Spring 2024 COP 3503 Exam #1 2/6/2024 – Part B (Backtracking) Solution

3) (5 pts) The method to find the "Magic Sum" for the Hexagram program (Program 2) was to take the 12 input numbers, add them, and divide this sum by three. Why does this work?

Every number was part of exactly two row sums. So, when you sum the six rows, each item, a_1, a_2, …, a_{12}, appears exactly twice, so the sum of six rows of the hexagram is equal to $2\sum_{=1}^{12} a_i$. Thus, if X is the magic row sum, then we have $6X = 2\sum_{=1}^{12} a_i$. Dividing this equation by 6, we get:

$$X = \frac{1}{3}\sum_{i=1}^{12} a_i$$

As the formula above says, to get the magic row sum, just add all 12 numbers and then take that sum and divide by 3. This is why that works! **Grading: Holistic, up to grader based on clarity of explanation.**

4) (5 pts) In class, code to solve the Tentaizu problem was executed, which ran fairly quickly. A single line of code was commented out and then the solution ran quite slowly (it solved one puzzle in several seconds, while we waited.) Conceptually, what did that line of code do?

If we completed square (r+1, c+1), but the number of bombs surrounding square (r, c) was not correct, that line of code said the board was not viable or possible to produce any valid solutions. This works because no new bombs adjacent to (r, c) will ever be placed after we complete our selection for square (r+1, c+1).

Grading: Holistic, up to grader based on clarity of explanation. Note that there are may ways to express this idea.

5) (5 pts) In class, we visualized a solution to the Eight Queens puzzle as a permutation of the integers from 1 to n, where the i^{th} integer in the permutation represented which row to place the queen in column i. Using this same visualization technique, draw the solution that corresponds to the first permutation lexicographically that correctly solves the five queens problem below.

Q				
			Q	
	Q			
				Q
		Q		

Grading: 2 pts for any valid 5Q solution, 0 pts if Queens attack each other or there aren't 5 of them, 5 pts if it's perfectly correct and 5 pts for the transpose of this picture (placing [1,3,5,2,4] in the rows instead of the columns.

6) (15 pts) A prefix composite number is one such that each prefix of it is also a composite number. For example, 44052 is a prefix composite number that is 5 digits long because 4, 44, 440, 4405 and 44052 are all composite numbers. Complete the program on below so that it prints out all prefix composite numbers that are 5 digits long.

```
import java.util.*;

public class prefixcomposite {

     public static void main(String[] args) {
          go(0, 0, 5);
     }

     // cur is a valid prefix-composite of k digits
     // n is the # of digits in the prefix-composites to be printed
     public static void go(int cur, int k, int n) {

          if (k == n) {                          // 1 pt
               System.out.println(cur);          // 1 pt
               return;                           // 1 pt
          }

          for (int i=0; i<10; i++)               // 1 pt
               if (!isprime(10*cur+i))           // 2 pts
                    go(10*cur+i, k+1, n);        // 3 pts
     }

     // Returns true if n is prime or if n is less than 2.
     // Must run in O(sqrt(n)) time and not use any doubles for
     // full credit.
     public static boolean isprime(int n) {
         for (int i=2; i*i<=n; i++)              // 2 pts
             if (n%i == 0)                       // 2 pts
                 return false;                   // 1 pt
         return true;                            // 1 pt
     }
}
```