

Red Black Trees (really similar to 2-4 trees) 2/11/2022

- (1) Valid BST, and each node is assigned a color red or black.
- (2) Root is black.
- (3) All the children of red nodes must be black.
- (4) Every external node is black.
- (5) All of the external black nodes have the same "black depth". The black depth of an external node is the # of black nodes on the path from the root to it.

How to convert 2-4 tree $\Rightarrow$ equiv R-B tree

each 2 node $\Rightarrow$ black node

each 3 node $\Rightarrow$ 2 nodes (parent black, child red)

each 4 node $\Rightarrow$ black parent, 2 red children.

$$\begin{aligned}\text{Height (R-B)} &\leq 2 \text{ Height (2-4 Tree equiv)} \\ &\leq 2 \lg n, n = \# \text{ ~~node~~ values} \\ &= O(\lg n)\end{aligned}$$

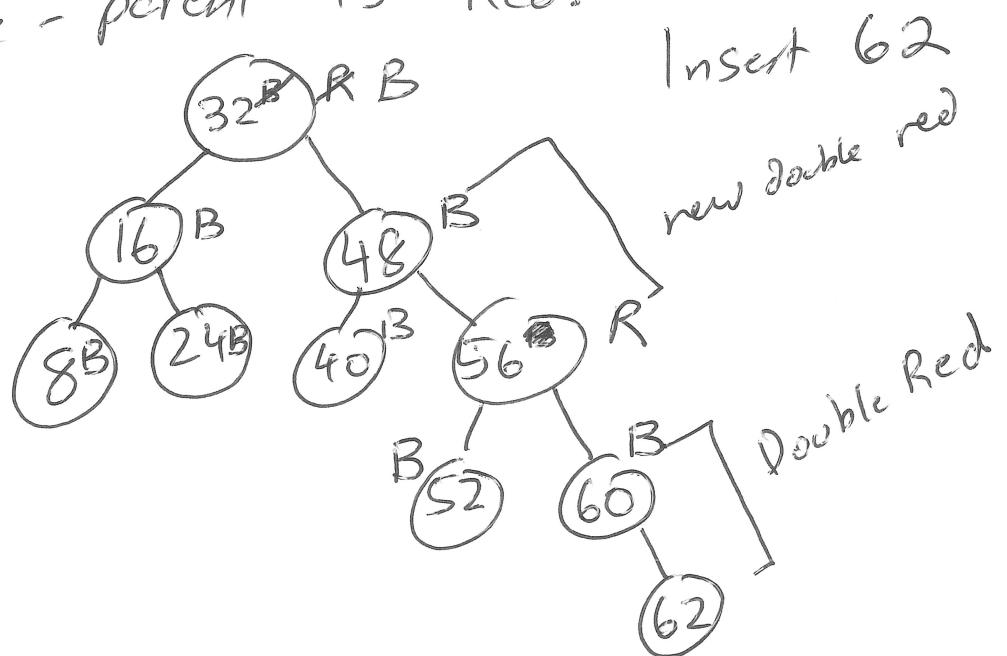
Insertion into R-B tree

Initially, when we insert a node, we'll do a regular BST insertion + color node red.

We make it red because that's guaranteed to NOT mess up the black height rule.

Easy case - inserted node's parent is black.
(similar to inserting into a 2 or 3 node in a 2-4 tree.)

Harder case - parent is Red.



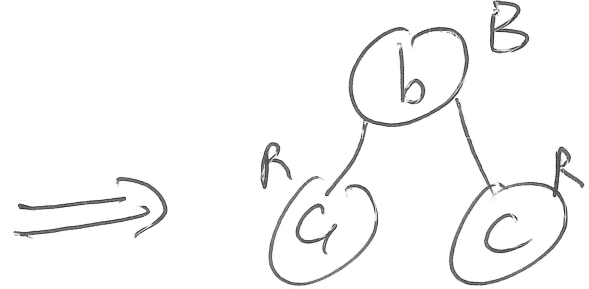
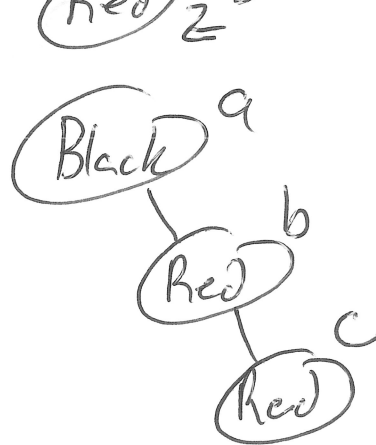
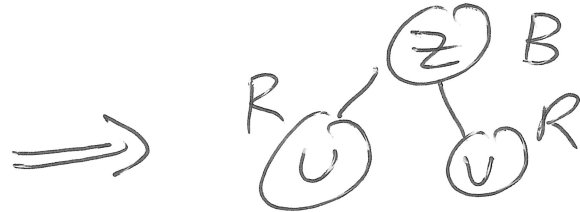
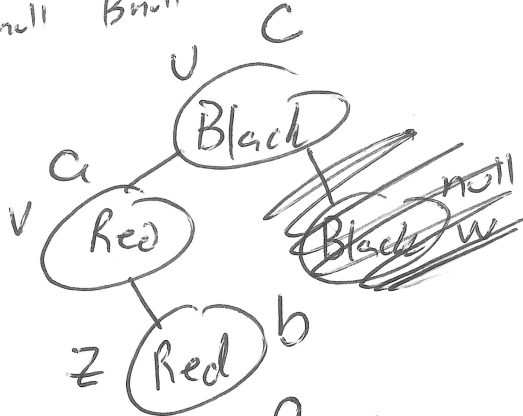
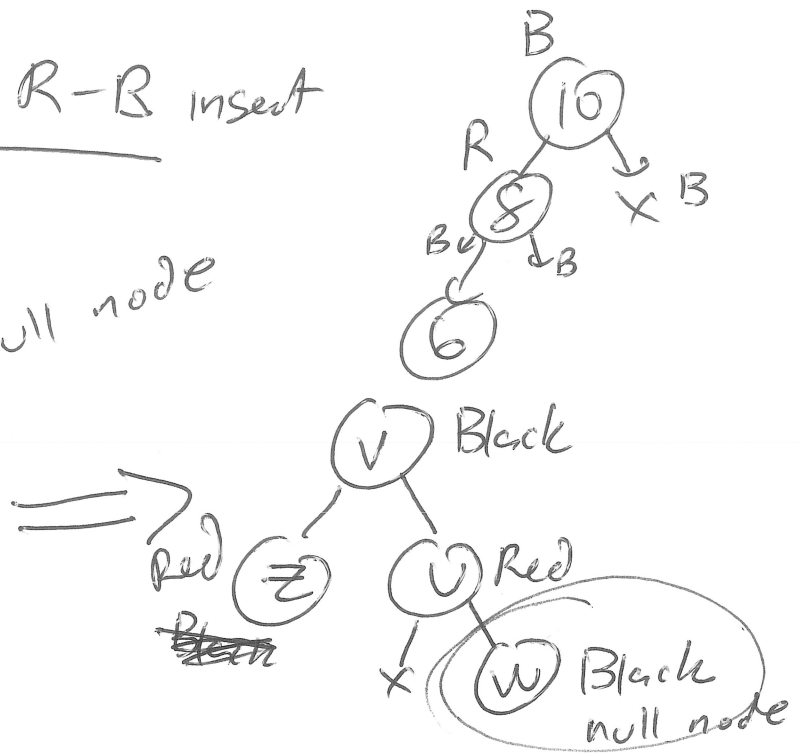
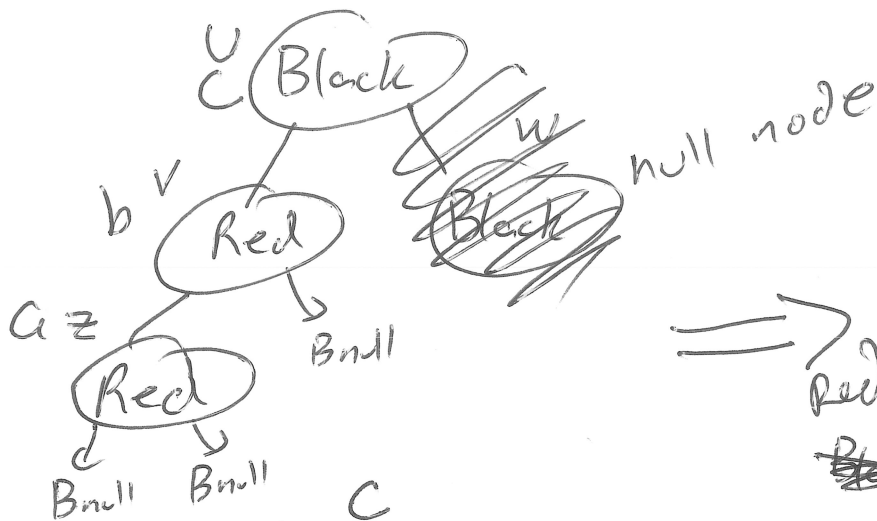
Push black node above 60 down.

Push black node above 48 down.

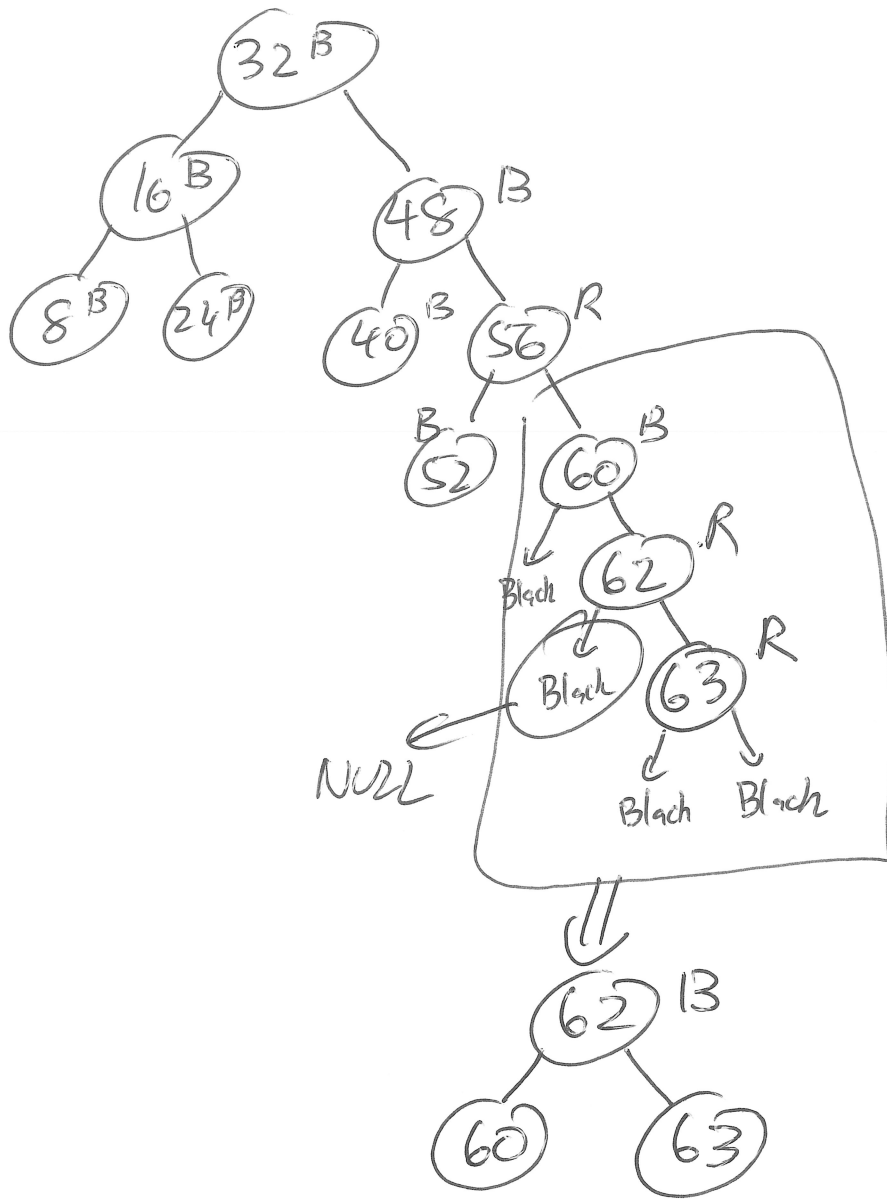
Oops! Root is Red.

SOLN $\Rightarrow$ TURN IT BLACK

Case 1 of R-B insert



If you label them a, b, c w/ $a \leq b \leq c$,
 then you always fix a as root black,
 a and c red



⇒
We can't push
black down if
the other side
(uncle) is already
black.

