

~~Adj. Adjacent~~
Adj. Sorts

Bubble sort

Insertion sort



Look at inversions

a pair of indices that
are out of order

Value 8, 6, 7, 5, 3, 0, 1

index 0 1 2 3 4 5 6

(0, 1) (0, 2) (0, 3) (0, 4) (0, 5) (0, 6)

(1, 3) (1, 4) (1, 5) (1, 6) ...

Dont
have
(1, 2)
(5, 6)

$$\binom{N}{2} = \frac{N(N-1)}{2}$$

max inversion is $\binom{N}{2} \propto N^2$

How many inversions are
"removed" when swapping
adj. elements?

At most 1

A sorted Array has
0 inversions.

Worst case is $\binom{N}{2}$ swaps.

What is the average #
of inversions for a
random permutation?

Consider value i and value j

$i < j$

How many inversion does i, j create on avg?

$\frac{1}{2}$ i and j are
 $\dots i \dots j \dots$ swapped in just as
 $\dots j \dots i \dots$ many perms as they are
 Unswapped.

Total possible Inversions is

$$\binom{N}{2}$$

Average $\binom{N}{2} \cdot \frac{1}{2}$

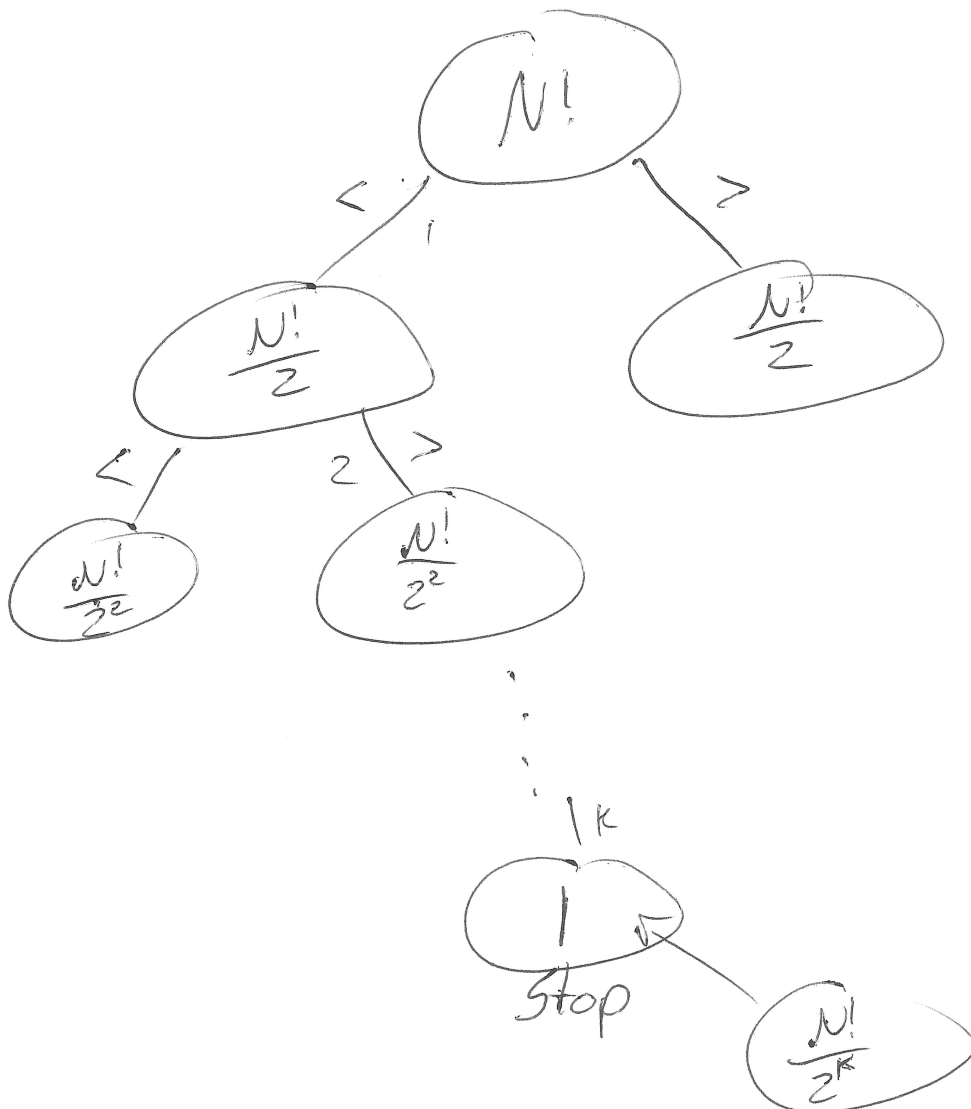
$$\frac{N^2}{4} - \frac{N}{4} \in O(N^2)$$

We don't use $a d j$. swaps.

We use Merge or Heap

If we know what our permutation
is we can unsort in linear
time

What is the ^{Average} ~~least~~ Amount
of information needed to sort
my array (know the permutation)



$$1 = \frac{N!}{2^k}$$

$$2^k = N!$$

$$k = \log_2(N!)$$

↑↑↑

$$k = \sum_{i=1}^N \log_2(i) < \sum_{i=1}^N \log_2(N)$$

$$N \log_2(N)$$

$$O(N \log(N))$$

$$\sum_{i=N/2}^N \log_2(i) < \sum_{i=N/2}^N \log_2(N)$$

↓

$$\frac{N}{2} \log_2\left(\frac{N}{2}\right)$$

$$\Omega(N \log(N))$$

$$\Theta(N \log(N))$$

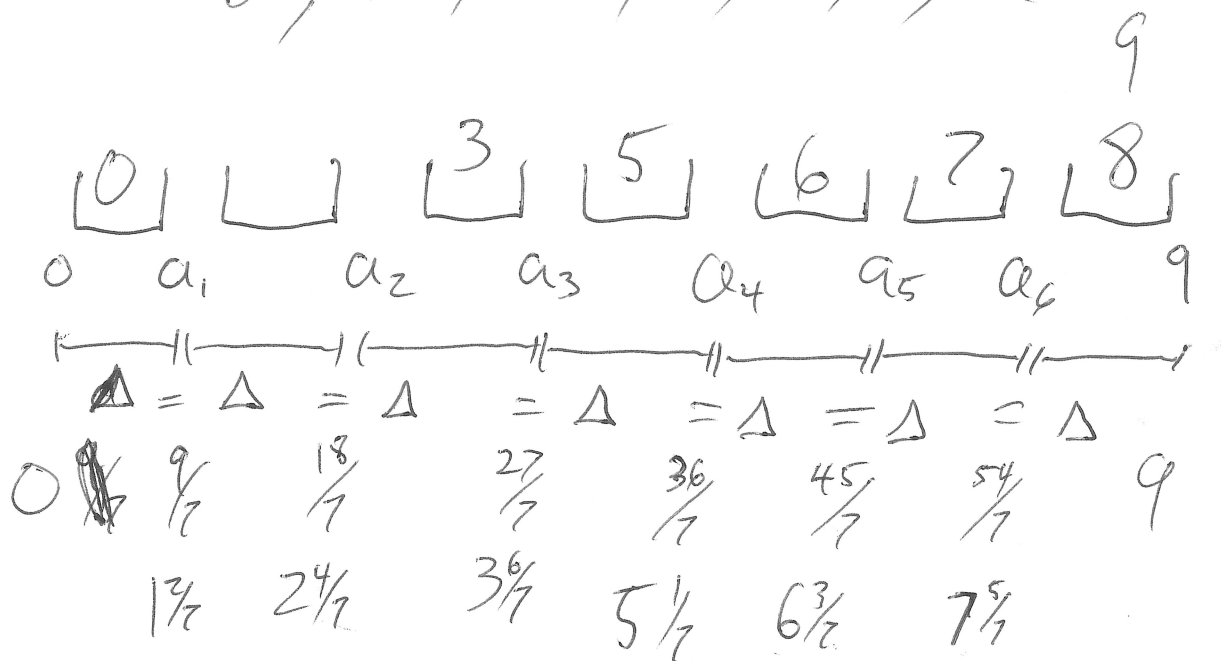
	B	A	W
O	$N \log(N)$	$N \log(N)$	N^2
$\sim$	$N \log(N)$	$N \log(N)$	N^2
Ω	$N \log(N)$	$N \log(N)$	N^2

Bucket Sort Counting Sort

Radix Sort

~~Bucket~~ Bucket

8, 6, 7, 5, 3, 0, 9



Worst case

everything in one bucket

$O(N^2)$ for insertion sort

$O(N \log N)$ merge or Heap.

Average case

each bucket will

have $O(1)$ element

hash collision argument

$O(N)$ to place all values on avg.

Merge buckets ~~also~~ takes
 $O(N)$

Counting Sort
(frequency)

for Array of N integers
in range of 0 to MAX

8, 6, 7, 5, 3, 0, 9

Frequency Array

0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0
1			1		1	1	1	1	1

Final

0	3	5	6	7	8	9
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Routine

$2N + MAX \in \Theta(N + MAX)$
BWA

RADIX

LSD

loop from ~~LSD~~ to MSD

sort by current Digit

235, 162, 734, 175, 237,

674, 628

162, 734, 674, 235, 175,

237, 628

628, 734, 235, 237, 162,

674, 175

162, 175, 235, 237, 628, 674,

734

$$2. \# \text{ Digits} \cdot N \in O(\# \text{ Digits} \cdot N)$$