COP 3503 4/4/24

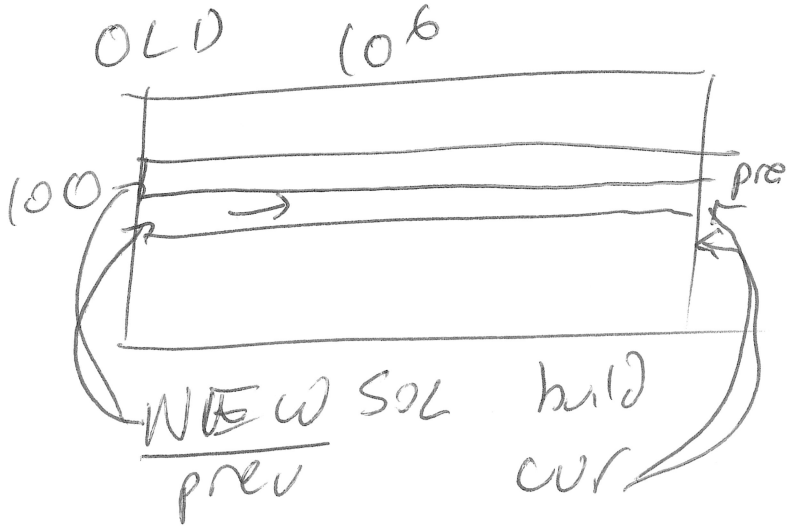
(0) Collect Com. Serv Forms
(1) Look @ Schedule
(2) Look @ Grid Path, Coins Code from last lec
(3) Today's DP: fewest coins, 0-1 knapsack, world series problem, if time live code random DP.

ways make change fix

$$dp[i][j] = dp[i-1][j] + dp[i][j - den[i]]$$

Only ever look @ my current row or previous row

Store only the prev row + cur row



mem $10^2 \times 10^6 = 10^8$

Space Saving DP

new mem $2 \times 10^6 =$ 2,020,000
prev = cur

Grid Problem w/o blocked sq LS Pascal's Trig

1	1	1	1	1	1
1	2	3	4	5	6
1	3	6	10	15	21
1	4	10	20	35	56
1	5	15	35	70	126

Fewest Coins

Input: Same as # ways DP

Denom = 1, 2, 5, 6, ~~#~~ ~~10~~ 9

n = 17

What is fewest # coins need to make change for n cents

$f(n) = \min(1+f(n-d[0]), 1+f(n-d[1]), 1+f(n-d[2])\ldots)$

$f(17) = \min(1+f(16), 1+f(15), 1+f(12), 1+f(11), 1+f(8))$

```
int[] dp = new int[n+1];
dp[0] = 0;
for (int i = 1; i <= n; i++) {
    dp[i] = MAX;
    for (j = 0; j < numden; j++) {
        if (den[j] > i) continue;
        dp[i] = Math.min(dp[i], 1 + dp[i - den[j]])
}   }
```

0-1 Knapsack

List of Items

I_1 $w=3$ $v=10$

I_2 $w=8$ $v=4$

I_3 $w=9$ $v=9$

I_4 $w=8$ $v=11$

$W=15$

most value I can get w/ total weight of 15 or less

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Item	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Item 1	0	0	0	10	10	10	10	10	10	10	10	10	10	10	10	10
Item 2	0	0	0	10	10	10	10	10	10	10	10	14	14	14	14	14
Item 3	0	0	0	10	10	10	10	10	10	10	10	14	19	19	19	19
Item 4	0	0	0	10	10	10	10	10	11	11	11	21	21	21	21	21

Is it better for me not to take the item OR to take it

$$\text{new } f(15) = \max\big(\underbrace{f(15)}_{\text{old}}, \underbrace{f(15-3)}_{\text{old}} + \underbrace{10}_{\text{item 1 value}}\big)$$

(15−3 ↑ weight)

If I take Item 1, then I only have $15-3=12$ free pounds in my knapsack but I gain value 10

Outer loop ⇒ goes through items

Inner loop ⇒ backwards to avoid duplicates

If infinite copies of each item, inner loop ⇒ forwards!

Example 2:

I_0 $w=4$, $v=6$

I_1 $w=2$, $v=4$

I_2 $w=3$ $v=5$

I_3 $w=1$ $v=3$

I_4 $w=6$ $v=9$

I_5 $w=4$ $v=7$

$W=10$

Item	0	1	2	3	4	5	6	7	8	9	10
	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	6	6	6	6	6	6	6
1	0	0	4	4	6	6	10	10	10	10	10
2	0	0	4	5	6	9	10	11	11	15	15
3	0	3	4	7	8	9	12	13	14	15	18
4	0	3	4	7	8	9	12	13	14	16	18
5	0	3	4	7	8	10	12	14	15	16	19

World Series Problem

teams play n games

team A has probability p of winning an individual game. (team B has prob $1-p$ of winning an ind. game)

Independence btw games.

What's the probability team A wins exactly k games?

$$\underbrace{\binom{n}{k}}_{\text{\# of orders of results w/ k wins}} \underbrace{p^k (1-p)^{n-k}}_{\text{prob of a fixed sequence of k wins.}}$$

let $dp[i][j]$ = probability that team A has i wins team B has j wins

$$dp[i][j] = d[i-1][j] * p + d[i][j-1] * (1-p)$$

d: team A won last game

$(1-p)$: team B wins last game

reduce floating pt error

$$\binom{7}{4} .6^4 .4^3$$