1/22 Big Omega and Big Theta

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Big Omega Formal

$f(n) \in \Omega(g(n))$
iff
$\exists c>0, n_0$ such that

$\forall n > n_0 \; f(n) \geq g(n) \cdot c$

Examples

1. $4n^2+2n \in \Omega(n^2)$
2. $n^2 \in \Omega(4n^2+2n)$

Yeah no duh...

Big Omega informally

$f(n) \in \Omega(g(n))$
means $g(n)$ is a lower bound of $f(n)$

Calculus of Ω

$f(n) \in \Omega(g(n))$ iff

$\lim_{n \to \infty} \frac{f(n)}{g(n)} > 0 \leftarrow$ zero

$\lim_{n \to \infty} \frac{\quad}{g(n)}$

Meaning

For large cases we can usually say that a program that can be considered the execution of $f(n)$ constant time operations will be no worse than a second program that use $g(n)$ constant time operations if $f(n) \in O(g(n))$ or $g(n) \in \Omega(f(n))$

Note that we do not say,

$$f(n) = O(g(n))$$

or

$$f(n) = \Omega(g(n))$$

$O()$ is a function that returns a set of functions, so saying $f(n) = O(g(n))$ would be incorrect.

Formally Big Theta

$$f(n) \in \Theta(g(n)) \text{ iff}$$

$$f(n) \in O(g(n))$$

$$f(n) \in O(g(n))$$

AND

$$f(n) \in \Omega(g(n))$$

Examples

1. $4n^2 + 2n \in \Theta(n^2)$
2. $2n^2 \notin \Theta(2^n)$

As an exercise prove 1.