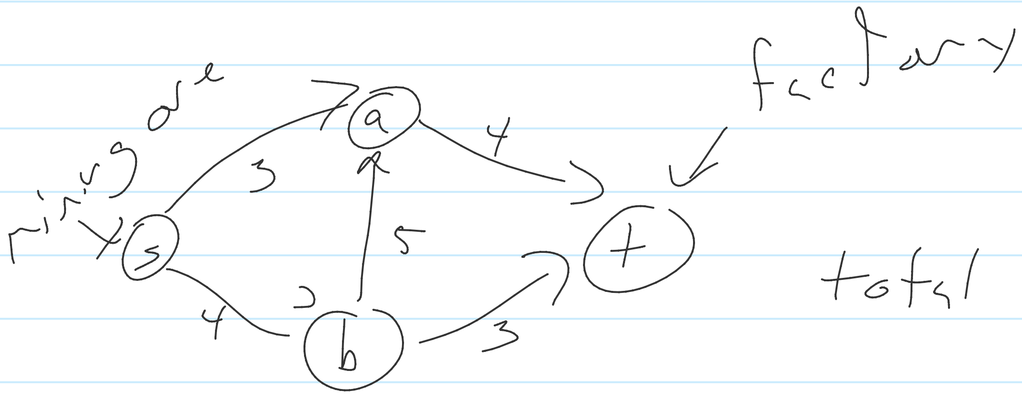
3/7 Flows

Wednesday, March 7, 2018 5:57 PM



total of 7 units

units

starting node called source

Ending node call sink

Definition of a flow

weighted graph (most of the time directed).
typically edge weights are referred to capacities.

A flow from s to t is as follow

1) Each edge is assigned a non-negative value, flow of edge, that each flow is no greater than the edge capacity

$$\forall((u,v)\in E)(\underset{\uparrow}{f_{uv}} \le \underset{\uparrow}{w_{uv}})$$

flow capacity

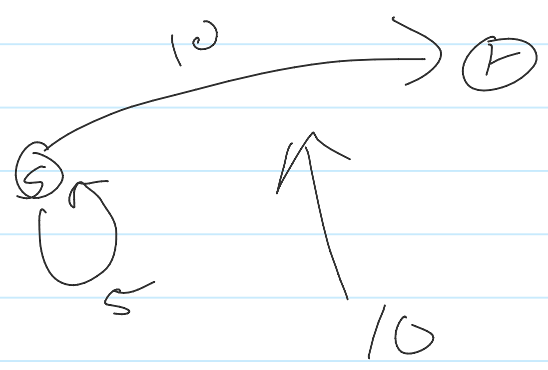
2) Aside from source (s) and sink (T) each node needs to conserve the flow. The inflow is out flow

$$\forall(v \in V \underset{\uparrow}{\setminus} \{s,t\})\left(\underset{\uparrow}{\sum_{(u,v)\in E} f_{uv}} = \underset{\uparrow}{\sum_{(w,v)\in E} f_{wv}}\right)$$

set difference out flow inflow

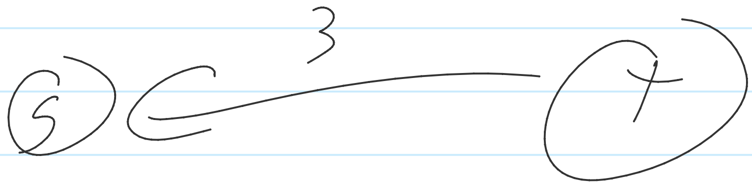
The flow of the graph is

out flow of the source node minus the inflow of the source node



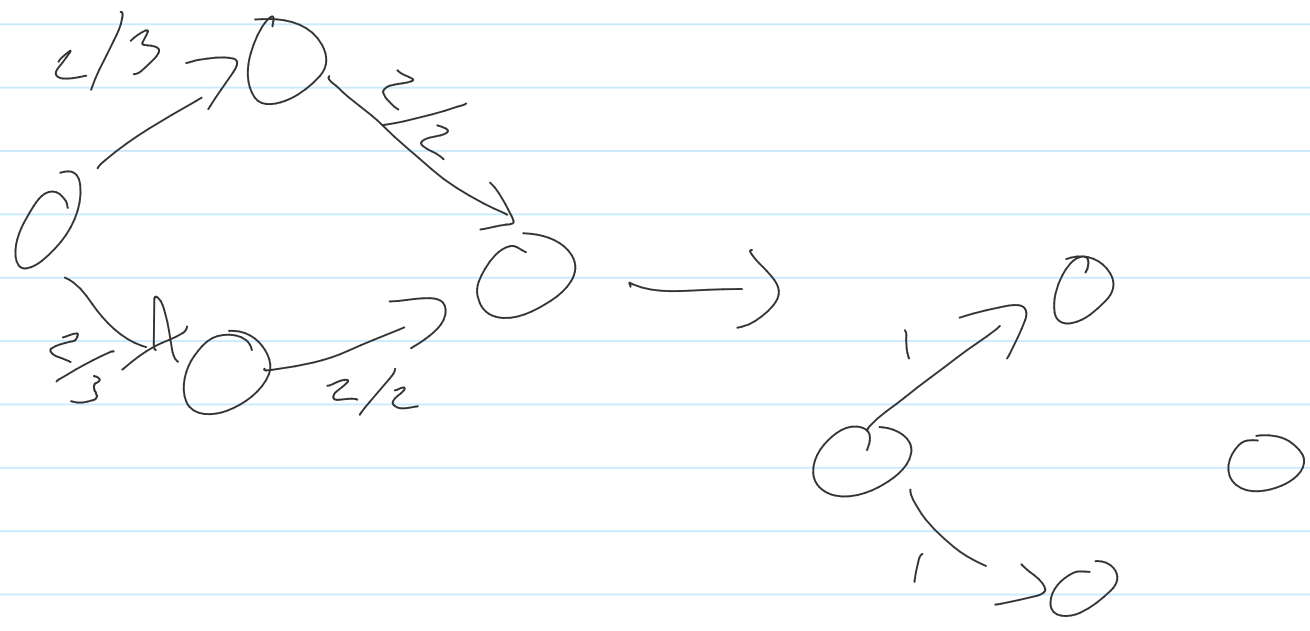
15 – 5

leaving entering

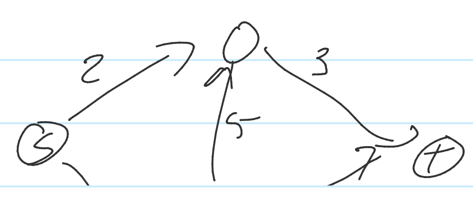


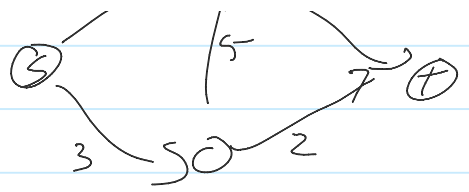
max flow for this graph is 0

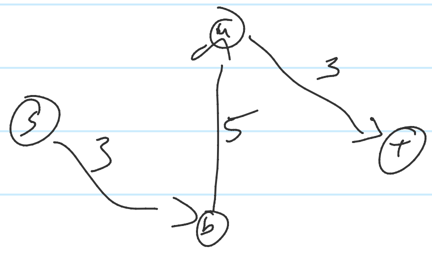
but you could have -3 flow

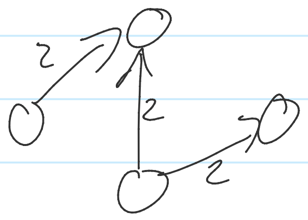


Find a path send flow through the path (the flow of which is the min capacity along path) update graph and remove full edges







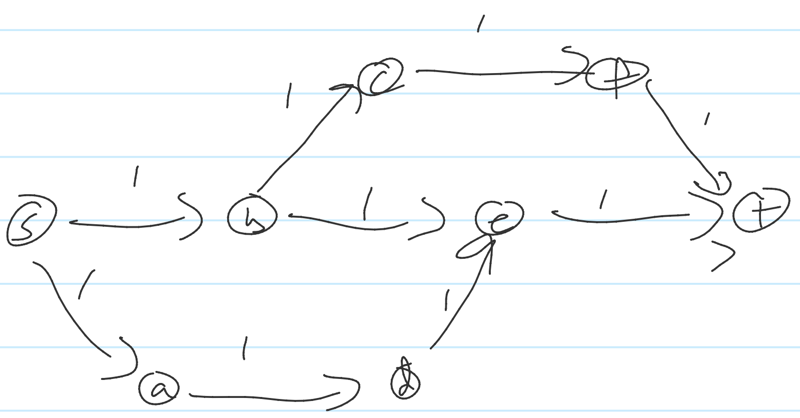


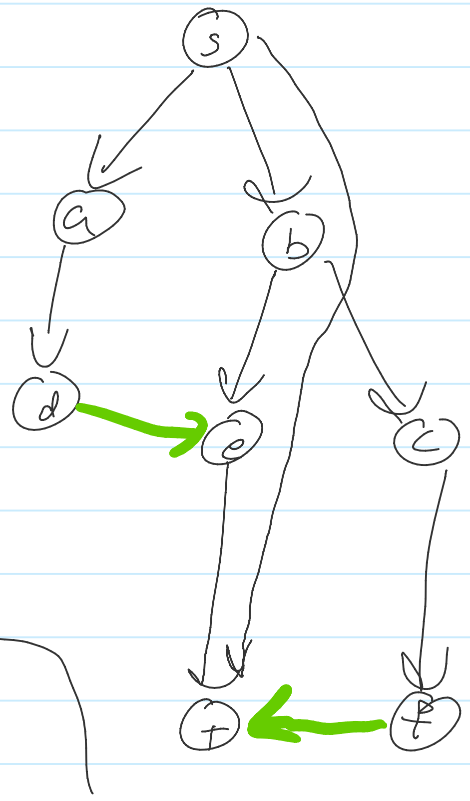
found
flow is 3

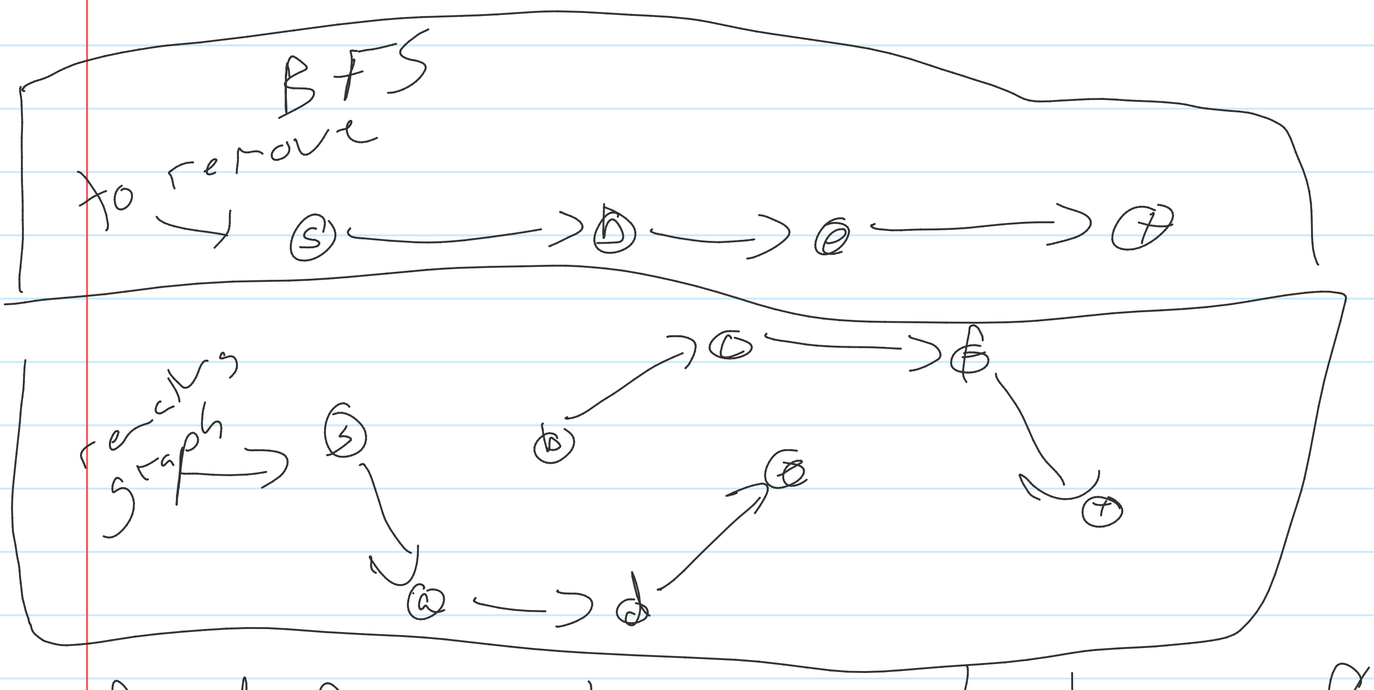
actual max flow is 5

try a BFS; this did not work,

Other tricky case

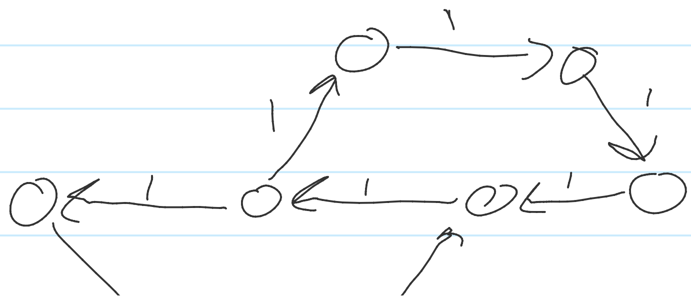




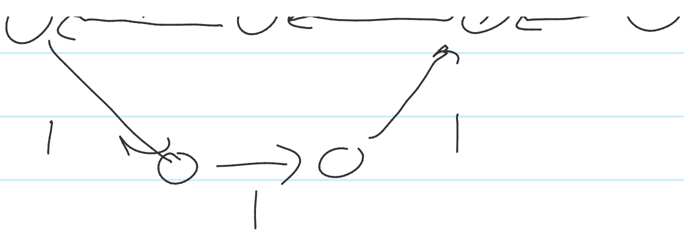


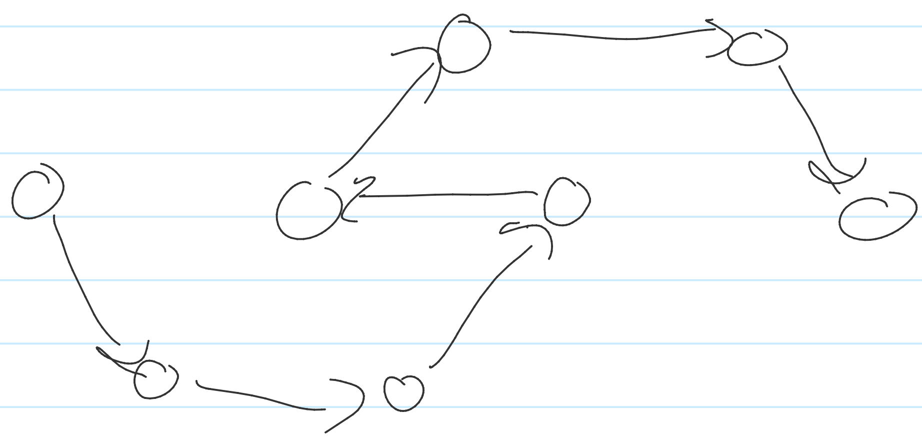
found flow is 1 actual max flow is2

Residual Graph

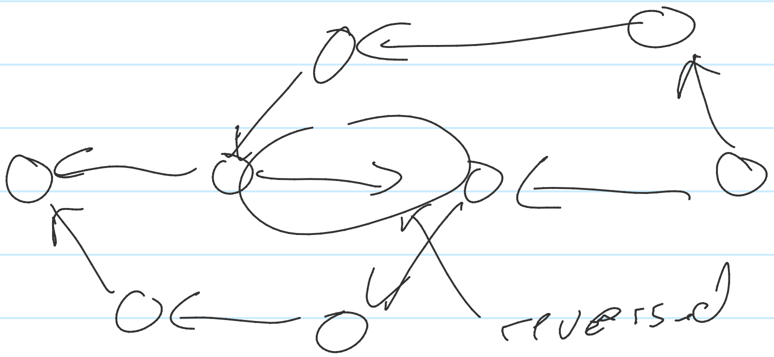


After first flow





second



After second flow

Run time of Algorithm   The algorithm is Ford-Fulkerson Algorithm

$O(F \times$ time to find path$)$
↑ DFS $= O(|V|+|E|)$
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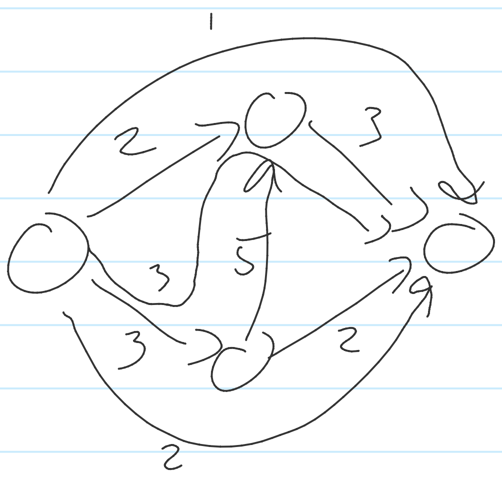
$O(F (|V|+|E|))$

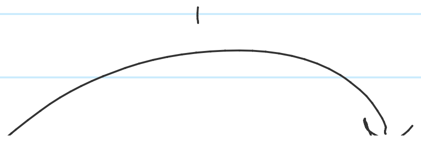
Dinitz's (Yefim Dinitz)

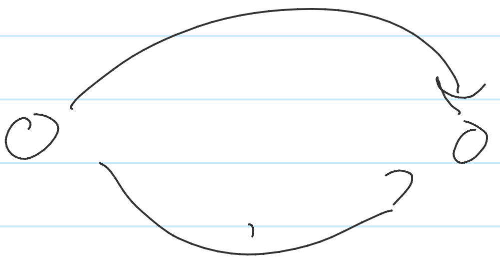
Dinic's

Better algorithm $O(|V|^2|E|)$



FFA





Correctness of FFA

Suppose FFA did not find an optimum solution.. Let F' be the flow graph of the optimum ~~graph~~ flow.

Sum of flow for FFA (F) nodes is zero and sum of flow for F' nodes is zero

Consider F' - F

We know that a positive path exists in F' - F we should have found that path in FFA we reached a contradiction.