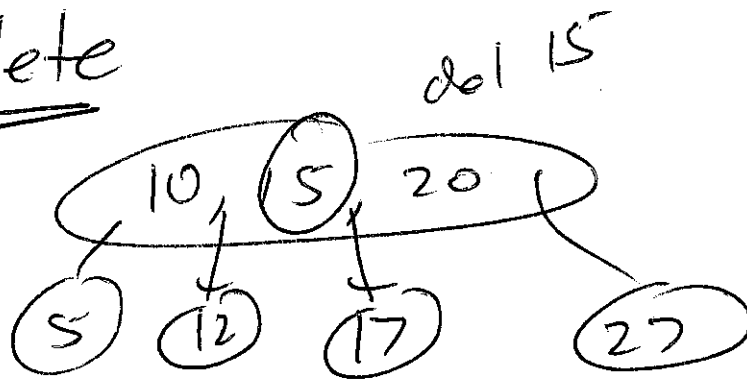


2-4 Trees

2/11/18 ①

Delete



Sweep 15 and 12 (largest value < 15)

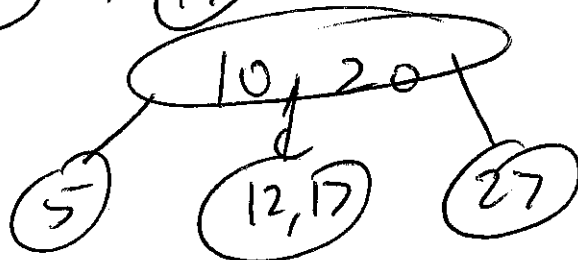


Delete this node (it's on the bottom level)

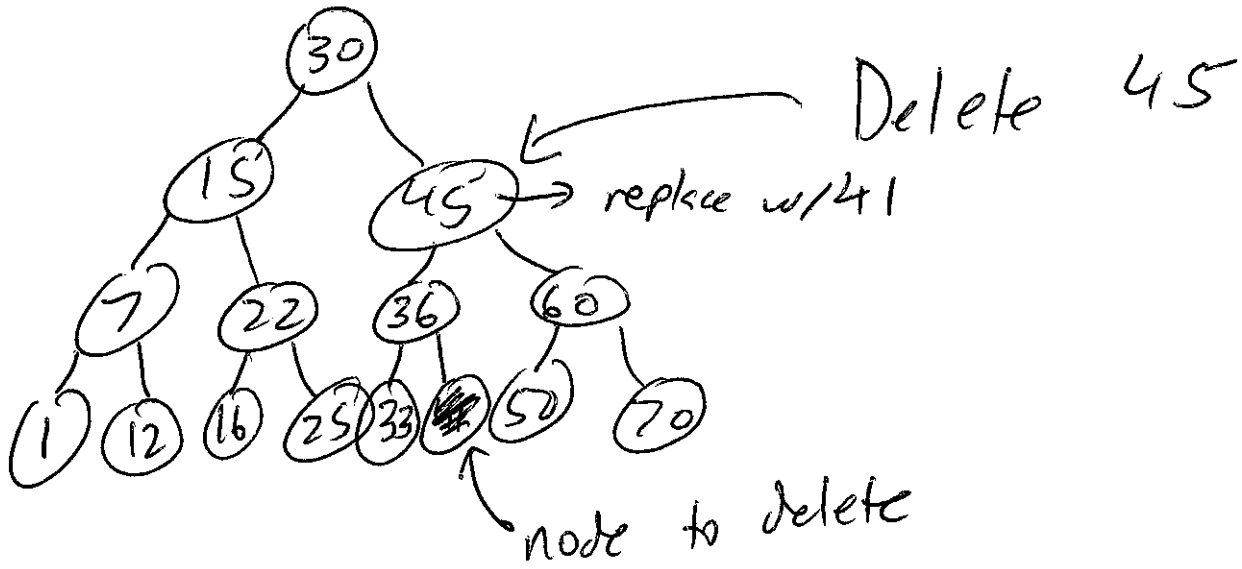
If adjacent sibling has > 1 value,
do a transfer

But, if no adjacent sibling has > 1 value,
we are safe to drop a value from the
parents into the fused node.

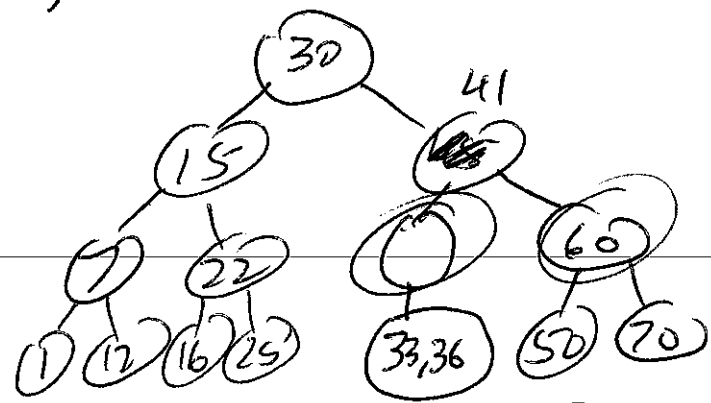
fused = $\emptyset + 17$



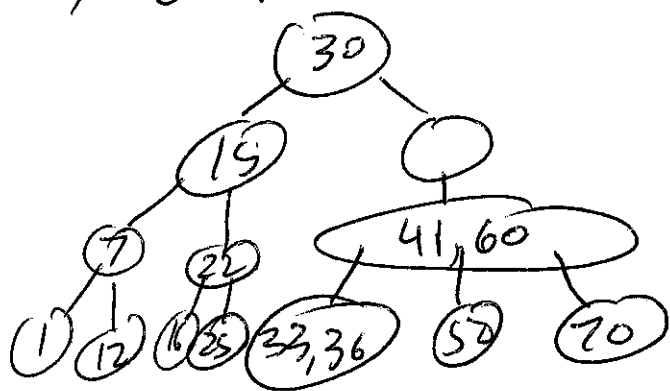
If we drop a value down, the top may become empty! 2/1/18 (2)



- 1) fuse 33 + ~~33~~
- 2) drop 36 down



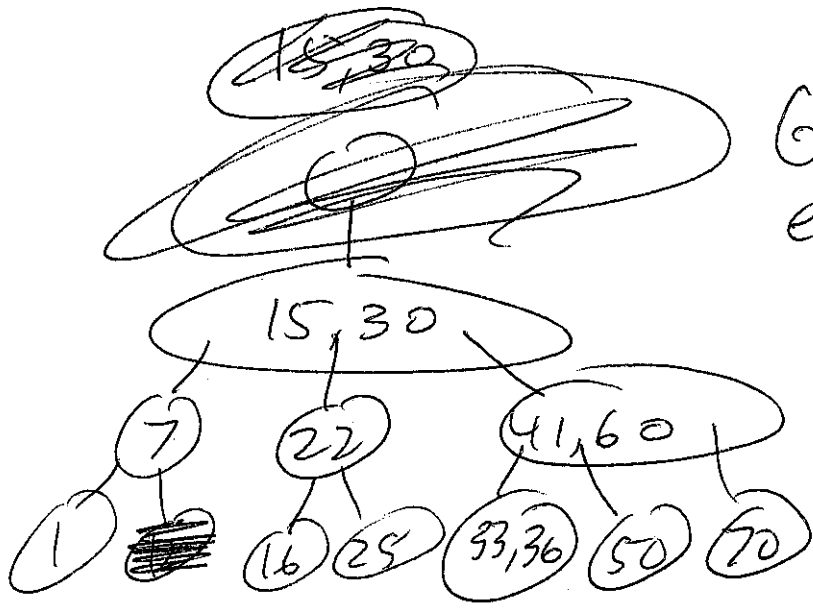
- 3) fuse 0 + 60
- 4) drop 41 down



- 5) fuse 15 + 0
- 6) drop 30

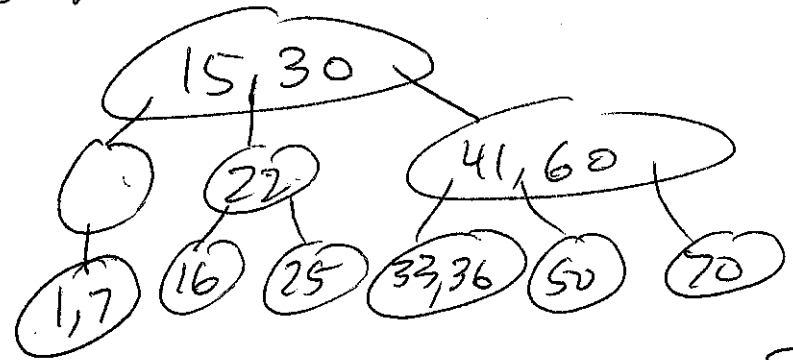
2/1/18 ③

Get rid of empty root

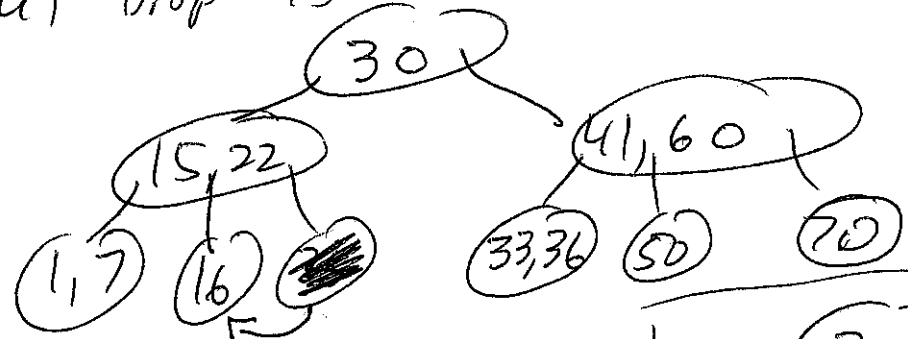


Delete 12

- 1) fuse ① + ~~②~~
- 2) Drop 7 down

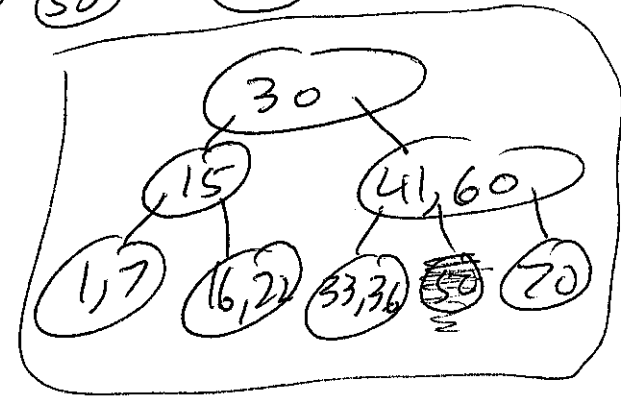


- 3) Sibling has 1 val, fuse $\emptyset + 22$
- 4) Drop 15



Delete 25

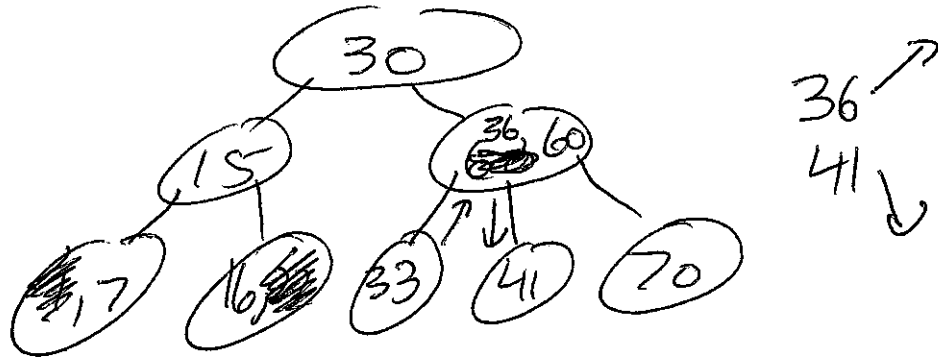
- 1) fuse ⑬ + ~~⑭~~
- 2) Drop 22



Delete 50

2/1/18 (4)

(1) Can transfer because 33,36
adjacent sibling has > 1 value



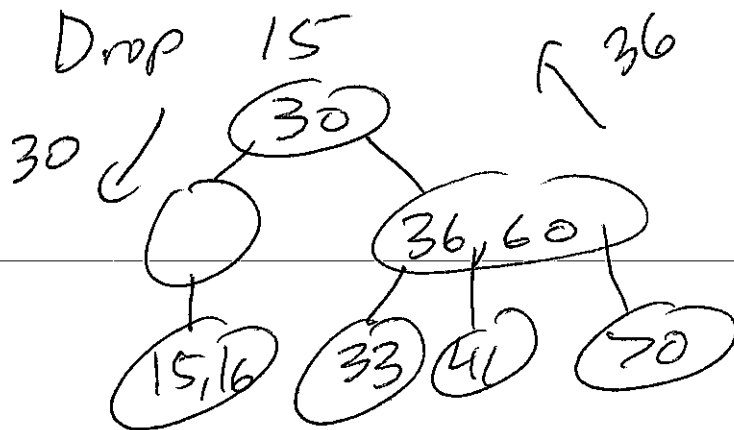
Delete 22

Delete 1

Delete 7

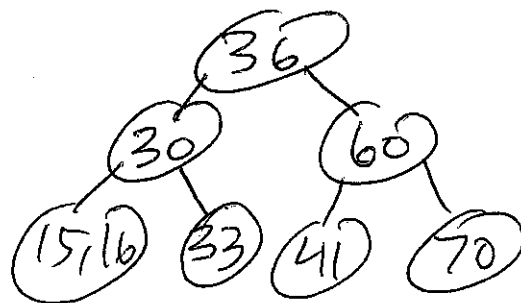
1) fuse $\emptyset + 16$

2) Drop 15



3) Siblings of \emptyset has > 1 value
Adj. 36,60

transfer \rightarrow send 36 up
send 30 down

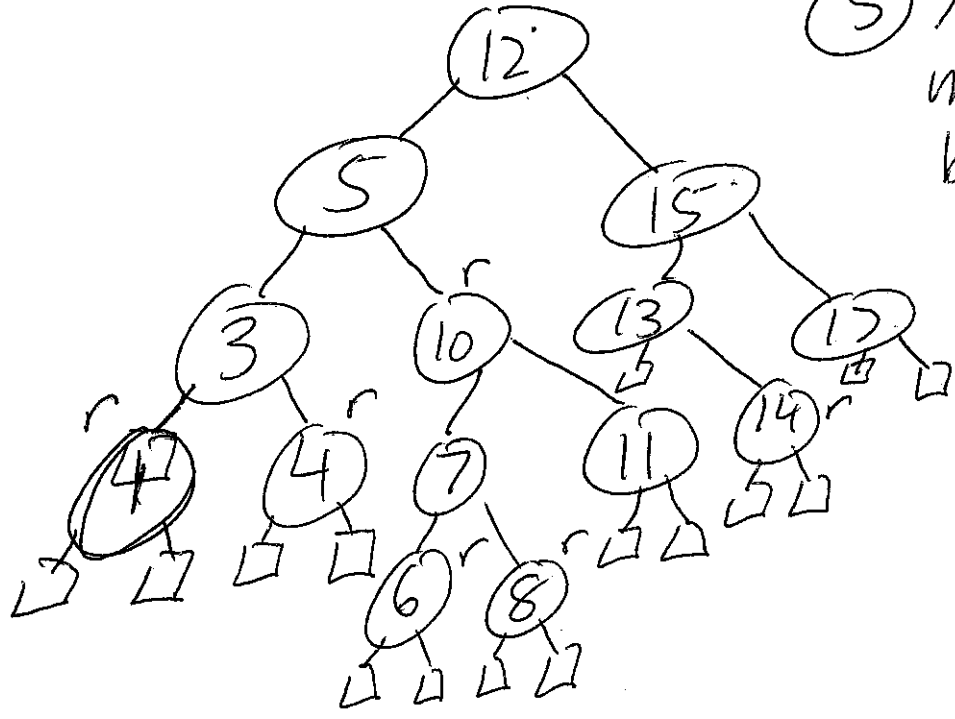


Red - Black Trees

Structure wise exactly like a binary search tree with some added restrictions =

- (1) each node has a color (red/black)
- (2) root is black
- (3) every external node (null children) is black
- (4) The "black depth" of each external node is the same.

(5) All red nodes must have black children.



Insert

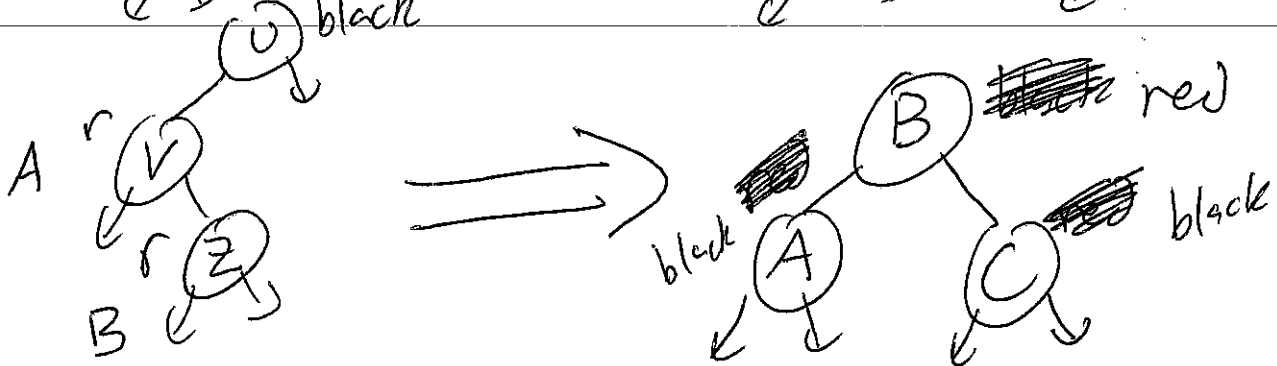
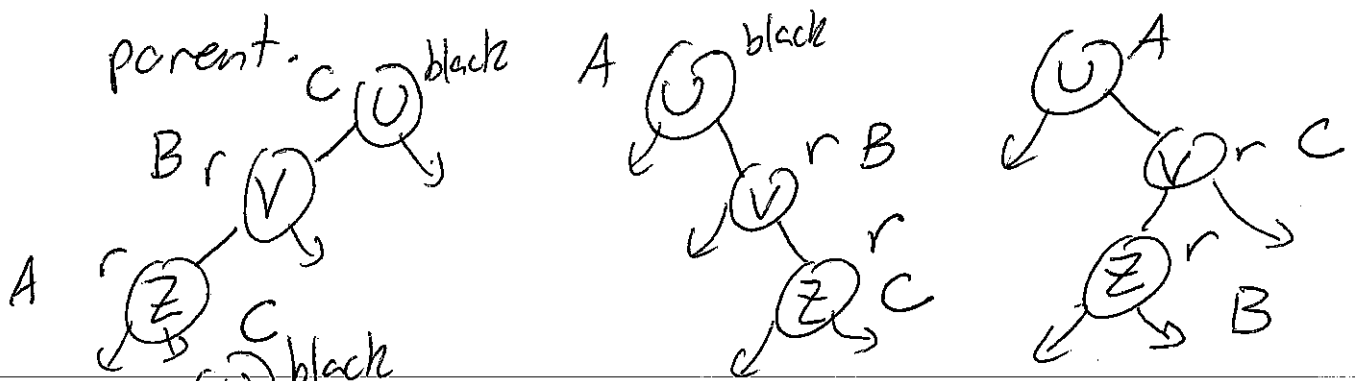
2/1/18⑥

- 1) Easy case - insert as child of a black node
Just make red + stop.

Insert 1

- 2) Harder case - parent is red.
Still make node red at first.

Let z be inserted node, u be parent.



Do a recoloring + restructuring that pushes the red up a level. If it fixes the problem great! If not, repeat!