

0-1 Knapsack Addendum

Inner loop goes backward to prevent using an item more than once.

```
for (Item)
  for (i = W; i >= w[item]; i--)
    dp[i] = max(dp[i], dp[i - w[item]] + v[item])
```

← this hasn't been updated yet based on item.

What if I want multiple copies of an item to allow

```
for (Item)
  for (i = w[item]; i <= W; i++)
    dp[i] = max(dp[i], dp[i - w[item]] + v[item])
```

← this was updated a minutes ago! (or less)

Floyd-Warshall's Alg

All Pairs Shortest Path

Neg Edge Weights

Detect Neg Cycles

Input: Directed
Weighted
Graph

Build up shortest paths (between all pairs of vertices) as we allow intermediate vertices.

(1) Store adjacency matrix of our graph

$adj[i][j]$ = edge weight from i to j .

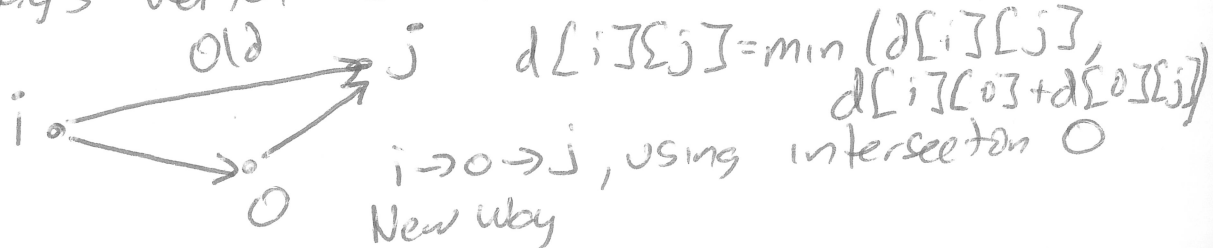
If no edge either store null or large # -
watch out for overflow!

This represents shortest distances when no intermediate vertices are allowed.

Let $d[i][j]$ represent our estimate for the shortest distance from i to j .

Initially $d[i][j] = adj[i][j]$.

Mom says vertex 0 is safe to cross



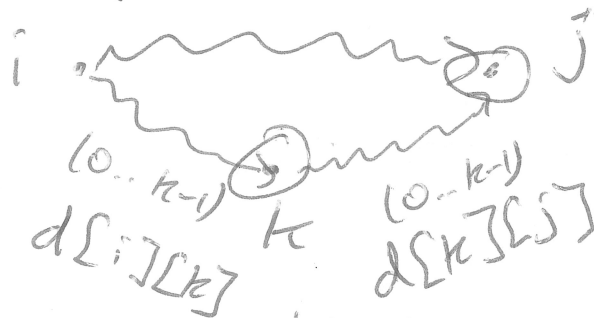
After this update for all i, j we'll
shortest distances that ONLY go through
vertex 0.



$$d[i][j] = \min(d[i][j], d[i][0] + d[0][j])$$

More generally, once vertex k is deemed to be
safe, we want to see if using it helps:

$$(0 \dots k-1) d[i][j]$$



$$d[i][j] = \min(d[i][j], d[i][k] + d[k][j])$$

// k intermediate vertex.

for ($k=0$; $k < n$; $k++$)

for ($i=0$; $i < n$; $i++$)

for ($j=0$; $j < n$; $j++$)

} $i = \text{start}$
 $j = \text{end}$

$$d[i][j] = \min(d[i][j], d[i][k] + d[k][j])$$

How to reconstruct the shortest path?

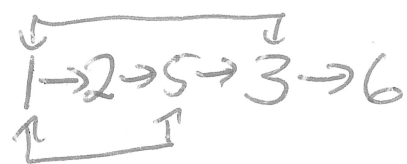
$\text{path}[i][j] = \text{last vertex on the}$
 $\text{shortest path from}$
 $i \text{ to } j$

$$\text{path}[1][6] = 3$$

$$\text{path}[1][3] = 5$$

$$\text{path}[1][5] = 2$$

$$\text{path}[1][2] = 1$$

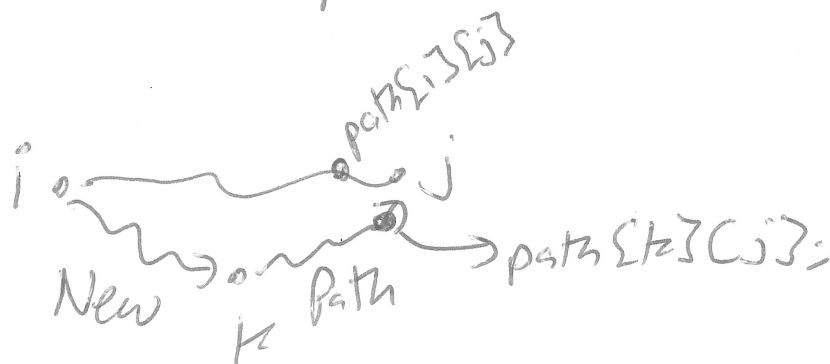


How to use

How to build?

Initially $\text{path}[i][j] = i$ for each edge that exists.
-1 otherwise

if $(d[i][k] + d[k][j] < d[i][j]) \{$
 $d[i][j] = d[i][k] + d[k][j];$
 $\text{path}[i][j] = \text{path}[k][j];$ // NOT $k!!!$
 $\}$



Runtime $O(U^3)$, max U in practice
 ~ 500 . Memory $O(U^2)$.

Neg cycle detection

1000 queries
 $\times 150$ k

