

3/27/18 ①

## E2 Histogram

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90-100	15
80-89	28
70-79	33
60-69	36
50-59	29
<hr/>	
40-49	10
30-39	7

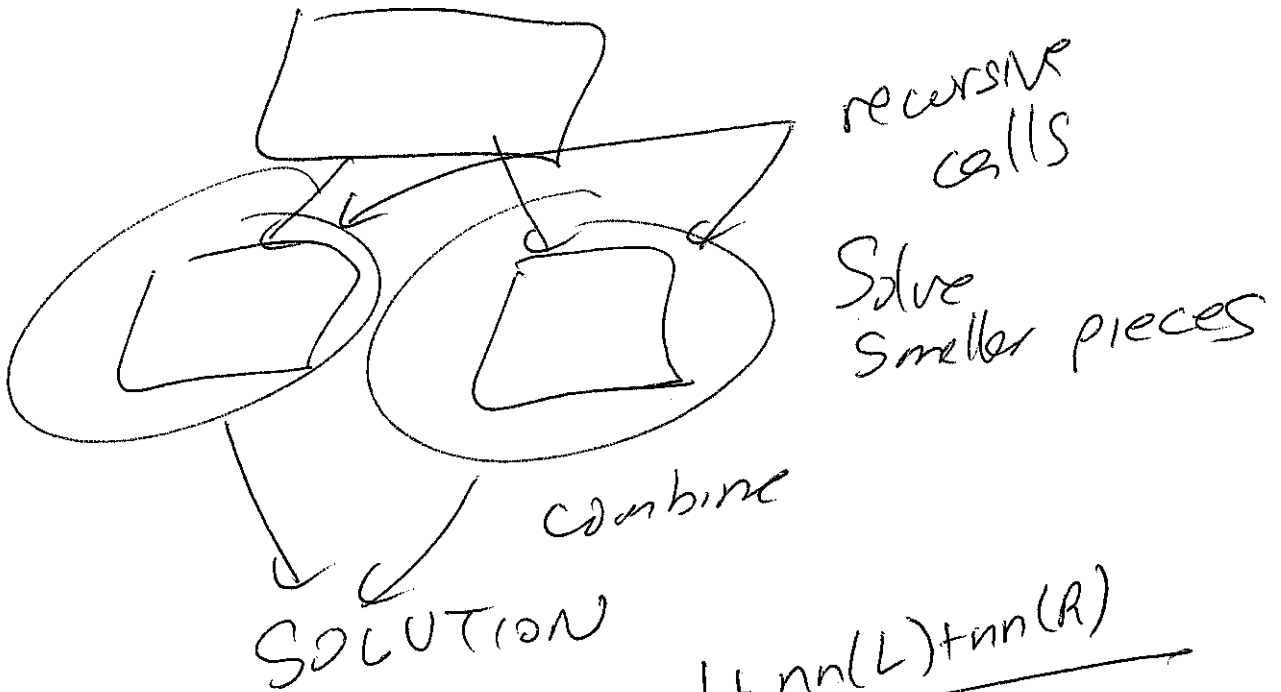
↑ OK  
↓ Need to Improve

A :  $\geq 85$   
B :  $\geq 70$   
C :  $\geq 55$

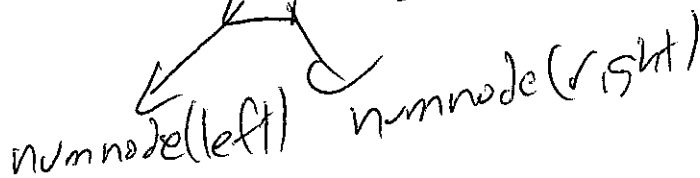
# Divide + Conquer

LCS, ~~Choice~~, ~~Subset Sum~~

Int Mult, Towers, Skyline



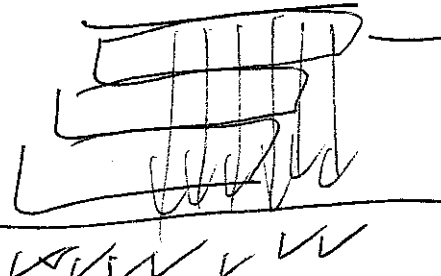
$$\frac{1 + nn(L) + nn(R)}{}$$



## Int Mult

~~3299~~  
x ~~208~~

n-digits  
m-digits



$O(n)$   
 $\rightarrow O(m)$

$O(nm)$

$O(m)$  rows

adding  $O((n+m)m)$

# Poly Mult

3/27/18 (3)

$$(3x^3 + 2x^2 + 9x + 5)(2x^2 + 7x + 8)$$

int mult special case of poly mult  $x=10$ .



$$A = A_H \times 10^k + A_L$$

$$B = B_H \times 10^k + B_L$$

$$\begin{aligned}
 A \times B &= (A_H \times 10^k + A_L)(B_H \times 10^k + B_L) \\
 &= A_H B_H \times 10^{2k} + (A_L B_H + A_H B_L) \times 10^k + A_L B_L
 \end{aligned}$$

$$T(n) = 4T\left(\frac{n}{2}\right) + O(n)$$

#  
rec  
calls

for addition  
(~~2~~ 3 adds)

## Master Thm

$$T(n) = A T\left(\frac{n}{B}\right) + O(n^k)$$

if  $B^k < A$ :  $O(n^{\log_B A})$  ✓

if  $B^k > A$ :  $O(n^k)$

if  $B^k = A$ :  $O(n^k \lg n)$

$A=4$   $k=1$   
 $B=2$   
 $B^k = 2$   
 $\downarrow$   
 $n^{\log_2 4}$   
 $\neq n^2$

Can I reduce # of calls to 3?

$$A \times B = A_H B_H \times 10^{2k} + (A_C B_H + A_H B_L) 10^k + A_L B_L$$

3/27/18 (9)

$$\begin{aligned} \text{Prod \#1} &: (A_H + A_L)(B_H + B_L) \\ &= A_H B_H + A_L B_H + B_L A_H + A_L B_L \end{aligned}$$

$$\text{Prod \#2}: A_H B_H \quad T(n) = 3T\left(\frac{n}{2}\right) + O(n)$$

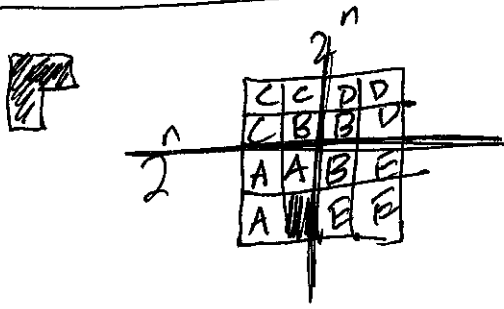
$$\text{Prod \#3}: A_C B_L$$

$$\text{Mid Term} = \text{Prod \#1} - \text{Prod \#2} - \text{Prod \#3}$$

$$\text{new runtime} = O(n^{\log_2 3})$$

$$\log_2 3 \approx 1.57$$

# Tromino Tiling



- ① Subdivide ~~the~~ input into 4 quadrants of size  $2^{n-1} \times 2^{n-1}$ .
- ② One of the four has a hole, recursively tile this square.
- ③ In the other 3 quadrants, in the middle, place a tile.
- ④ Recursively tile other 3 quadrants (they now have a "hole").

# Longest Common Subsequence

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String 1: ACGCTACTGAT

subseq: a subset of items in a sequence in the order they appear.

String 2: GTTACAGCATCATT

A common subsequence is a subsequence that appears in 2 sequences (strings).

Goal: What is the length of the LONGEST common subsequence?

$$\begin{aligned}
 & \text{lcs}(\text{ACGCTACTGAT}) \\
 & \text{GTTACAGCATCAT} = \left. \begin{array}{l} \text{if} \\ \text{match} \end{array} \right\} \\
 & 1 + \text{lcs}(\text{ACGCTACTGA}, \\
 & \quad \text{GTTACAGCATCAT})
 \end{aligned}$$

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$$\text{lcs}(\text{ACGTCA}, \text{CAGGTCA})$$

$$= \text{MAX} \left( \begin{array}{l} \text{lcs}(\text{ACGTCA}, \text{CAGGTC}), \\ \text{lcs}(\text{ACGTCA}, \text{CAGGTCA}) \end{array} \right)$$

↳ When letters don't match!