1/22 Big Oh

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Big Oh Notation

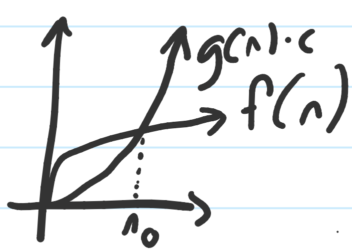
Formaly we can write

$f(n) \in O(g(n))$ iff

$\exists c, n_0$ such that
$\forall n > n_0 \; f(n) \leq g(n) \cdot c$

Informaly we can say that
$f(n) \in O(g(n))$
means that $g(n)$ is
an upper bound for
$f(n)$!

Pictorally we can have



Examples
1. $4n^2 + 2n \in O(n^2)$
2. $2n^2 \in O(2^n)$

Proof 1.
Let $n_0 = 1$ and $c = 7$

$$4n^2 + 2n \leq 4n^2 + 2n^2$$
$$= 6n^2 \quad (\text{since } n > n_0 = 1)$$
$$\leq 7n^2$$

n was arbitrarily large than n_0
so $4n^2 + 2n \leq 7n^2$ for all $n > n_0$ $\blacksquare$

Proof 2.

Let $n_0 = 7$ and $c = 1$

Consider the case when $n = 8$

$$2 \cdot n^2 = 2 \cdot 8^2 = 128 < 256 = 2^8$$

Thus $2n^2 \leq 2^n$ when n is 8.

Suppose $2n^2 \leq 2^n$ for some arbitrary $n \geq 8$.

Consider $2(n+1)^2$

$$2(n+1)^2 = 2n^2 + 4n + 2$$
$$= 2n^2 + 2(2n+1)$$
$$\leq 2n^2 + 2(2n + 6n) \quad (\text{since } n \geq 8 \geq 1)$$
$$= 2n^2 + 2(8n) \quad (\text{since } n \geq 8)$$
$$\leq 2n^2 + 2(n \cdot n)$$
$$= 2n^2 + 2n^2$$
$$= 2(2n^2)$$
$$\leq 2(2^n) \quad (\text{by hypothesis})$$
$$= 2^{(n+1)}$$

[illegible]

$= 2^{(n+1)}$

The hypothesis holds for $n+1$

Since n was arbitrarily chosen by Principle of induction it holds for all integer $n \geq 8$

$\blacksquare$

Calculus Definition of Big Oh

$f(n) \in O(g(n))$ iff

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = c$$

for some constant c.