

Red Black Trees 2/1/2024

Properties

- 1) Valid BST, each node colored Red or Black.
- 2) Root Black
- 3) Children Red ~~node~~ ^{node} are Black
- 4) Every external node is black (null)
(nulls off leaf nodes)
- 5) All external black nodes have the same black depth. (# of black nodes on path from root to each external node is the same)

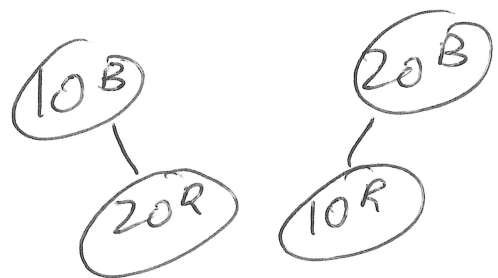
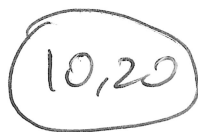
2 node 2-4 tree



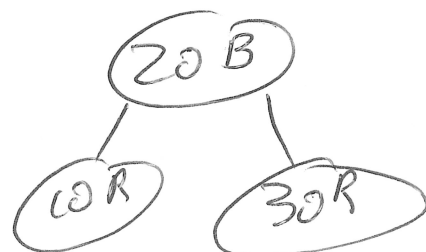
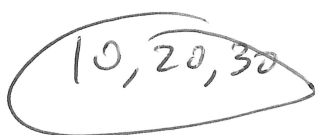
One Black in R-B tree

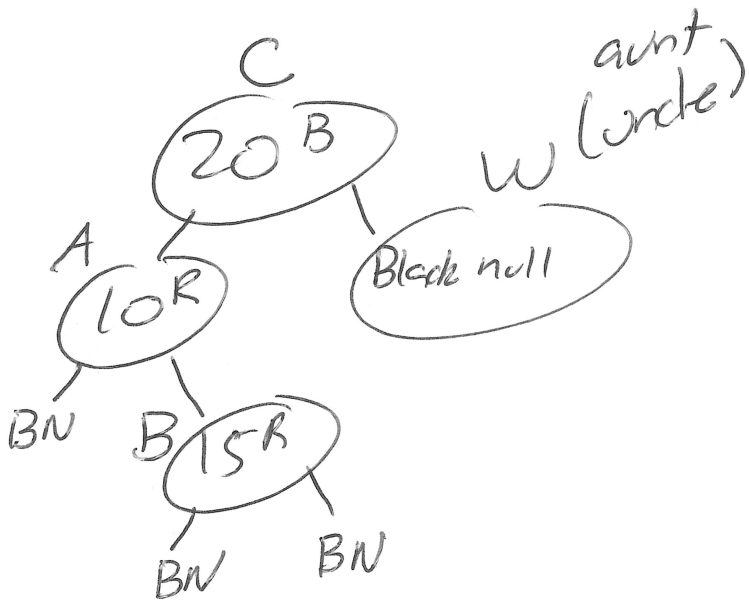


3 node 2-4 tree



4 node 2-4 tree

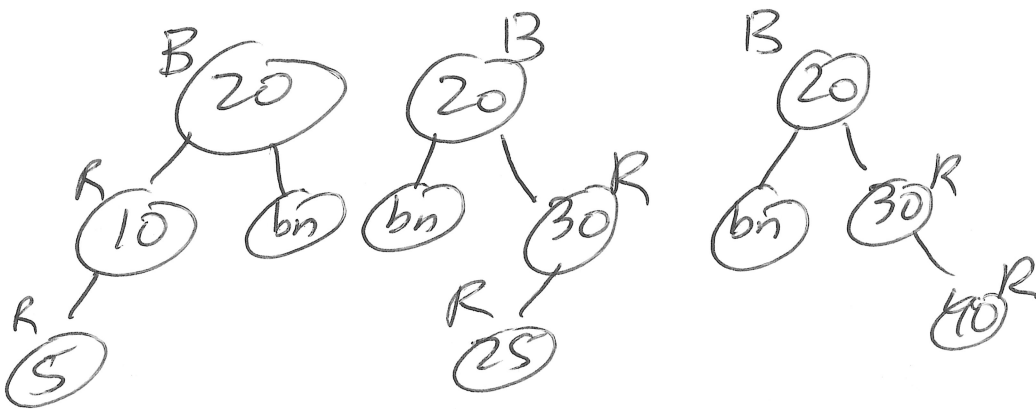




Insert 15 \Rightarrow Problem
2 reds

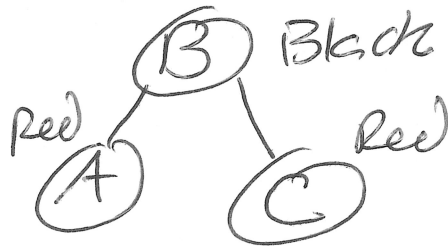
Case 1

Uncle/uncle is black node

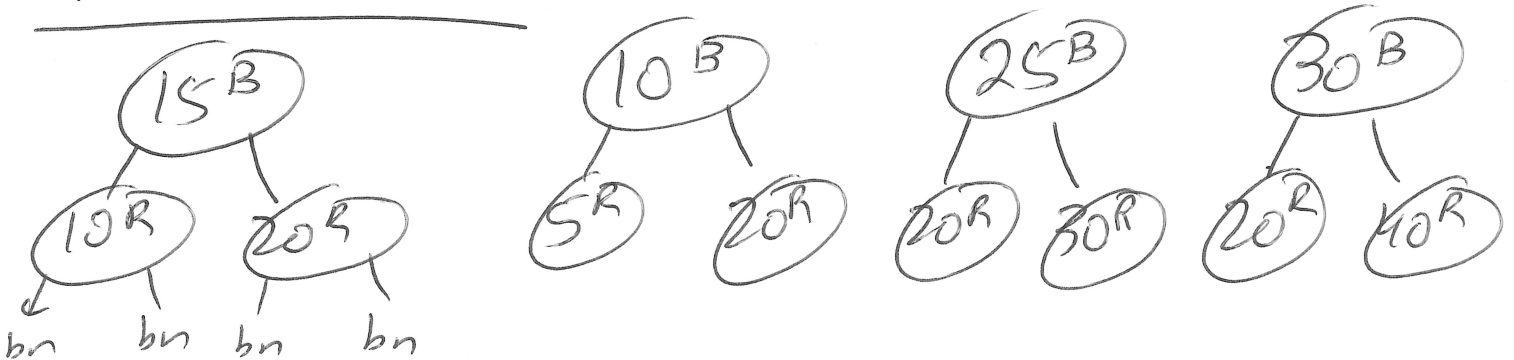


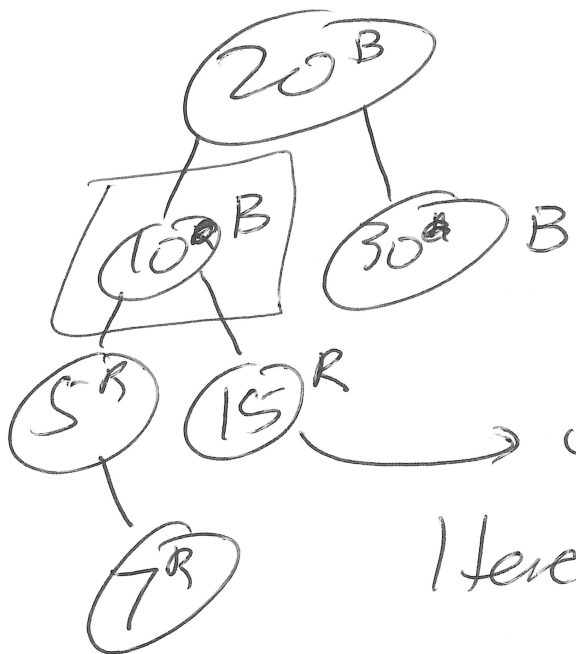
Uncle/uncle
doesn't have
to be a
null node

In all 4 cases, relabel nodes A, B, C
 $A < B < C$ and restroctive



New Rc





Insert ~~5~~

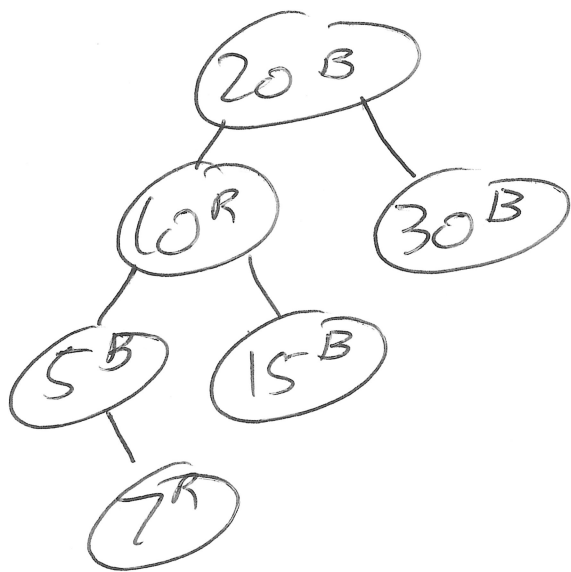
Sorted 20, 10, 30, 5, 15

Insert 7

uncle/grandparent = Red

Here: Recoloring

If uncle is ~~black~~^{red} node, then grandparent must be black node. Push black color down one node



Case 2

Push black color down grandparent to both children
 So grandparent = Red
 Both children of grandparent = Black

Here we're fine but if the parent of the new red node is red we have an issue.

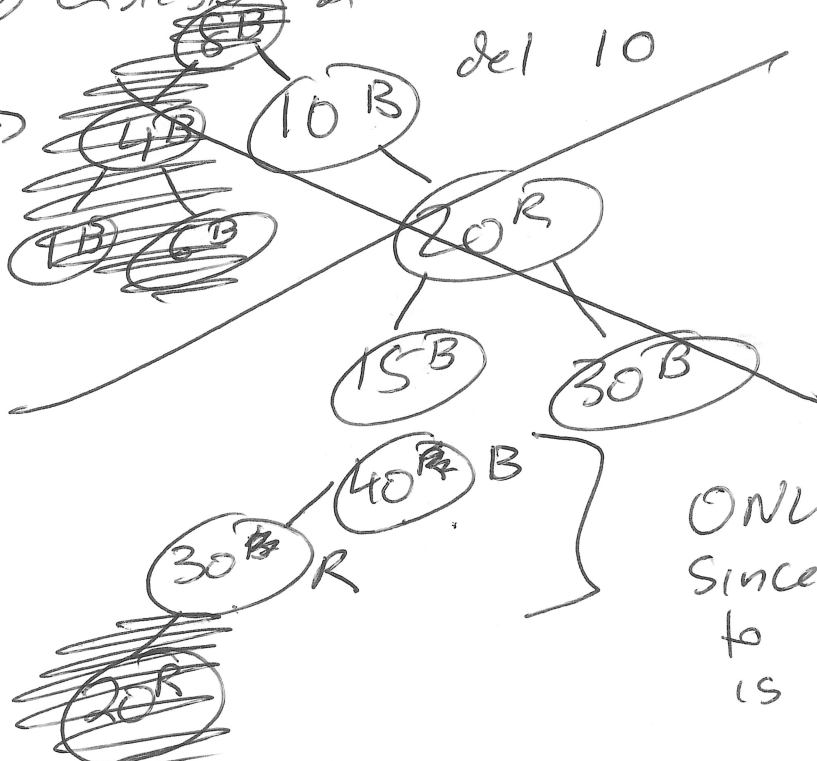
Deletion

① Red Leaf Node

② Black node w/ one red child (reason only 1 red child is that in reg BST delete we never delete a physical node w/ 2 children \Rightarrow max left or min right)

③ Black leaf node

\rightarrow easiest \rightarrow NO FURTHER WORK



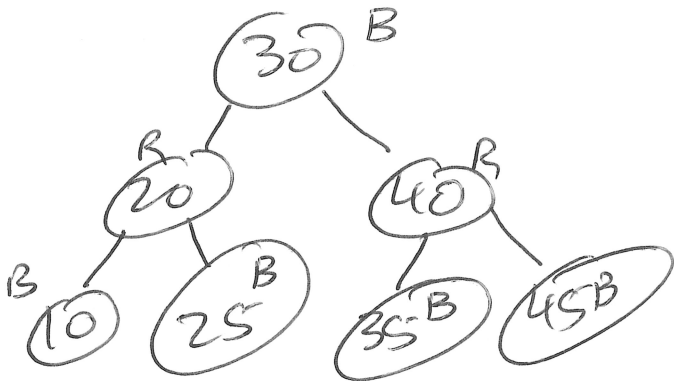
ONLY CASE
 Since black height have to be the same this is the only case

all we'll do is delete it color child black

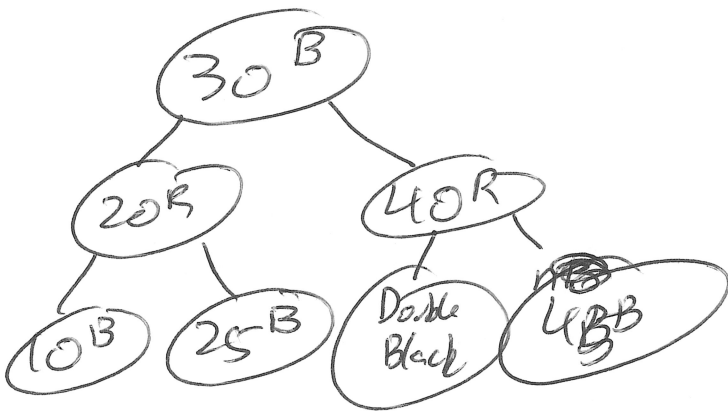


Black Leaf Node

Delete 35



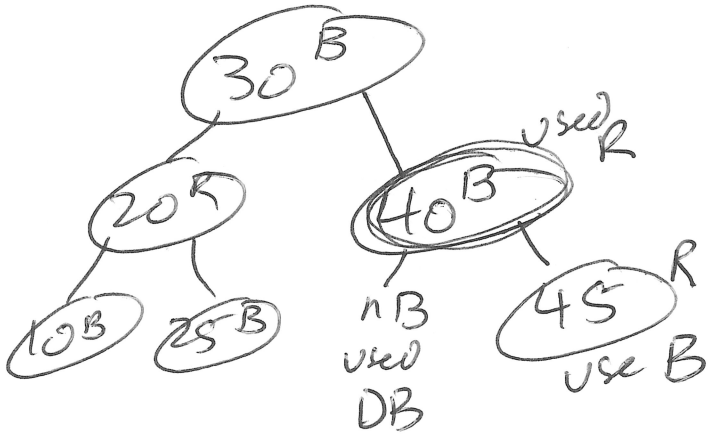
⇒



Step 1: Change the null node to be "double black"

Step 2: "Bubble up" black at the double black null node 1 level

⇒



fixed