we see that based on the values of $(p \wedge q) \vee r$ and $[(p \wedge q) \vee r] \Rightarrow$ (q ? r), we can definitively fill out five slots for our unknown column. These 5 slots specify the operation (q ? r) for all possibilities where at least one of the two values is 1. (Note, that for the last question mark, we can actually determine that the value is 0, based upon the third row of the truth table.) Thus, the only ambiguity possible for the operator is that 0?0=0 OR 0?0=1. Thus, we have the two following truth tables for ?:

?	r=0	r=1
q=0	0	1
q=1	0	1

?	r=0	r=1
q=0	1	1
q=1	0	1

(Note the first table is equivalent to r, and the second is equivalent to $r \vee \neg q$.)