Fall 2016 COT 3100 Section 1 Homework 4
Assigned: 10/4/2016
Due: 10/14/2016

Note: For all of these questions, please solve without a calculator or computer. Use the methods shown in class to solve these problems manually. You may double check basic arithmetic with a calculator.

1) What is the sum of an arithmetic sequence with 100 terms with the first term equal to 17 and a common difference of 4?

2) Let $a_1, a_2, \ldots, a_{65}$ be an arithmetic sequence such that $a_{22} = 46$ and $a_{35} = 267$. Find $\sum_{i=1}^{65} a_i$.

3) What is the sum of an infinite geometric sequence with a first term of 7 and a common ratio of $\frac{2}{5}$?

4) Consider a geometric sequence $a_1, a_2, \ldots, a_{15}$ such that $a_5 = 48$ and $a_9 = 768$. Find $\sum_{i=1}^{15} a_i$.

5) Determine the following sum in terms of n: $\sum_{i=1}^{n}(i(i+1)(i+2))$.

6) By noticing that $\sum_{i=1}^{n}(2i+1) = \sum_{i=1}^{n}((i+1)^2 - i^2) = \sum_{i=1}^{n}(i+1)^2 - \sum_{i=1}^{n} i^2$, and noticing the telescoping nature of the sum on the right, determine $\sum_{i=1}^{n}(2i+1)$.

7) What is $\sum_{i=1}^{\propto} i(\frac{3}{5})^{i-1}$?

8) What is $\begin{bmatrix} 1 & 3 & 4 \\ 6 & 2 & -3 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 1 \\ 4 & -3 \end{bmatrix}$?

9) Let $M_1 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, $M_2 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, and $M_3 = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}$. Calculate the (a) join of M_1 and M_2, (b) meet of M_1 and M_2, and (c) the Boolean product of M_1 and M_3.

10) Express the system of equations below as a matrix multiplication:

$$3x + 4y + 5z = 16$$
$$2x - 5y + 11z = 3$$
$$x + 6y + 2z = 15$$

(Note: no need to solve the system. If you want to, you may, however.)

11) Using mathematical induction, prove that $\sum_{i=1}^{n}\left(i(i!)\right) = (n+1)! - 1$, for all positive integers n.

12) Note that the n^{th} Harmonic number, denoted H_n, equals $\sum_{i=1}^{n}\frac{1}{i}$. Using mathematical induction, prove that $\sum_{i=1}^{n} H_i = (n+1)H_n - n$, for all positive integers n.

13) Using mathematical induction, prove that $21 \mid (4^{n+1} + 5^{2n-1})$ for all non-negative integers n.

14) Using mathematical induction, prove that $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ for all positive integers n.

15) Using mathematical induction, prove that $\sum_{i=1}^{n} i^2 < n^3$, for all positive integers $n \geq 2$. (Please do this proof as written instead of proving the equality and then arguing that the real formula is less than n^3. My goal here is to have you practice the mechanics of using induction on an inequality.)