LINEAR VS BINARY SEARCH

COP 3502

```
// Pre-conditions:  exponent is >= to 0
// Post-conditions: returns base^exponent

int Power(int base, int exponent) {

      if (exponent == 0)
          return 1;
      else
          return (base*Power(base, exponent – 1);
}
```

- To convince you that this works, let's look at an example:
  - Power(5,2):

Power(5,0) : return 1

Power(5,1) : return 5 * Power(5,0)

Power(5,2) : return 5 * Power(5,1)

STACK

1

5 * 1 = 5

5* 5 = 25

Stack trace back to the original function call

Recursion

- Why use recursion?
  - Some solutions are naturally recursive.
    - In these cases there might be less code for a recursive solution, and it might be easier to read and understand.

- Why NOT user recursion?
  - Every problem that can be solved with recursion can be solved iteratively.
  - Recursive calls take up memory and CPU time
    - Exponential Complexity – calling the Fib function uses 2^n function calls.
  - Consider time and space complexity.

Recursion Example

- Let's do another example problem – Fibonacci Sequence
  - 1, 1, 2, 3, 5, 8, 13, 21, …
- Let's create a function **`int Fib(int n)`**
  - we return the nth Fibonacci number
  - Fib(1) = 1, Fib(2) = 1, Fib(3) = 2, Fib(4) = 3, Fib(5) = 5, …
- What would our base (or stopping) cases be?

Fibonacci

- 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, …

- Base (stopping) cases:
  - Fib(1) = 1
  - Fib(2) = 1,
- Then for the rest of the cases: Fib(n) = ?
  - Fib(n) = Fib(n-1) + Fib(n-2), for n>2

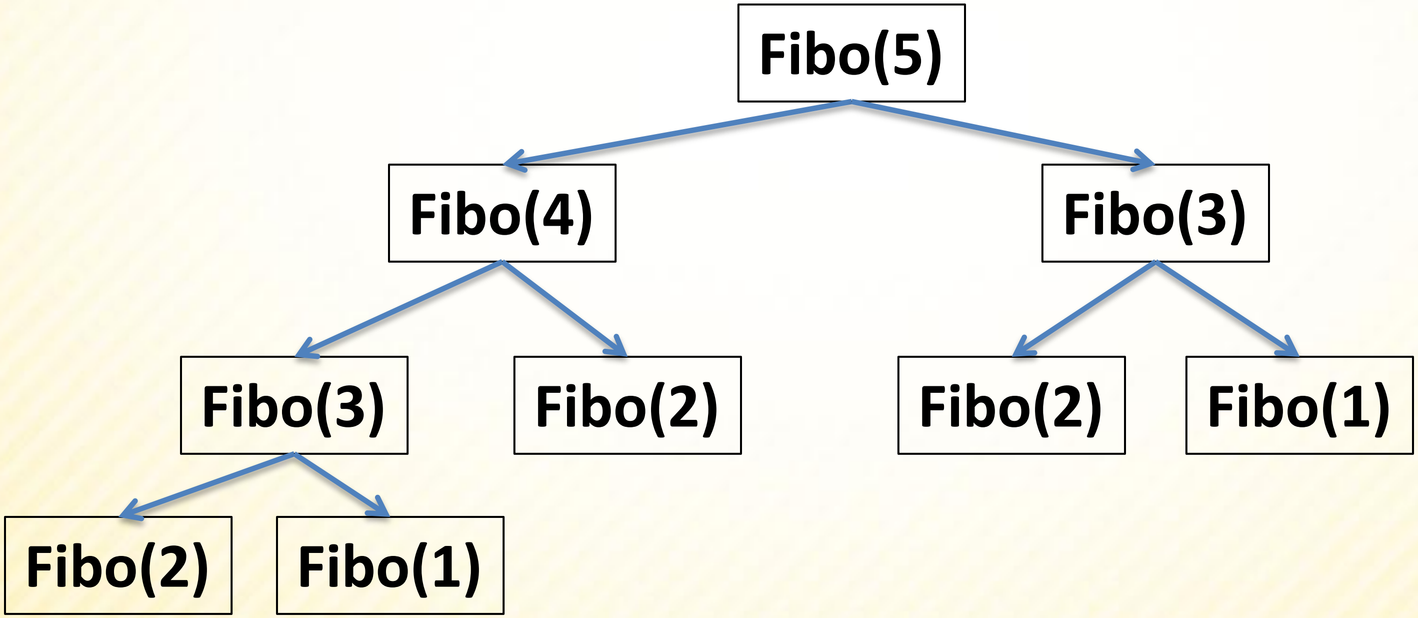
- So Fib(9) = ?
  - Fib(8) + Fib(7) = 21 + 13

Recursion - Fibonacci

- See if we can program the Fibonacci example…

Recursion - Fibonacci

- Let's say we called Fibo(5), we can visualize the calls to Fibo on the stack as a tree:



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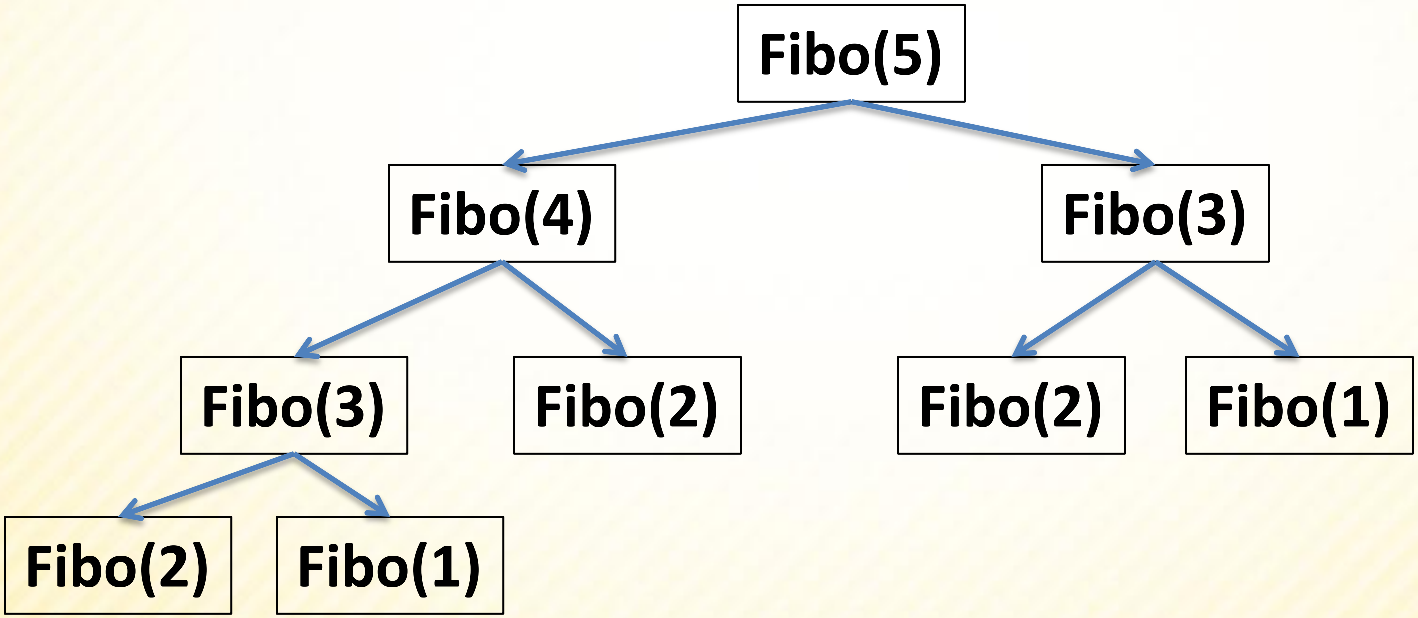

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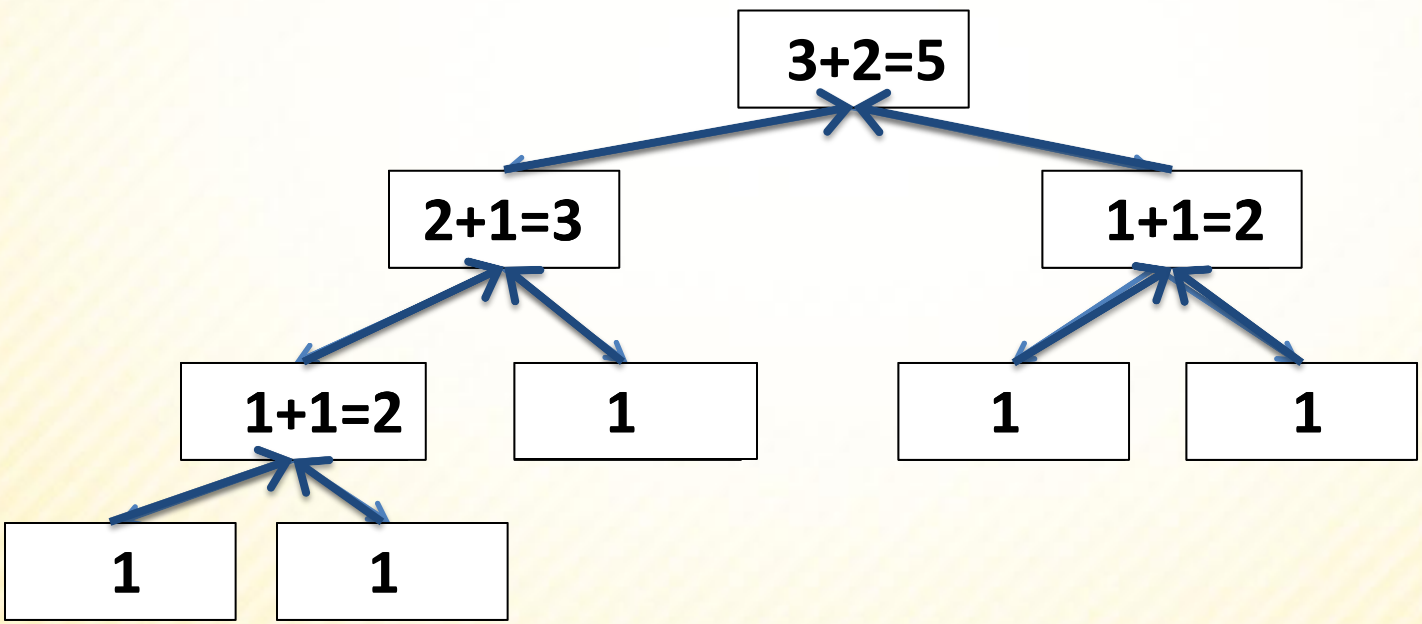


Practice Problem

- Write a recursive function that:
  - Takes in 2 non-negative integers
  - Returns the product
    - Does NOT use multiplication to get the answer

```
int Multiply(int first, int second) {
    if (( second == 0 ) || ( first = 0))
        return 0;
    else
        return (first + Multiply(first, second – 1));
}
```

Linear Search

- In C Programming, we looked at the problem of finding a specified value in an array.
  - The basic strategy was:
    - Look at each value in the array and compare to x
      - If we see that value, return true
      - else keep looking
      - If we're done looking through the array and still haven't found it, return false.

```
int search(int array[], int len, int value) {
    int i;
    for (i = 0; i < len; i++) {
        if (array[i] == value)
            return 1;
    }

    return 0;
}
```

Linear Search

- For an unsorted array, this algorithm is optimal.
  - There's no way you can definitively say that a value isn't in the array unless you look at every single spot.

- But we might ask the question, could we find an item in an array faster if it were already sorted?

```
int search(int array[], int len, int value) {
    int i;
    for (i = 0; i < len; i++) {
        if (array[i] == value)
            return 1;
    }

    return 0;
}
```

Binary Search

- Consider the game you probably played when you were little:
  - I have a secret number in between 1 and 100, make a guess and I'll tell you whether your guess is too high or too low.
  - Then you guess again, and continue guessing until you guess right.
- What would a good strategy for this game be?

Binary Search

- If you divide your search space in half each time,
  - you won’t run the risk of searching ¾ of the list each time.
  - For instance, if you pick 75 for your number, and you get the response “too high”,
  - Then your number is anywhere from 1-74…
- So generally the best strategy is:
  - Always guess the number that is halfway between the lowest possible value in your search range and the highest possible value in your search range.

Binary Search

- How can we adapt this strategy to work for search for a given value in an array?
- Given the array:

	LOW				MID				HI
Index	0	1	2	3	4	5	6	7	8
Value	2	6	19	27	33	37	38	41	118

- Search for 19
  - Where is halfway in between?
  - One guess would be (118+2) / 2 = 60
    - But 60 isn't in the list and the closest value to 60 is 41 almost at the end of the list.
  - We want the middle INDEX of the array.
    - In this case: The lowest index is 0, the highest is 8, so the middle index is 4!

Binary Search

- Searching for 19:

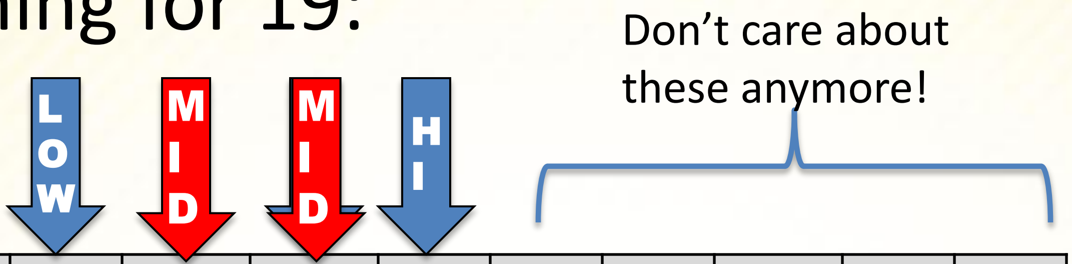
MID

Index	0	1	2	3	4	5	6	7	8
Value	2	6	19	27	33	37	38	41	118

- Now we ask,
  - Is 19 greater than, or less than, the number at index 4?
    - It is Less than, so now we only want to search from index 0 to index 3.

Binary Search

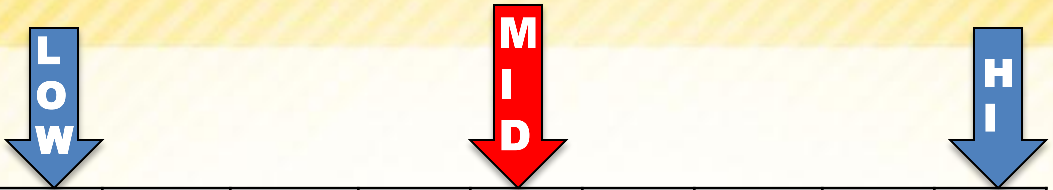
- Searching for 19:



Index	0	1	2	3	4	5	6	7	8
Value	2	6	19	27	33	37	38	41	118

- The middle of 0 and 3 is 1 (since (3+0)/2 = 1)
  - So we look at array[1]
  - And ask is 19 greater than or less than 6?
    - Since it’s greater than 6, we next search halfway between 2 and 3, which is (2+3)/2 = 2
    - At index 2, we find 19!

Binary Search



Index	0	…	…	…	n/2	…	…	…	n
Value	x_1	x_2	$x_{\dots}$	$x_{\dots}$	$x_{\dots}$	$x_{\dots}$	$x_{\dots}$	$x_{\dots}$	x_n

```
int binsearch(int array[], int n, int value) {
    int low = 0, high = n - 1;

    while (low <= high) {

        int mid = (low + high)/2;

        if (value < array[mid])
            high = mid - 1;
        else if (value > array[mid])
            low = mid + 1;
        else
            return 1;
    }
    return 0;
}
```

Efficiency of Binary Search

- Now, let’s analyze how many comparisons (guesses) are necessary when running this algorithm on an array of ***n*** items.

- First, let’s try ***n = 100***:
  - After 1 guess, we have 50 items left,
  - After 2 guesses, we have 25 items left,
  - After 3 guesses, we have 12 items left,
  - After 4 guesses, we have 6 items left,
  - After 5 guesses, we have 3 items left,
  - After 6 guesses, we have 1 item left,
  - After 7 guesses, we have 0 items left.

Also note that when n is odd,
such as when n = 25,
We search the middle element #13,
There are 12 elements smaller than it and 12 larger,
So the number of items left is slightly less than 1/2.

- The reason we have to list that last iteration is because the number of items left represent the number of other possible values to search.
  - We need to reduce this number to 0!

Efficiency of Binary Search

- In the general case we get something like:
  - After **1 guess**, we have **n/2 items** left,
  - After **2 guesses**, we have **n/4 items** left,
  - After **3 guesses**, we have **n/8 items** left,
  - ...
  - After **k guesses**, we have **$n/2^k$ items** left,

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- If we can find the value that makes this fraction 1, then we know that in one more guess we'll narrow down the item:
  - $\frac{n}{2^k} = 1$, ***now we just solve for k (the # of guesses)***

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  - $k = \log_2 n$

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  - $\frac{n}{2^k} = 1$, ***now we just solve for k (the # of guesses)***
  - $n = 2^k$
  - $k = \log_2 n$

- This means that a binary search roughly takes $\log_2 n$ comparisons when search for a value in a sorted array of n items.
  - **This is much much faster than searching linearly!**

Efficiency of Binary Search

- Let's look at a comparison of a linear search to a logarithmic search:

n	log n
8	3
1024	10
65536	16
1048576	20
33554432	25
1073741824	30

RECURSION

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Recursive Binary Search

- The iterative code is not the easiest to read, if we look at the recursive code
  - It's MUCH easier to read and understand

```
int binsearch(int *values, int low, int high, int searchVal) {
    int mid;
    if (!terminating condition){







    }
    return 0;
}
```

Recursive Binary Search

- We need a stopping case:
  - We have to STOP the recursion at some point
- Stopping cases:
  1. We found the number!
  2. Or we have reduced our search range to nothing – the number wasn't found ☹
     - ?? The search range would be empty when low > high

```
int binsearch(int *values, int low, int high, int searchVal) {
    int mid;
    if (low <= high){
        mid = (low+high)/2;
        if (searchVal == values[mid])
            return 1;
        else if (searchVal > values[mid])
            // Do something else
        else
            // Do something
    }
    return 0;
}
```

Recursive Binary Search

- What are our recursive calls going to be?
  - We need to change what low and high are
  - So we get the following:

```
int binsearch(int *values, int low, int high, int searchVal) {
    int mid;
    if (low <= high){
        mid = (low+high)/2;
        if (searchVal == values[mid])
            return 1;
        else if (searchVal > values[mid])
            // Do something else
        else
            // Do something
    }
    return 0;
}
```

Recursive Binary Search

- Binary Search Code summary (using recursion):
  - If the value is found,
    - return 1
  - Otherwise
    - `if (searchVal > values[mid])`
      - `Recursively call binsearch to the right`
    - `else if (searchVal < values[mid])`
      - `Recursively call binsearch to the left`
  - `If low is ever greater than high`
    - `The value is not in the array return 0`

Why Recursion?

- Recursion – behind the scenes
  - Every time we recurse, we are doing another function call, this results in manipulating the run-time stack in memory, passing parameters, and transferring control
    - So recursion costs us both in time and memory usage

Why Recursion?

Recursive Solution

```
int fact(int n) {
   if (n==1)
        return 1;
   return n*fact(n-1);
}
```

Iterative Solution

```
int fact(int n) {
   int result = 1;
   while (n > 1) {
        result *= n--;
   }
}
```

- More elegant – easier to read:
- But we aren't seeing the stack manipulations which require:
  - pushing a new n,
  - space for the function's return value, and updating the stack pointer register
  - and popping off the return value and n when done

Why Recursion?

- If recursion is harder to understand and less efficient, why use it?
  - It leads to elegant solutions – less code, less need for local variables, etc
  - If we can define a function mathematically, the solution is easy to codify
  - Some problems require recursion
    - Tree traversals
    - Graph traversals
    - Search problems
    - Some sorting algorithms (quicksort, mergesort)
      - Note: this is not strictly speaking true, we can accomplish a solution without recursion by using iteration and a stack, but in effect we would be simulating recursion, so why not use it?
    - In some cases, an algorithm with a recursive solution leads to a lesser computational complexity than an algorithm without recursion
      - Compare Insertion Sort to Merge Sort for example

Practice Problem

- Write a recursive function that:
  - Takes in 2 non-negative integers
  - Returns the product
    - Does NOT use multiplication to get the answer
  - So if the parameters are 6 and 4
    - We get 24
    - Not using multiplication, we would have to do 6+6+6+6