COP 3503 3/26/24

(1) Exam 2 - Returned in Recitation FRI
Try Grades Wed Night
Current Grade Formula, Current Grade Cutoffs

(2) Divide + Conquer (Recursion)

- Int Multiplication
- Tromino Tiling
- Skyline

0110 × 1011 $6 \times 11 = 66$

$(01 \times 2^2 + 10)(10 \times 2^2 + 11)$

$(01 \times 10) \times 2^4 + (01 \times 11) \times 2^2 + (10 \times 10) \times 2^2 + (10 \times 11)$

(01×10): Smaller mult; $\times 2^4$: bit shift
(01×11): Smaller mult; $\times 2^2$: bit shift
(10×10): Small mult; $\times 2^2$: bit shift
(10×11): Smaller mult

Let $T(n)$ = run-time a/s 2 n-bit #s

$$T(n) = 4 \times T\left(\frac{n}{2}\right) + O(n)$$

$4 \times T(\frac{n}{2})$: doing 4 rec mult
$O(n)$: adding 4 products

$T(n) = O(n^2)$

$$T(n) = AT\left(\frac{n}{B}\right) + O(n^k)$$

if $B^k < A$, then $O(n^{\log_B A})$

$2^1 < 4 \rightarrow O(n^{\log_2 4}) \rightarrow O(n^2)$

true

If we lower A, the new runtime $O(n^{\log_2 3})$

Real Name: Karatsuba's

$X = X_H \cdot b^{n/2} + X_L$ $\quad X = 0110,\ X_H = 01,\ X_L = 10$

$Y = Y_H \cdot b^{n/2} + Y_L$ $\quad Y = 1011,\ Y_H = 10,\ Y_L = 11$

$$X \cdot Y = (X_H \cdot b^{n/2} + X_L)(Y_H b^{n/2} + Y_L)$$

$$= \underbrace{(X_H Y_H)}_{M_1} \cdot b^n + \underbrace{(X_H Y_L + X_L Y_H)}_{\text{Can we recover this w/1 more mult.}} b^{n/2} + \underbrace{(X_L Y_L)}_{M_2}$$

$$M_3 = (X_H + X_L)(Y_H + Y_L) = \underbrace{X_H Y_H}_{M_1} + X_H Y_L + X_L Y_H + \underbrace{X_L Y_L}_{M_2}$$

$$M_3 - M_1 - M_2 = X_H Y_L + X_L Y_H$$

With these steps, we do 3 Rec calls size $\frac{n}{2}$, $O(n)$ extra work (larger constant)

$$T(n) = 3T(\tfrac{n}{2}) + O(n) \rightarrow O(n^{\log_2 3})$$

0110 1011

$X_H = 01, \quad X_L = 10$

$Y_H = 10, \quad Y_L = 11$

$X_H \times Y_H = 01 \times 10 = 10 \qquad M_1$

$X_L \times Y_L = 10 \times 11 = 110 \qquad M_2$

$X_H + X_L = 01 + 10 = 11$

$Y_H + Y_L = 10 + 11 = 101$

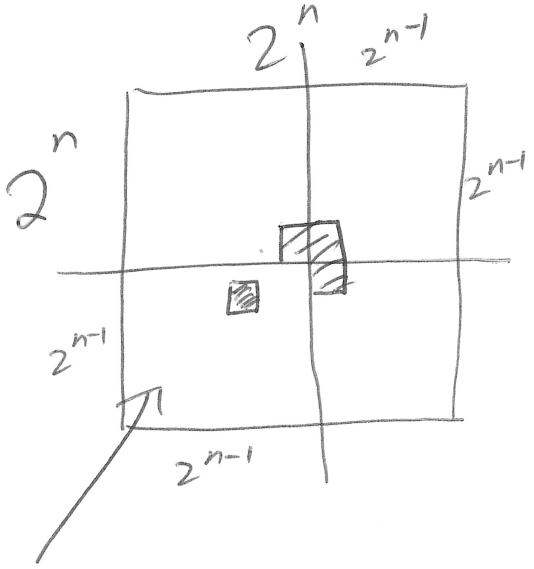
$(X_H + X_L)(Y_H + Y_L) = 11 \times 101 = 1111 \qquad M_3$

$M_3 - M_2 - M_1 = 1111 - 10 - 110 = \underline{111}$

$$M_1 2^4 + (M_3 - M_2 - M_1) 2^2 + M_2$$

$$\begin{array}{rll} 100000 & \rightarrow & M_1 \times 2^4 \\ 11100 & & (M_3 - M_2 - M_1) \times 2^2 \\ + \quad 110 & & M_2 \\ \hline 1000010 = 66 & & \end{array}$$

Tromino Tiling



fill this space with Tromino Tiles - no tiles can overlap and all space must be covered except the "hole".

① Recursively tiled!

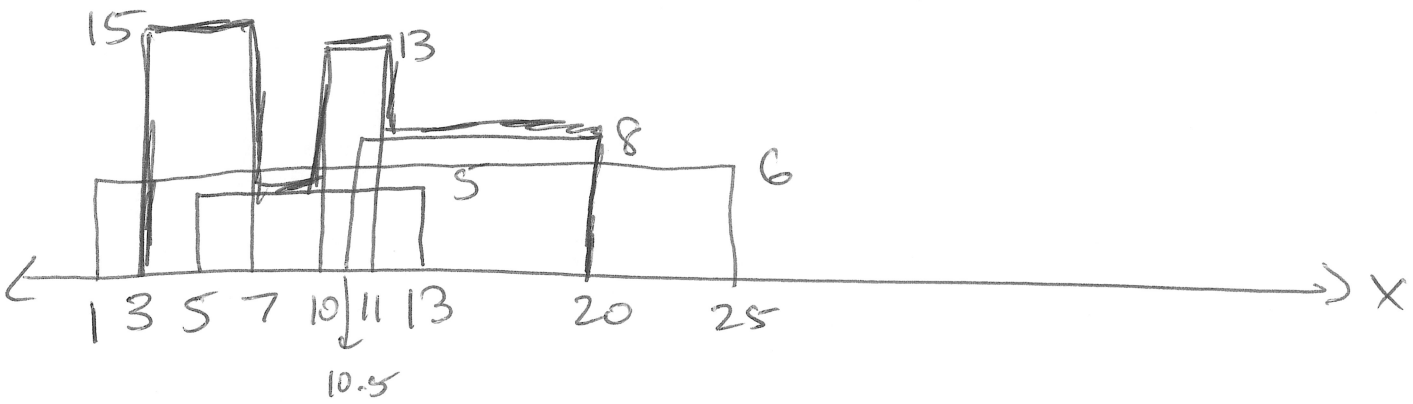
② Place 1 tromino in "the middle" covering 1 square each in the 3 bisser squares that don't have a small square missing

③ Recursively tile the other 3 Quadrants

$2^1 \times 2^1$ is B.C.

3	3	1	1
3	2	1	
4	2	2	5
4	4	5	5

Skyline



[3] 15 [7] 5 [10] 13 [11] 8 [20]

[1] 6 [3] 15 [7] 6 [10] 13 [11] 8 [20] 6 [25]

Merge in new building [1] 6 [25]

$O(n)$, n = # of old building

Run-time

$1+2+3+4+5+ \cdots +n = \frac{n(n+1)}{2}$

$= O(n^2)$

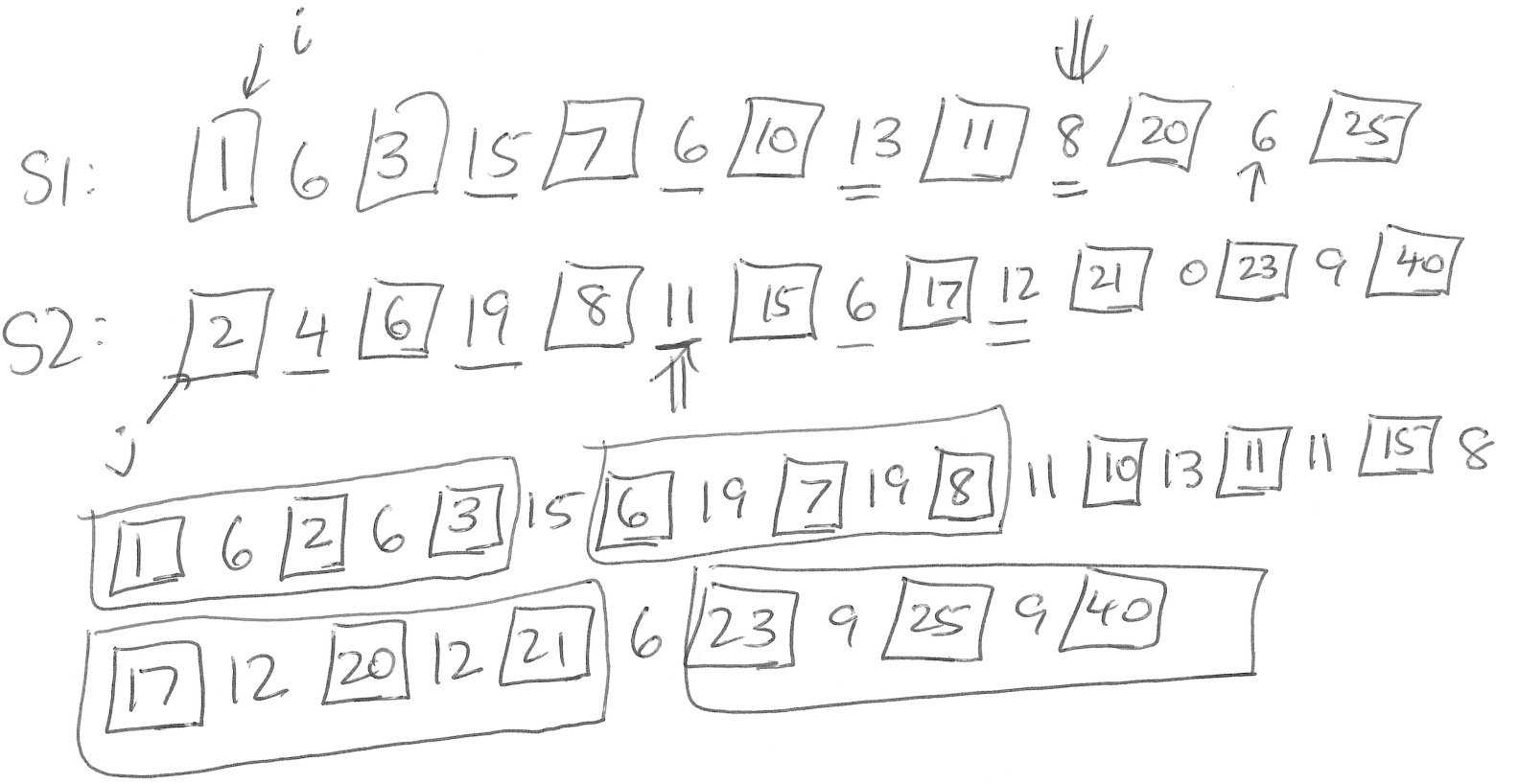
Can we do better?

Solve 1st $n/2$ buildings

||

Solve last $n/2$ buildings

THEN CAN WE MERGE?



11 6 3 15 6 19 8 11 10 13 11 11 15 8 17 12 21 6

23 9 40 Smooth/Shrink

$O(n)$ time

let $T(n)$ = runtime Skyline of n buildings

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{2}\right) + O(n)$$

- $T\left(\frac{n}{2}\right)$: left $\frac{n}{2}$ build
- $T\left(\frac{n}{2}\right)$: other $\frac{n}{2}$ build
- $O(n)$: skyline merge

$$T(n) = 2T\left(\frac{n}{2}\right) + O(n)$$

$$T(n) = O(n \log n)$$