COP 3530: Computer Science III
Summer 2005

Ford-Fulkerson Example For Labs

Instructor : Mark Llewellyn
markl@cs.ucf.edu
CSB 242, 823-2790

http://www.cs.ucf.edu/courses/cop3530/summer05

School of Computer Science
University of Central Florida

Ford-Fulkerson Algorithm - Example

Initialize all edges of the flow graph with zero flow
Loop while ∃ a path in ***Gr*** from ***s*** to ***t***
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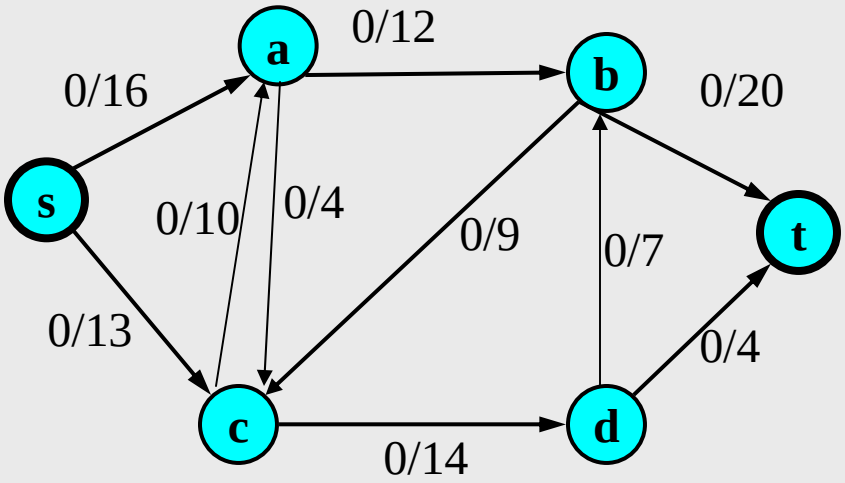
end Loop

Use an arbitrary path in this example – not necessarily a greedy implementation

G_f



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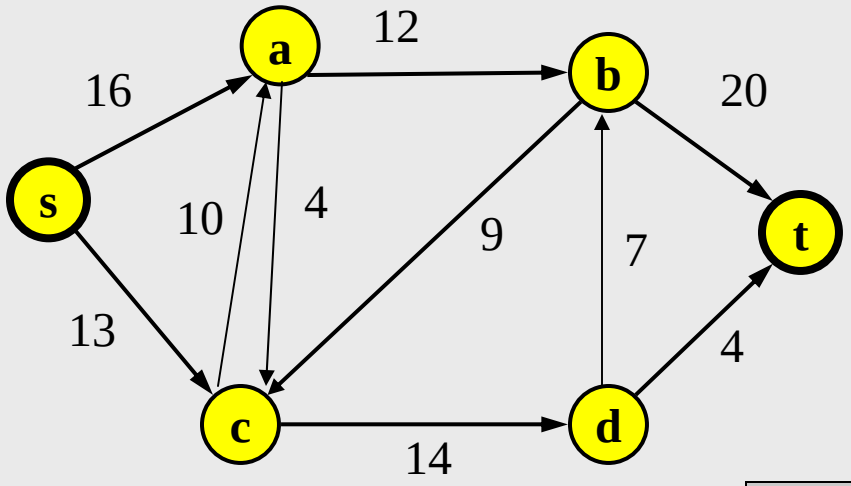

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Minimum residual capacity = 4

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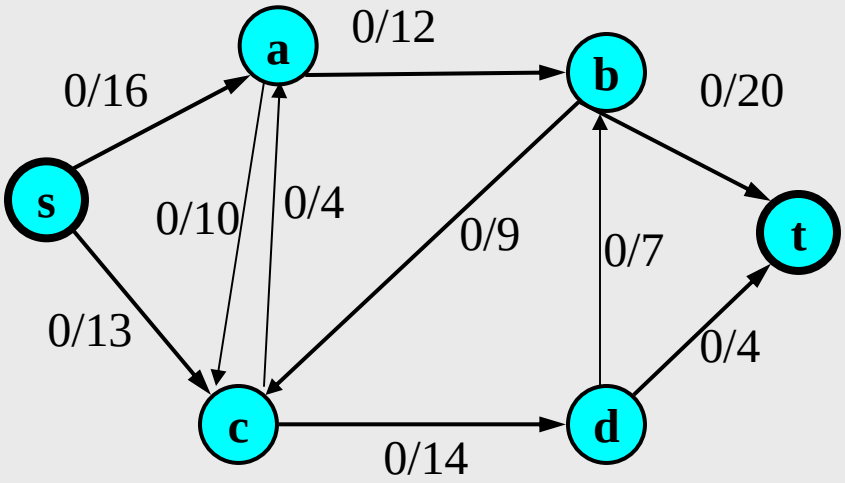

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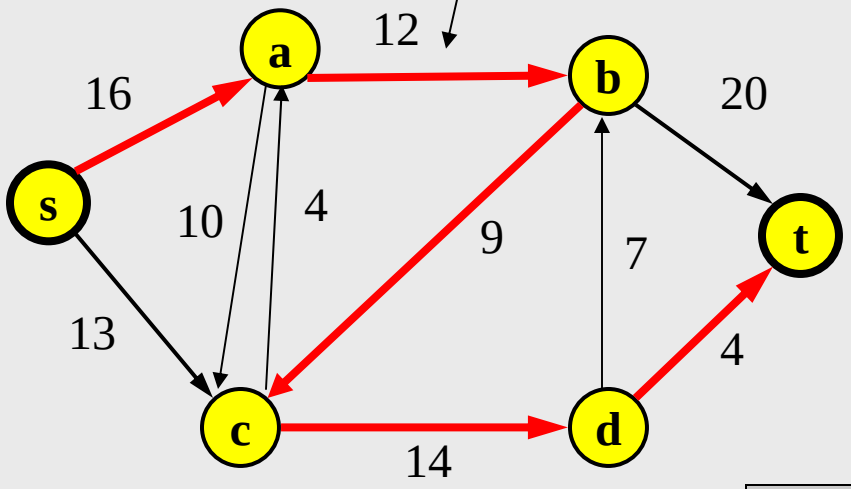

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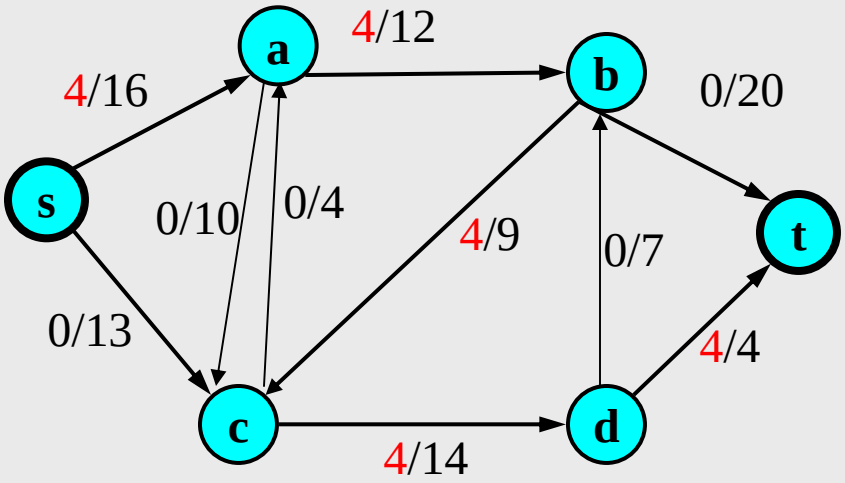

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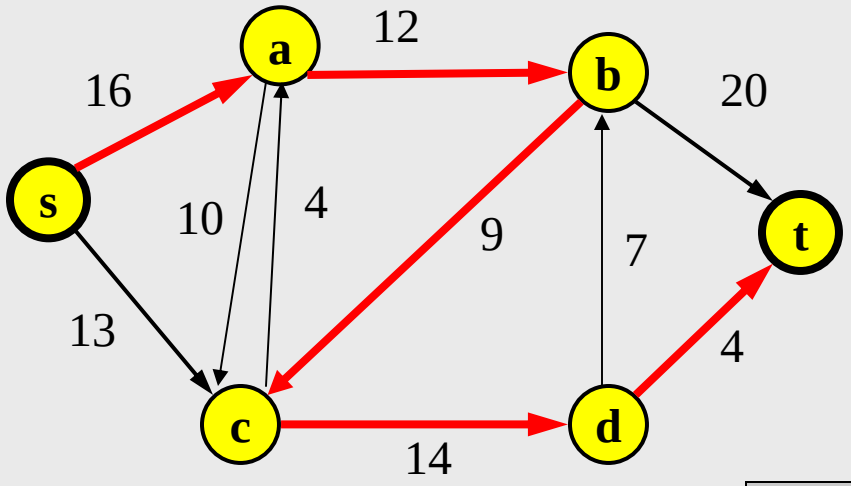

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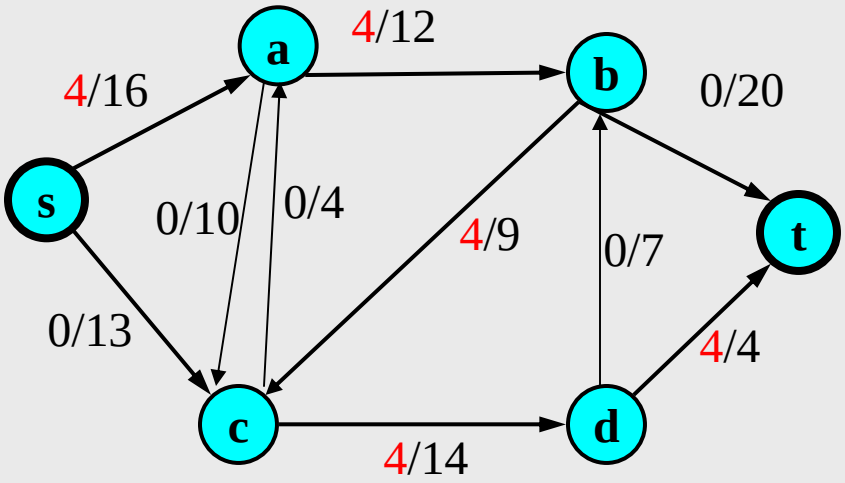

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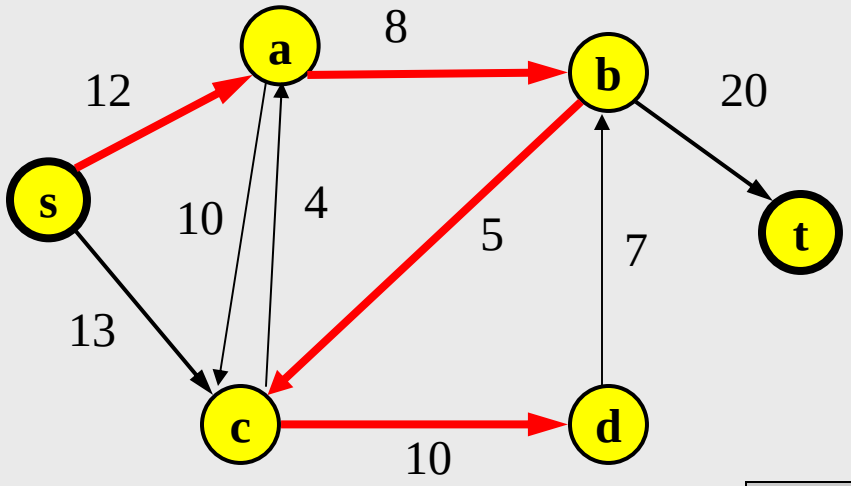

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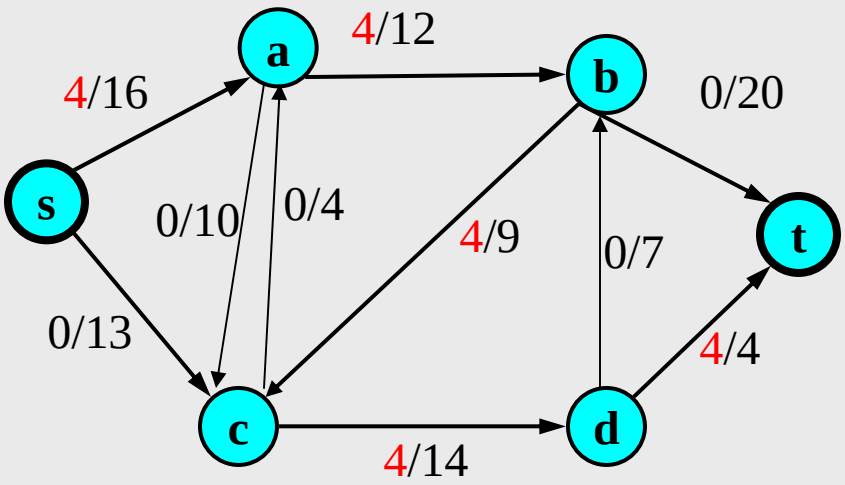

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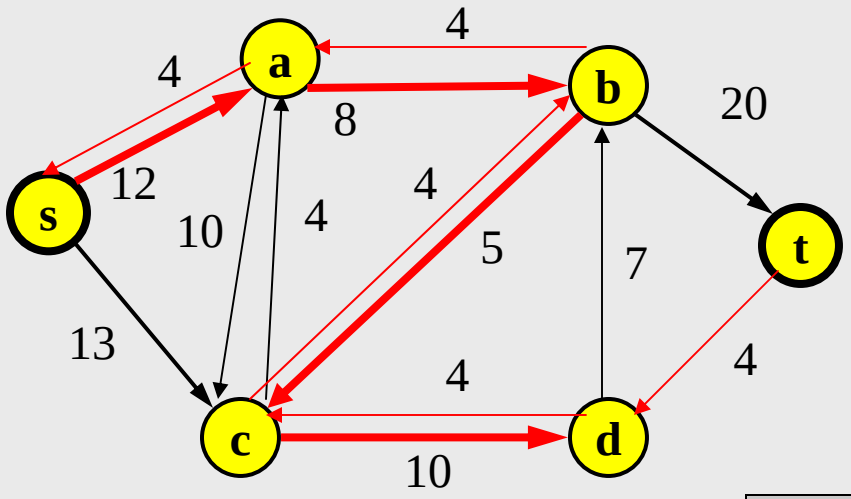

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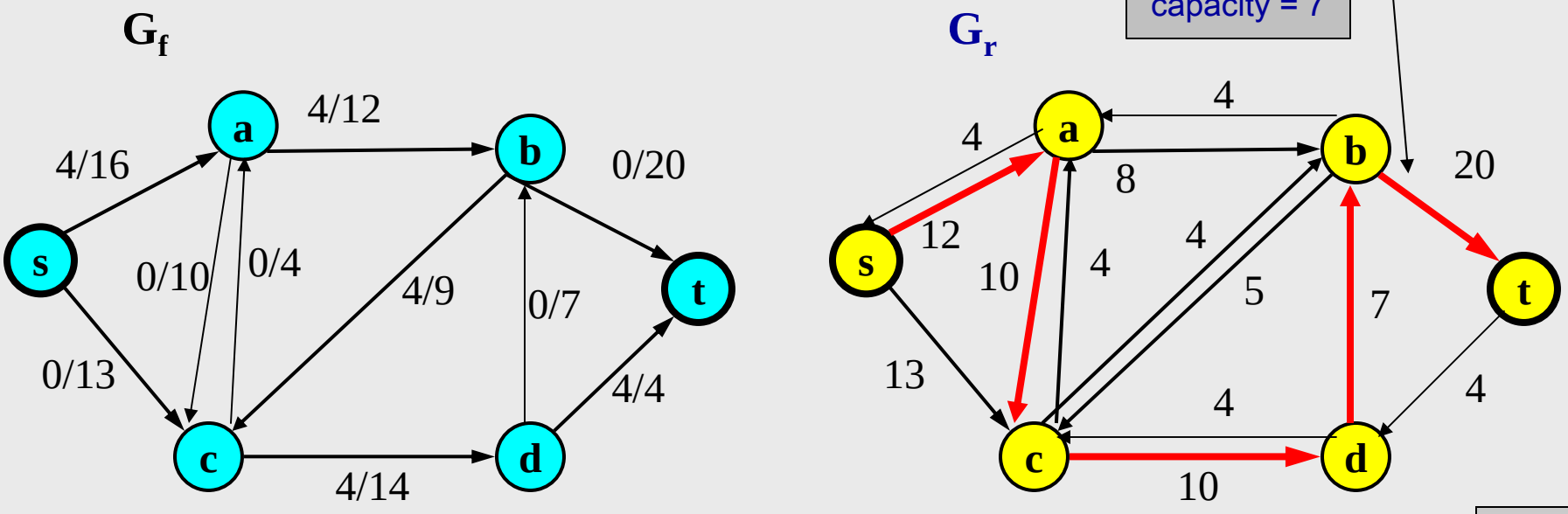
Minimum residual capacity = 7



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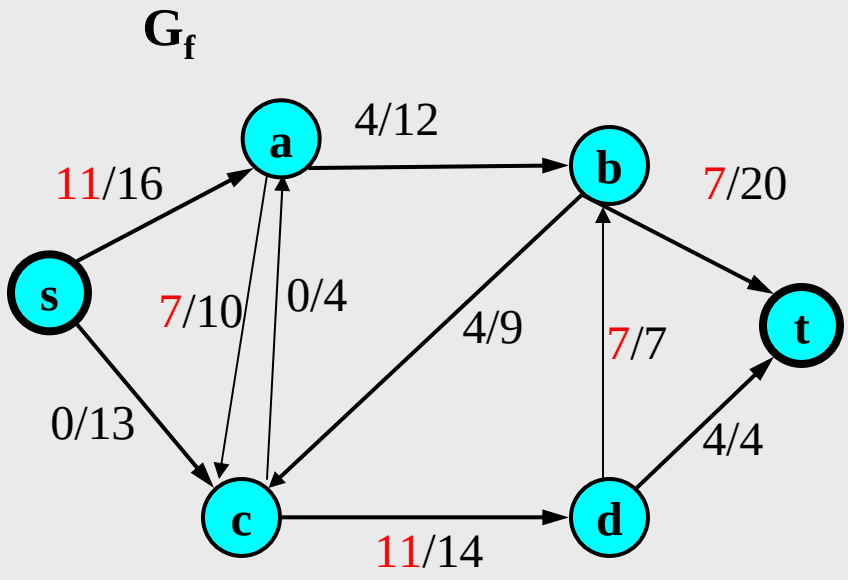

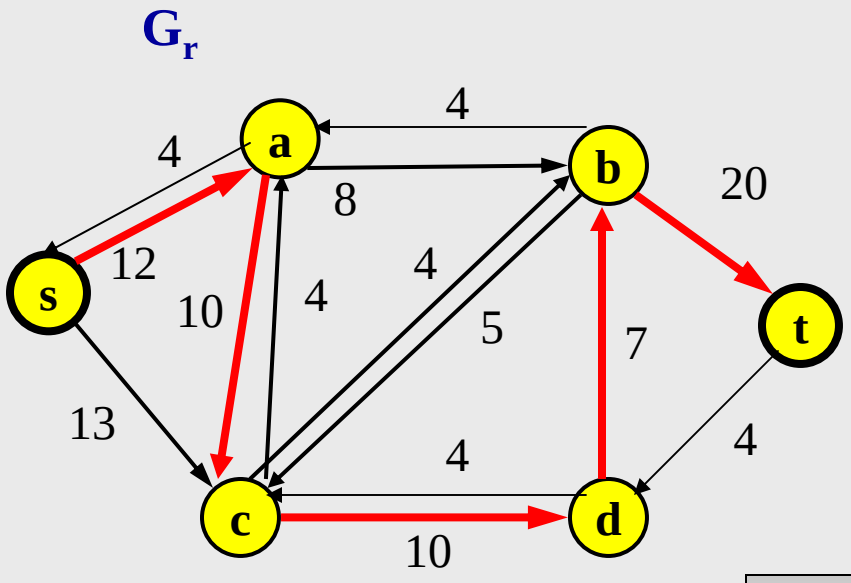

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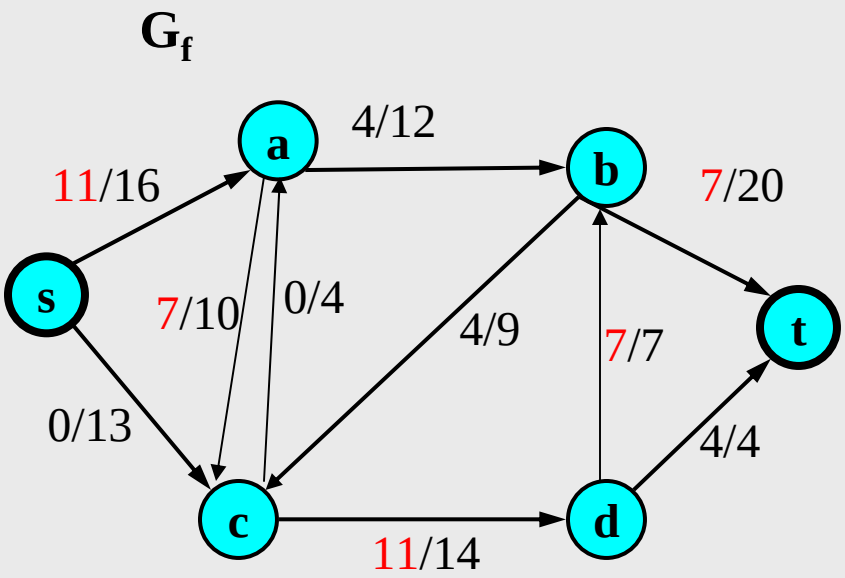

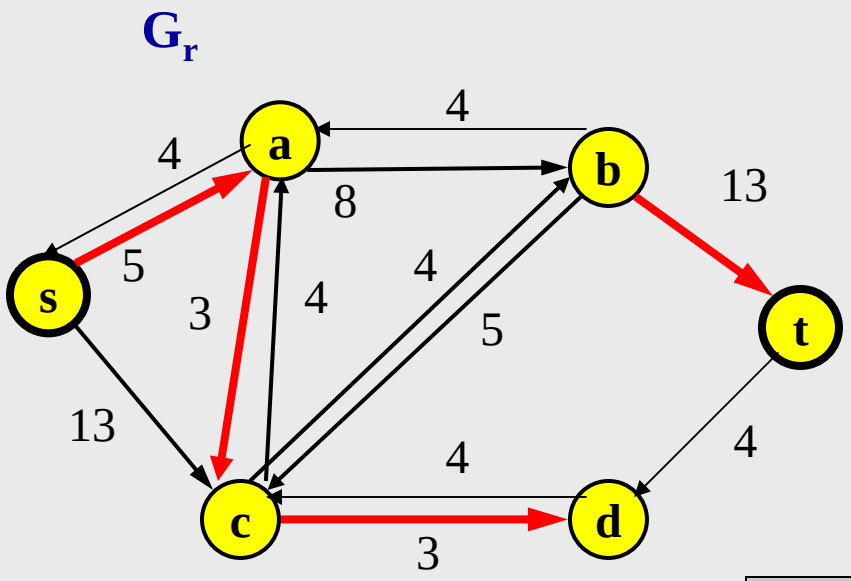

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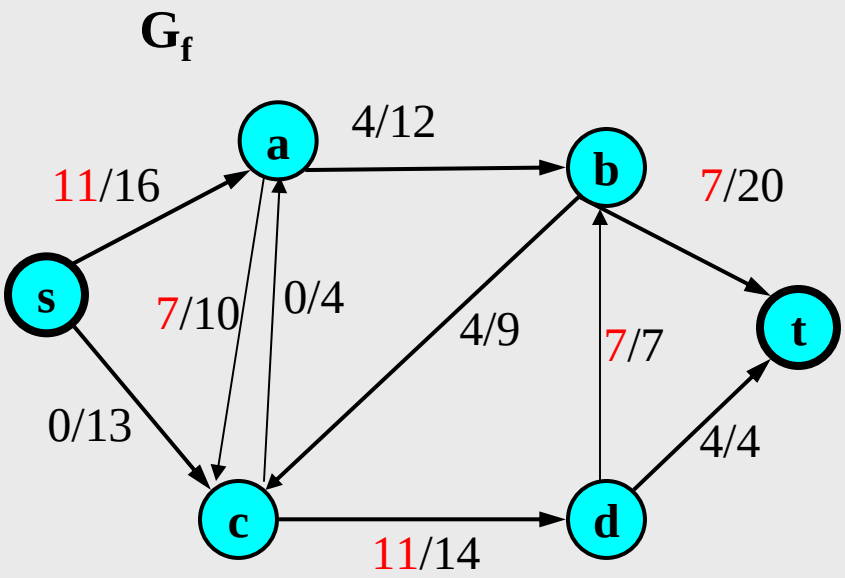

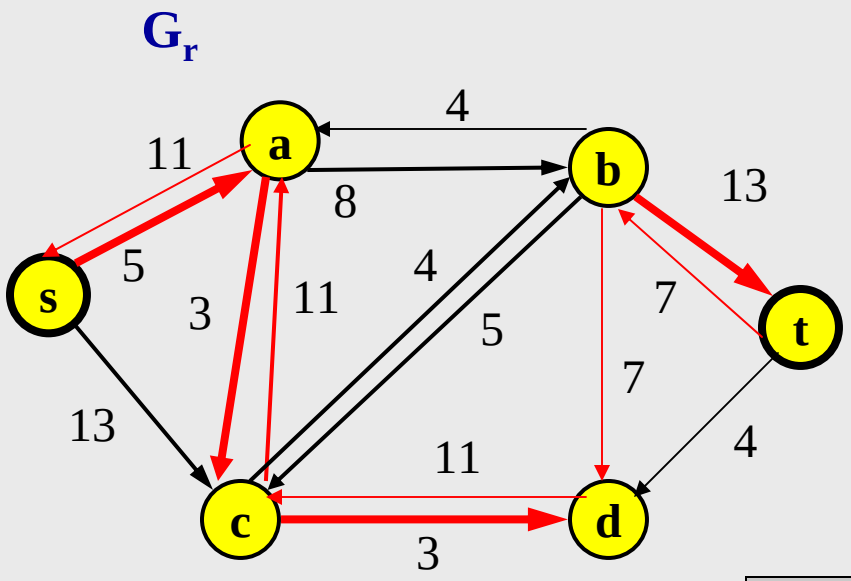

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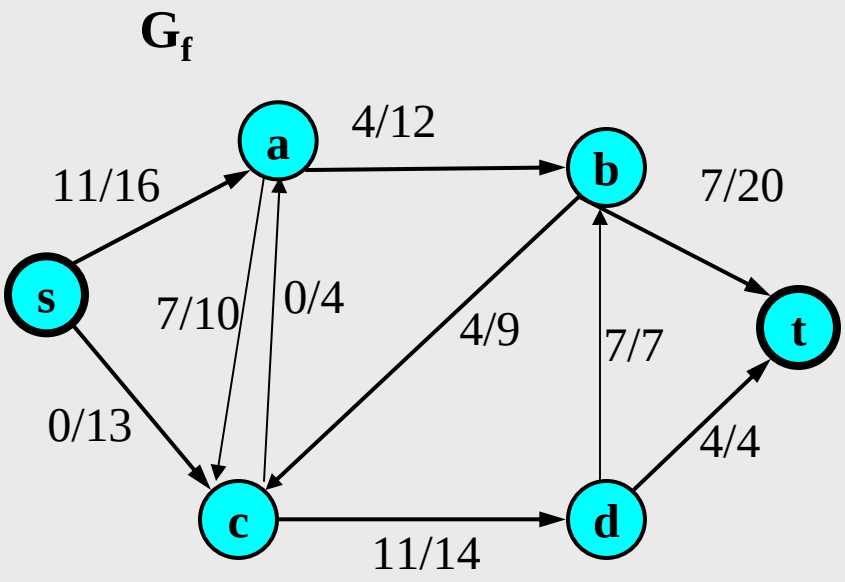
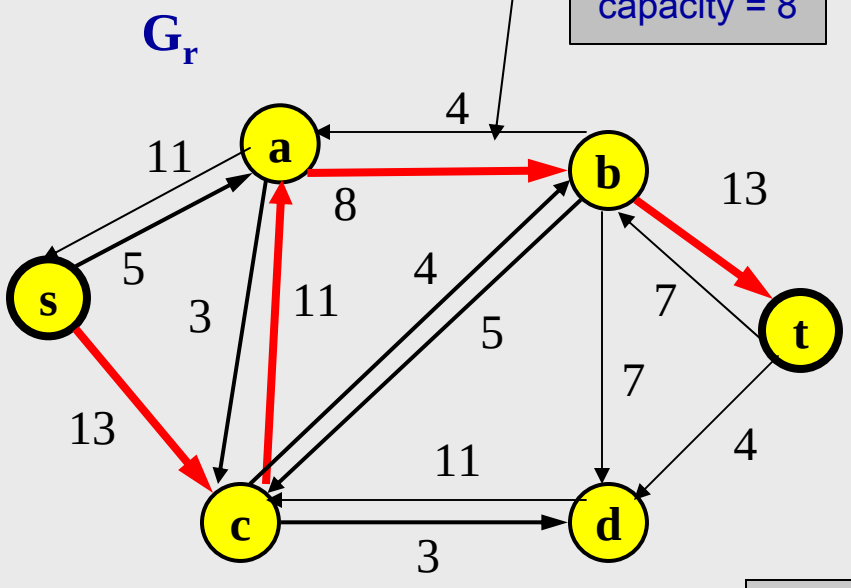
Minimum residual capacity = 8





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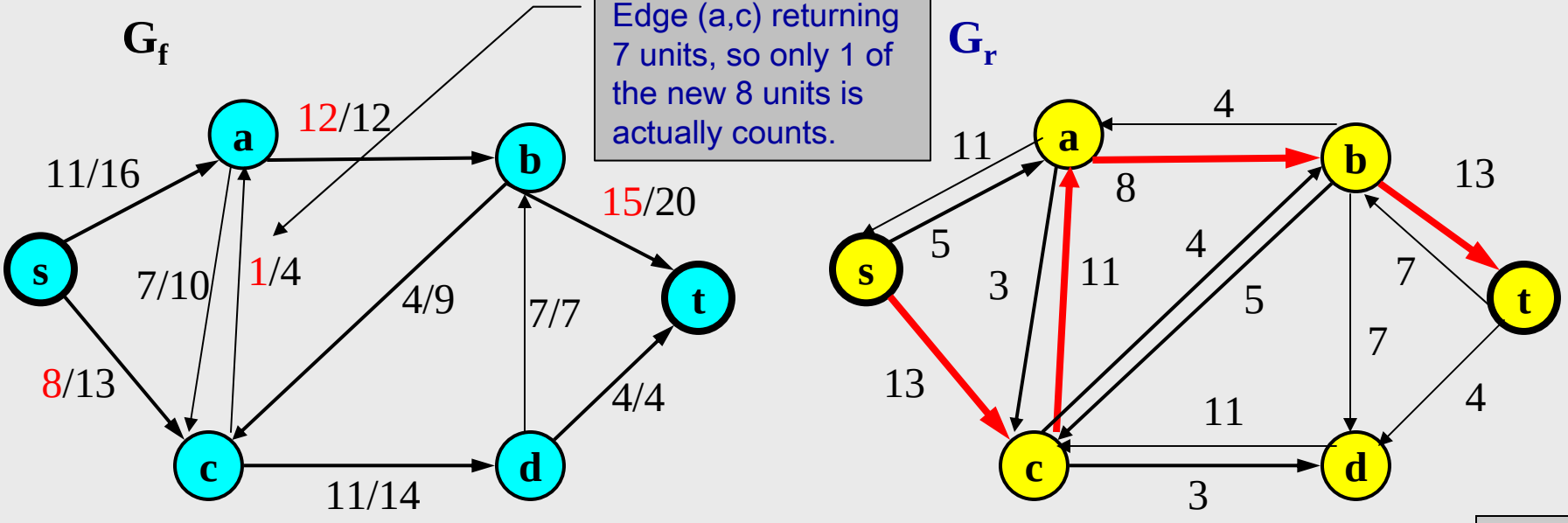
Edge (a,c) returning 7 units, so only 1 of the new 8 units is actually counts.



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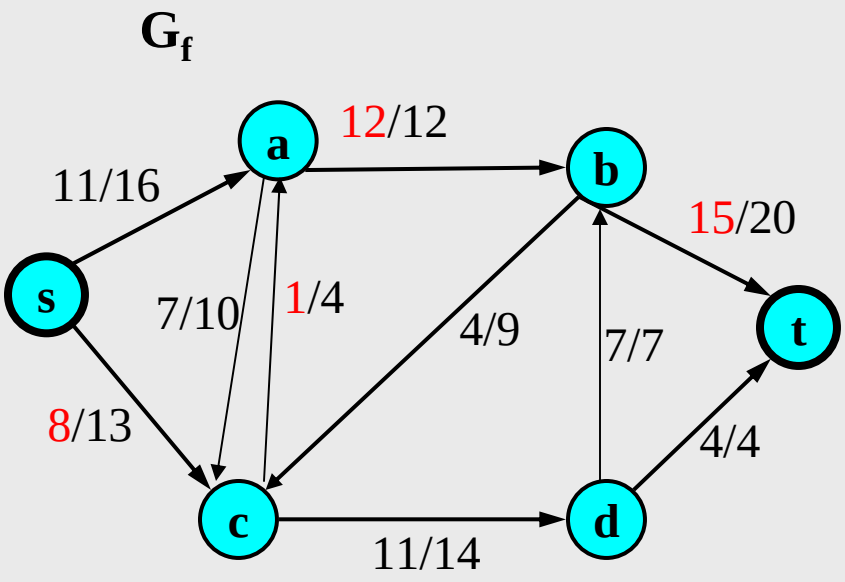

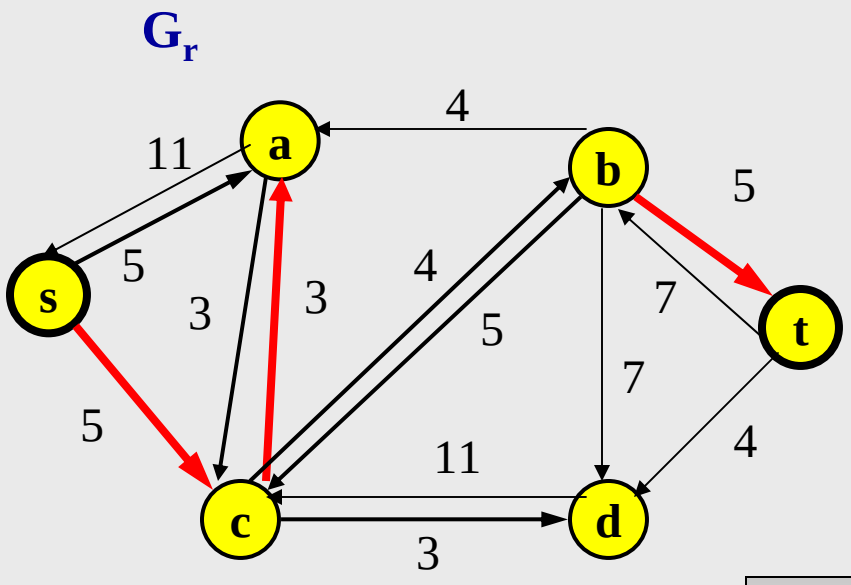

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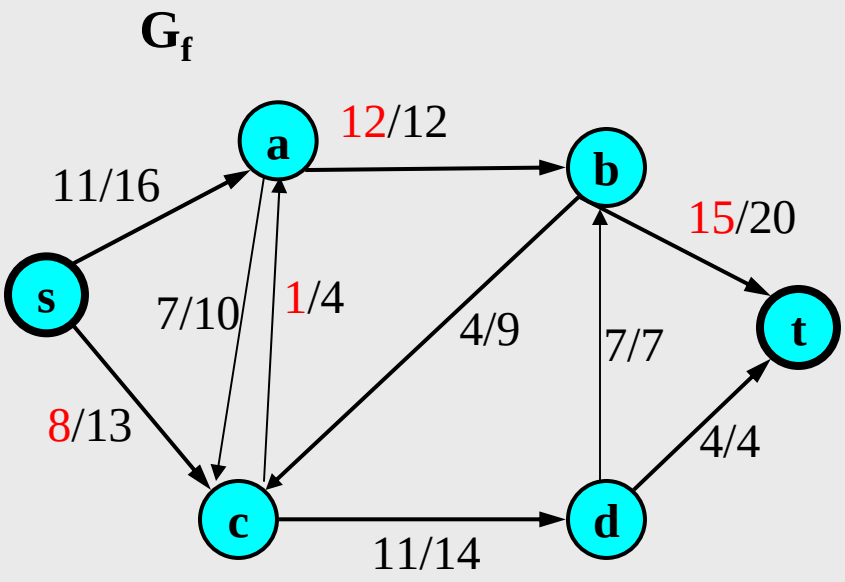

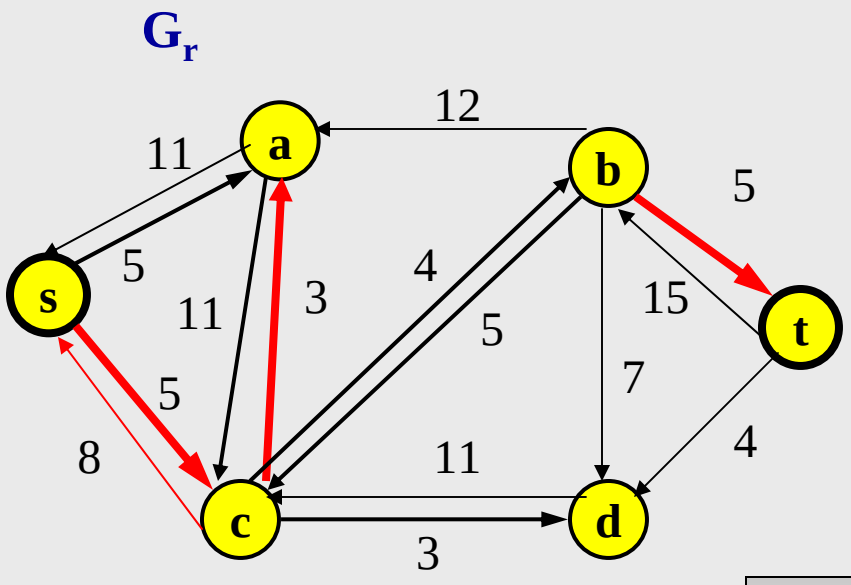

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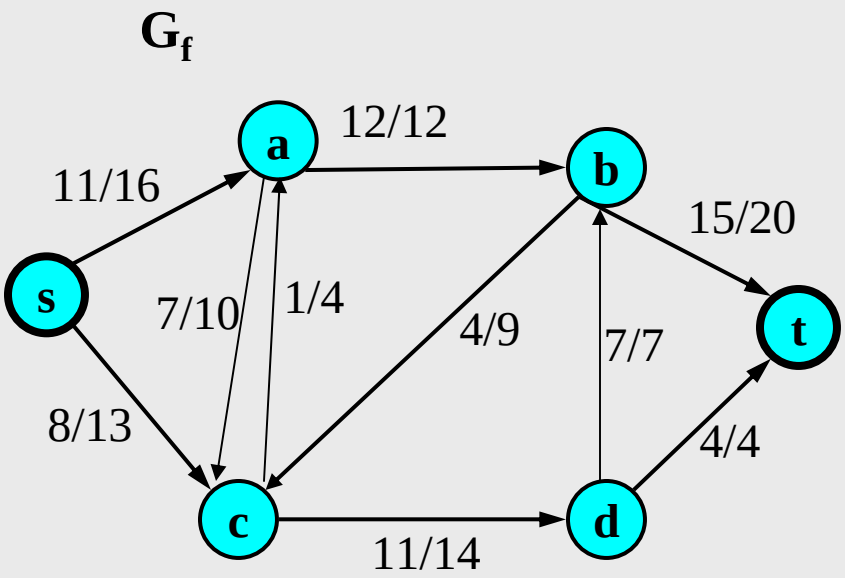
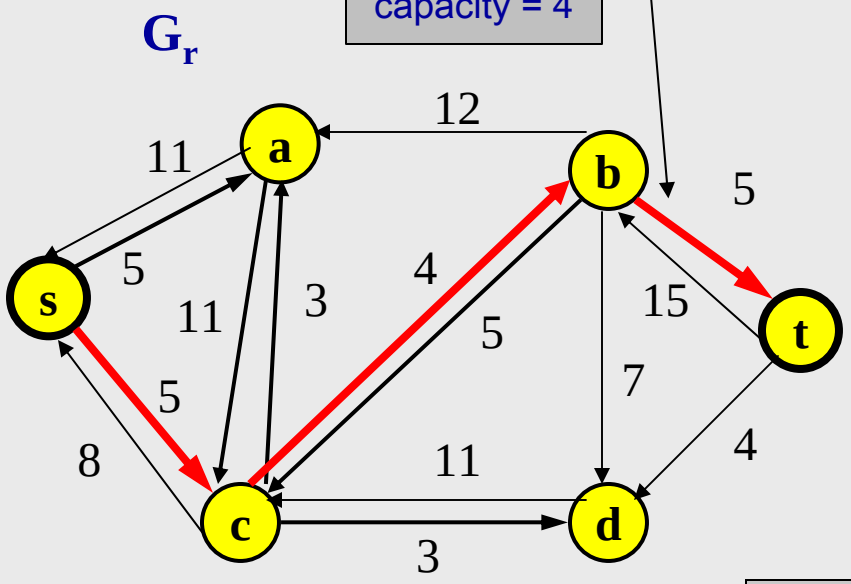
Minimum residual capacity = 4





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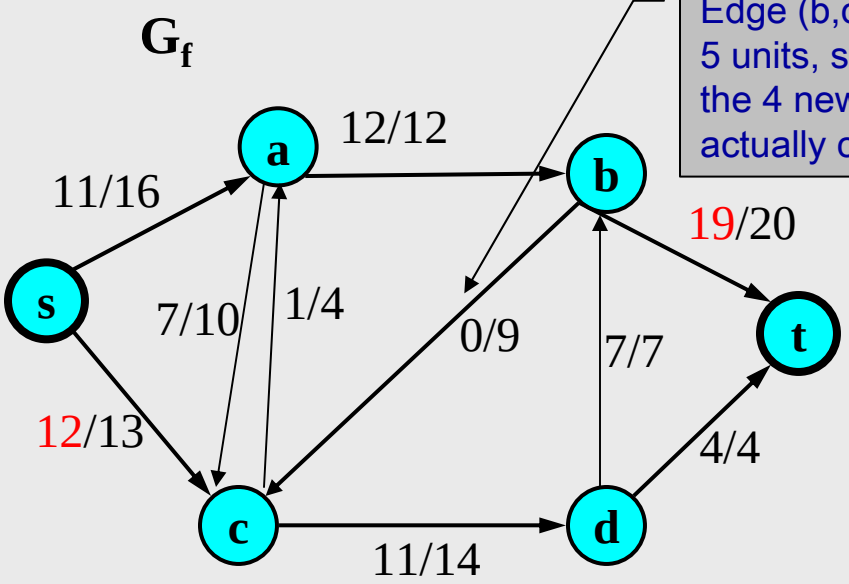
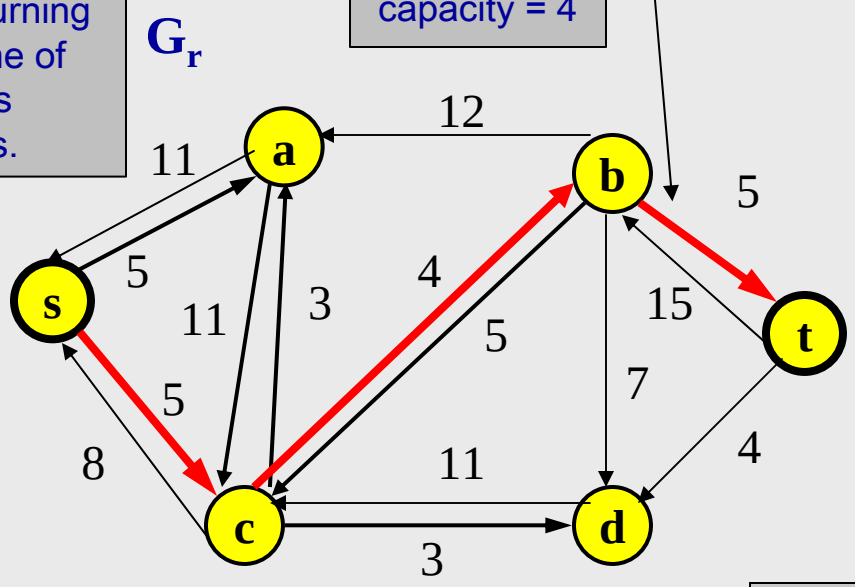
Edge (b,c) returning 5 units, so none of the 4 new units actually counts.





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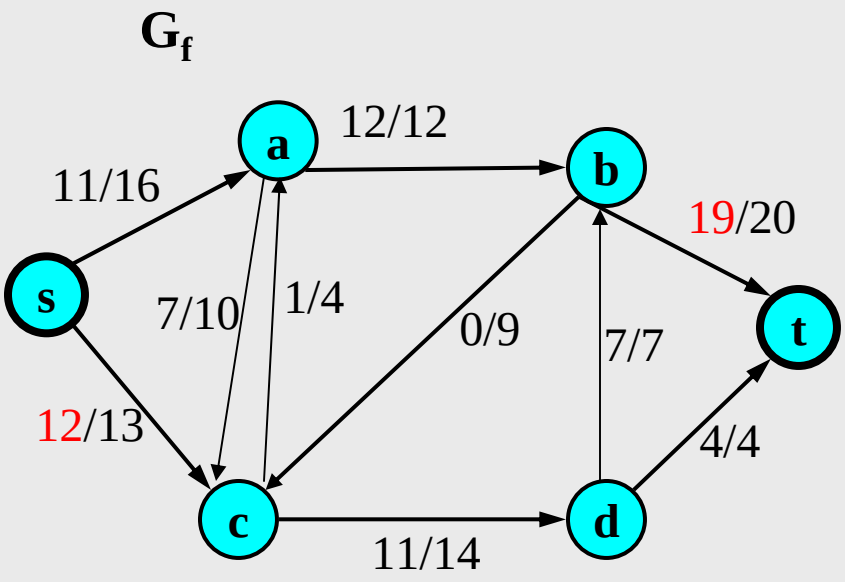

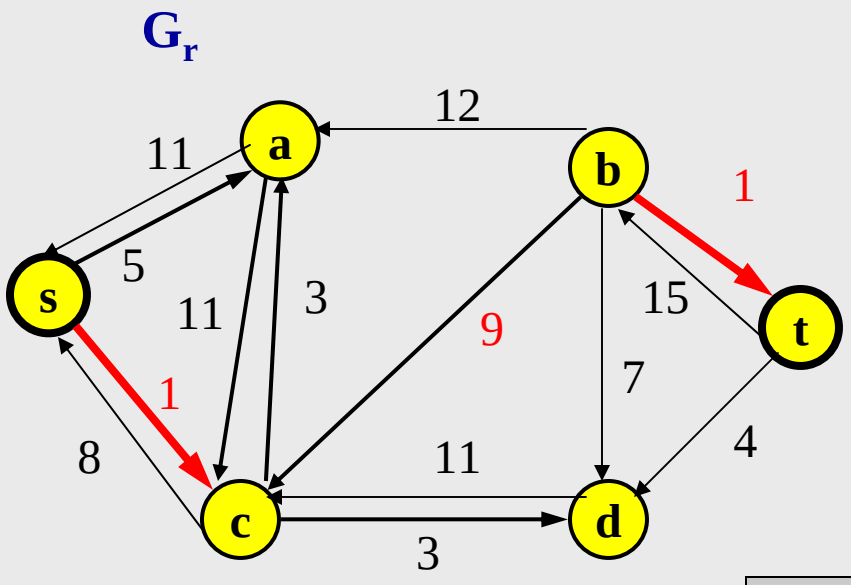

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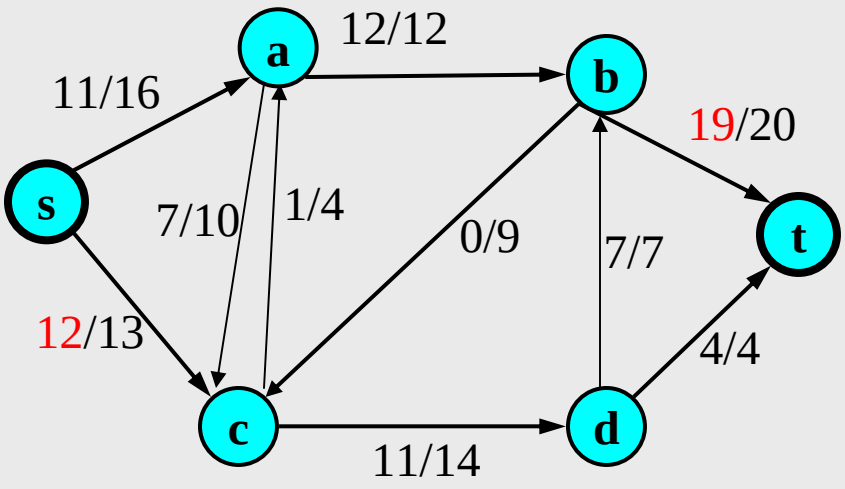

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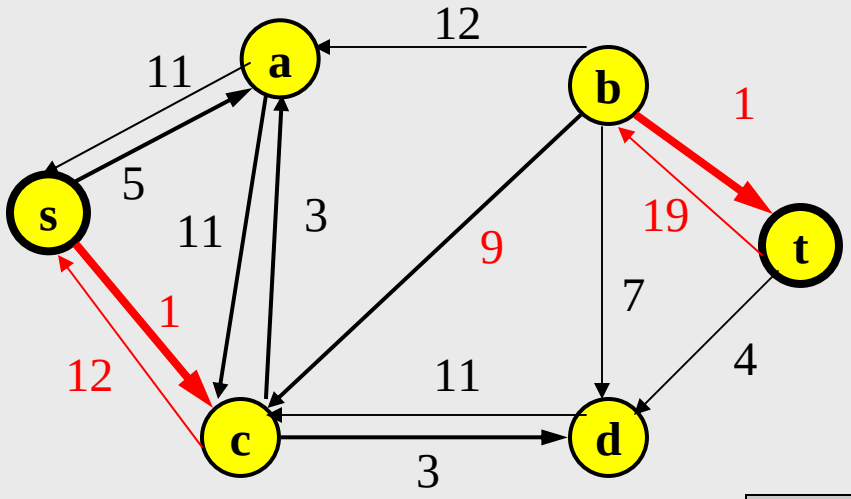

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Algorithm terminates as there are no more paths from source to sink in the residual graph.

Final graphs are shown below.