HEAPS

COP 3502

Binary Heaps

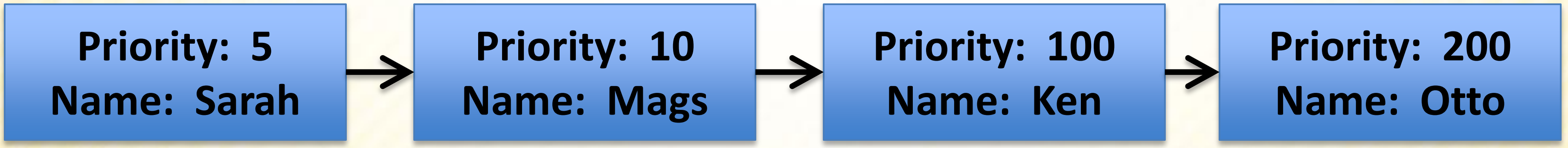
- Binary heaps are used for two purposes:
  - Priority Queues
  - Heap sort

Binary Heaps

- Priority Queue
  - A priority queue is where you always extract the item with the highest priority next.
- Priority Queue Example
  - Let’s say we are Google and we want an efficient way to do determine which applicant from our applicant pool to interview when a new position opens up.
  - So we assign a priority based on a particular formula – including application arrival time, GPA, and understanding of Heaps, ironically enough.

Binary Heaps

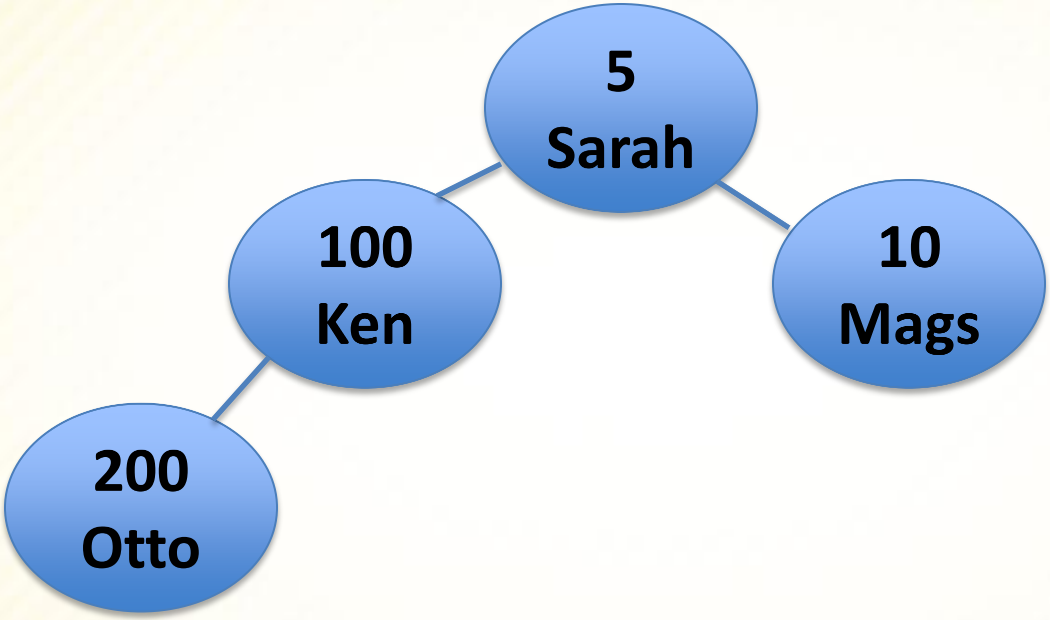
- How could we implement this using our existing methods?
  - We don't want just a normal queue, because that's FIFO, doesn't care about a priority value.
  - We could use a linked list sorted by priority.
    - Then we would have a long insertion time for insert, because we have to traverse the list to find where our element goes.



  - This isn't necessary, because all we care about is the next applicant to interview, not that the list is sorted.

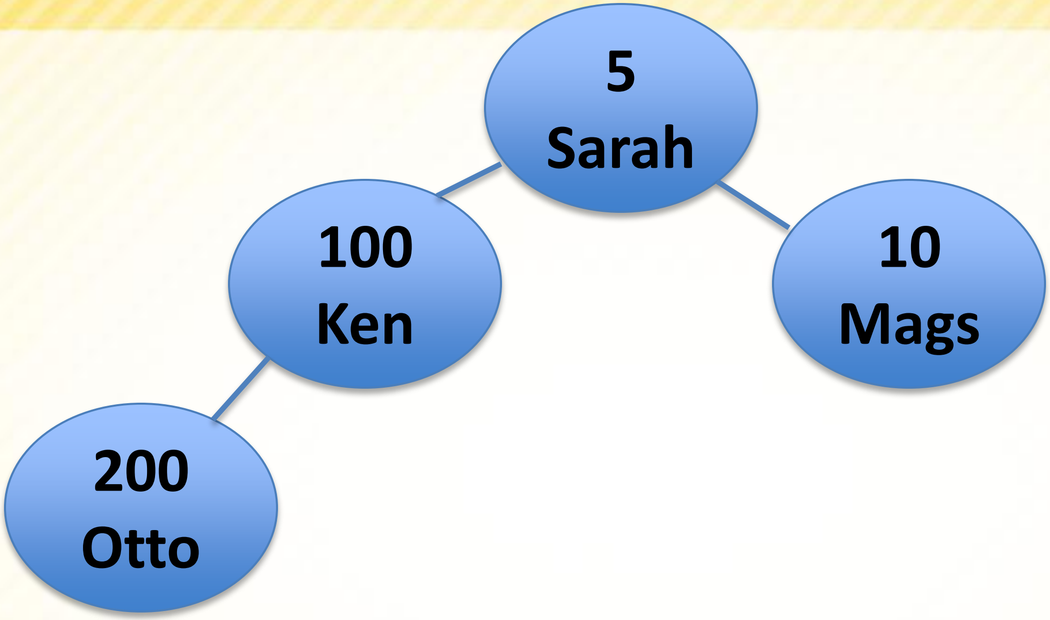
Binary Heaps

- Consider a minimum binary heap:



  - Looks similar to a binary search tree
  - BUT all the values stored in the subtree rooted at a node are greater than or equal to the value stored at the node.

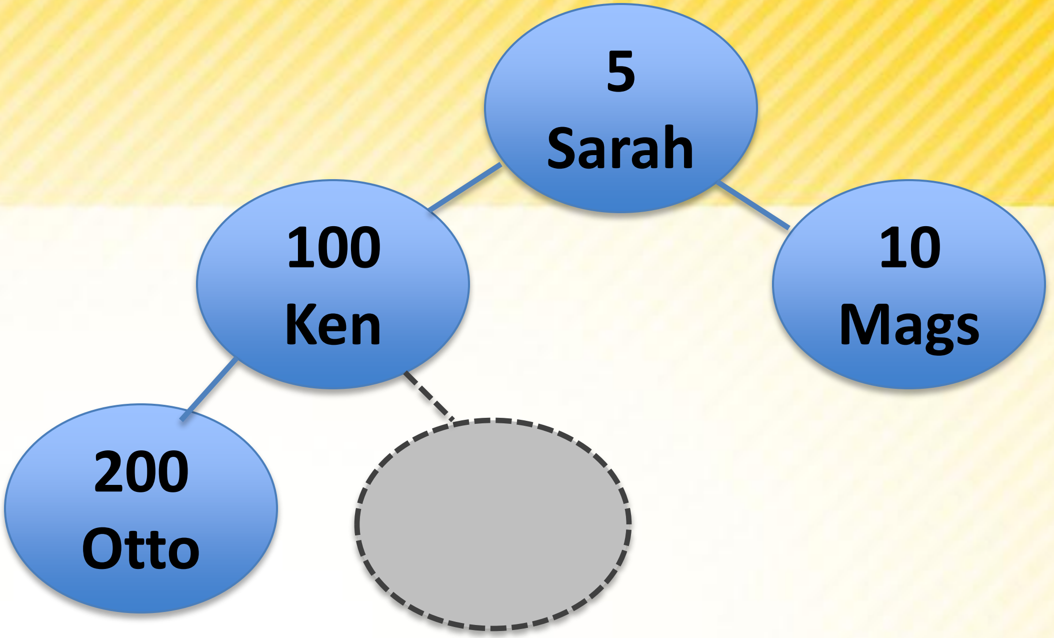
Binary Heaps



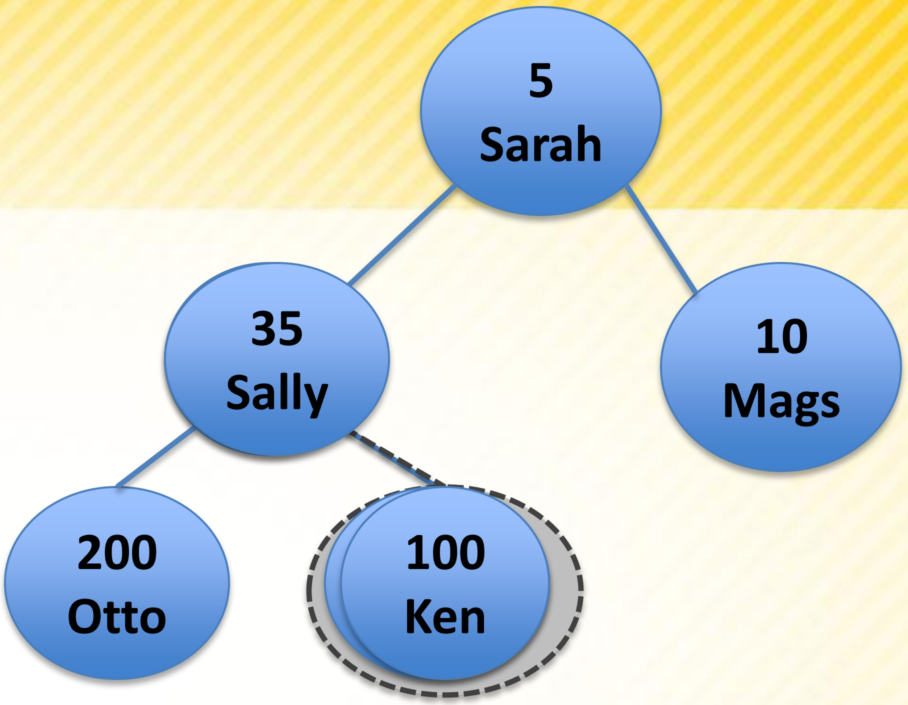
- The only operations we need are:
  - **Insert** and **RemoveMin**
  - We can implement a heap using a complete binary tree or an array as we will talk about later.
  - No matter how we implement it, we will visualize the data structure as a tree, like the one above.

Insert

- Since we want a complete binary tree
  - We insert the new node into the next empty spot
  - Filling each level from left to right
  - Then we need to worry about where this node should move to depending on its priority.



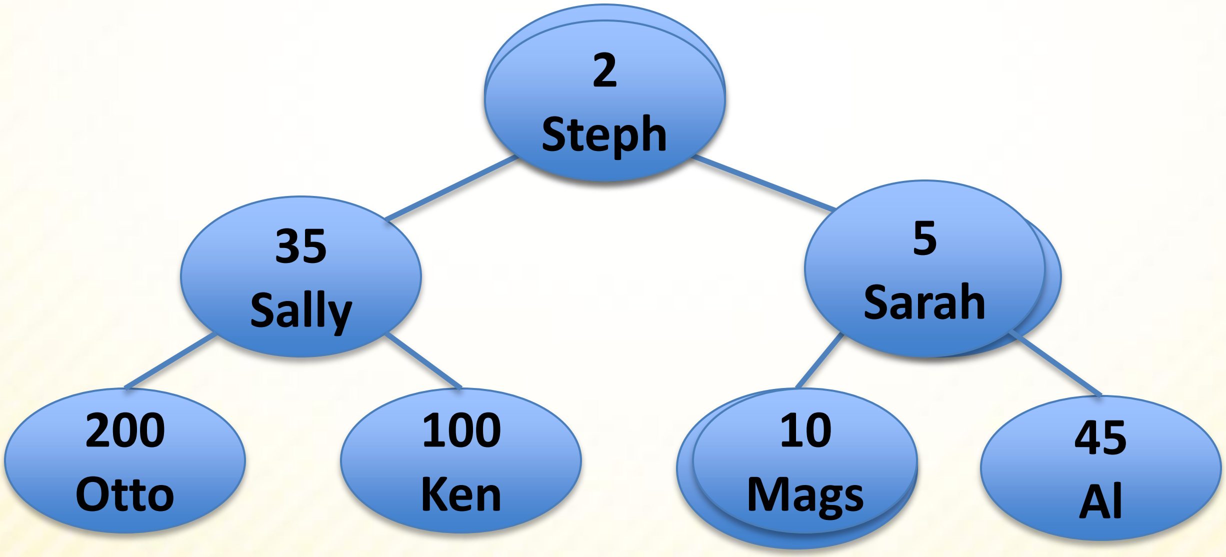
Insert



- The problem is in all likelihood, if the insertion is done in this location, the heap property will not be maintained.
- Thus, you must do the following "Percolate Up" procedure:
  - If the parent of the newly inserted node is greater than the inserted value, swap the two of them.
  - This is a single "Percolate Up" step.
  - Now, continue this process until the inserted node 's parent stores a number lower than it.

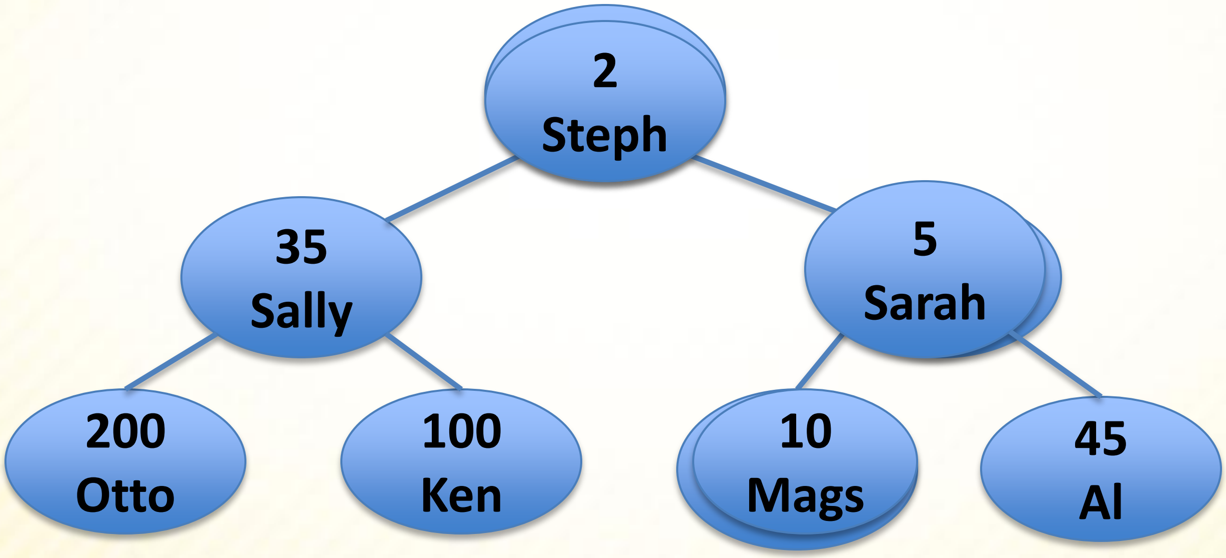
Insert

- **<u>Percolate Up:</u>**
  - If the parent of the newly inserted node is greater than the inserted value, swap the two of them.
  - Now, continue this process until the inserted node 's parent stores a number lower than it.



Heap Implementation

- <u>Array Implementation:</u>
  - Instead of using a binary tree implementation,
  - We can use an array implementation where the children of the node at index ***i*** are the nodes at indices ***2i*** and ***2i+1***.

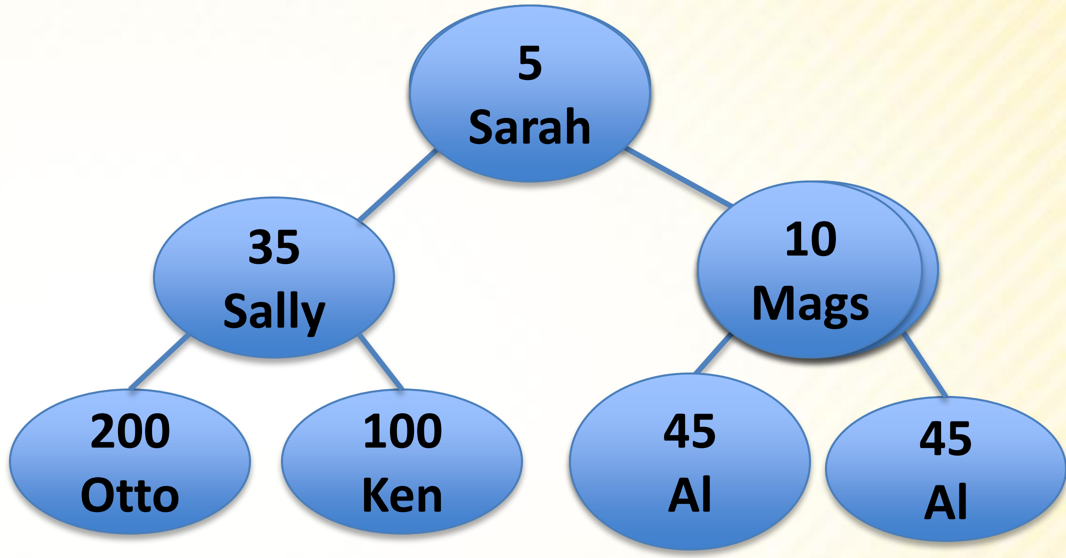


index	0	1	2	3	4	5	6	7	…n
node	X	2 Steph	35 Sally	5 Sarah	200 Otto	100 Ken	10 Mags	45 Al	

Delete Minimum

- Delete the min (which is always the root), and return:

- Now we need to replace it, but with what?
  - ➢Replace with the last element in the array, or the last node added to the tree.
- Then Percolate Down.



<u>Percolate Down:</u>

If the children of this node has children less than it swap it with the MIN of its 2 children, until the node has children that are larger than it.

Runtime of heap operations

- What is the height of a complete binary tree?
  - (Where we completely fill each level from left to right)
  - The maximum height is $h = \log_2((n+1)/2)$
- Insert
  - So for insert the maximum number of times we can swap a node up the tree in a tree with n nodes would be
    - $\log_2((n+1)/2)$
- DeleteMin
  - And for delete the maximum number of times we can swap a node down the tree in a tree with n nodes would be
    - $\log_2((n+1)/2)$
- So, Insert and Delete are both **O(log n)**

Heapify

- Bottom up heap construction
  - How to construct a heap out of an unsorted array of elements.
    - **1) Place all the unsorted elements in a complete binary tree.**
    - **2) Go through the nodes of the tree in backwards order running Percolate Down on each of these nodes. (Skip over all leaf nodes.)**
- Shown on the board

Heapify

- Bottom up heap construction
  - As this is done, one invariant we see is that each subtree below any node for which Percolate Down has already executed is a heap.
    - Thus, when we run Percolate Down on the root at the end of this algorithm, the whole tree is one heap.
  - Can you see why we can NOT go through the nodes in forward order?
    - Give an example where doing so produces a tree that is not a heap.

Heapify Analysis

- Shown on the board

Heapsort

- Shown on the board
  - Now that we have determined how to execute several operations on a heap, we can use these to sort values using a heap. Here is the idea:
    - 1) Insert all items into a heap
    - 2) Extract the minimum item n times in a row, storing the values sequentially in an array.
  - Since each inserting and extraction take O(lg n) time, this sort works in O(n lg n) time.