



# **SORTING**

COP 3502

# Sorting a List

- Let's say we have a list of the names of people in the class and we want to sort alphabetically
  - We are going to describe an algorithm (or systematic methods) for putting these names in order
  - The algorithms we will cover today:
    - Selection Sort      Insertion Sort      Bubble Sort



BOB



JOE



ABE



SAM



ANN





# Sorting a List

## ■ Selection Sort

- Finds the smallest element (alphabetically the closest to a)
  - Swaps it with the element in the first position
- Then finds the second smallest element
  - Swaps it with the element in the second position
- Etc. until we get to the last position, and then we're done!



BOB



JOE



ABE



SAM



ANN

# Sorting a List

## ■ Selection Sort

Min = "Abe"

C  
U  
R  
R

C  
H  
E  
C  
K  
E  
R

C  
H  
E  
C  
K  
E  
R

C  
H  
E  
C  
K  
E  
R

C  
H  
E  
C  
K  
E  
R

"Joe" < "Bob"?

"Abe" < "Bob"?

"Sam" < "Abe"?

"Ann" < "Abe"?



BOB



JOE



ABBE



SAM



ANN



# Sorting a List

## ■ Selection Sort

- Finds the smallest element (alphabetically the closest to a)
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ABE



JOE



BOB



SAM

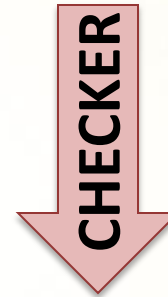
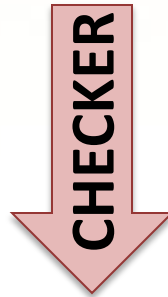
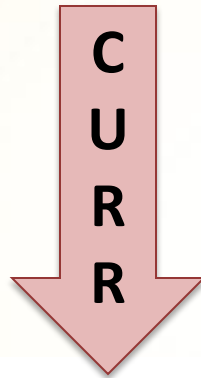


ANN

# Sorting a List

## ■ Selection Sort

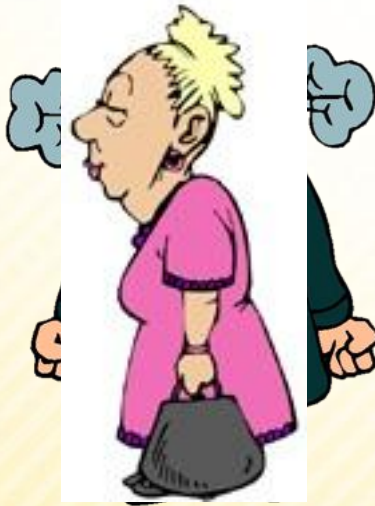
Min = "Ann"



"Bob" < "Joe"? "Sam" < "Bob"? "Ann" < "Bob"?



ABE



ANN



BOB



SAM



JOE



# Sorting a List

## ■ Selection Sort

Min = "Bob"

C  
U  
R  
R

C  
H  
E  
C  
K  
E  
R

C  
H  
E  
C  
K  
E  
R

"Sam" < "Bob"? "Joe" < "Bob"?"



ABE



ANN



BOB



SAM



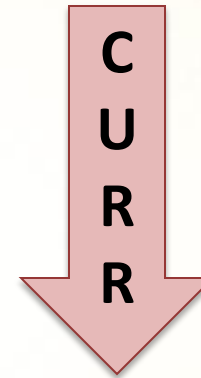
JOE

# Sorting a List

## ■ Selection Sort

Min = "Joe"

Notice that now the list is sorted!  
So we can stop when **Curr** is on the  
2<sup>nd</sup> to last element.



"Joe" < "Sam"?



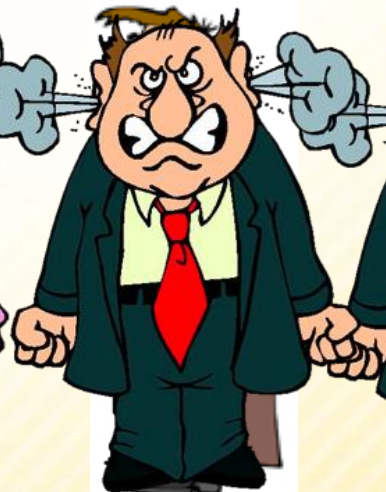
ABE



ANN



BOB



SAM



JOE



# Sorting a List

## ■ Insertion Sort

- Take each element one by one, starting with the second and “insert” it into the already sorted list to its left in the correct order.



BOB



JOE



ABE



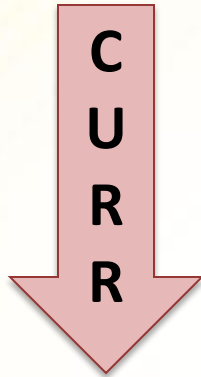
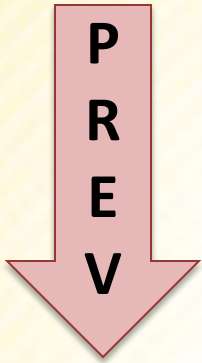
SAM



ANN

# Sorting a List

## ■ Insertion Sort



“Joe” < “Bob”?

Pos = 1



BOB



JOE



ABE



SAM



ANN



# Sorting a List

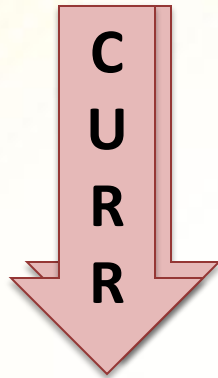
## ■ Insertion Sort



"Abe" < "Bob"?



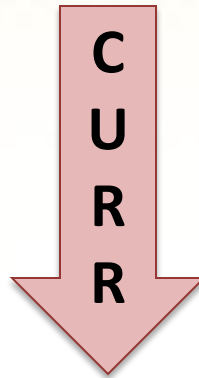
BBB



"Abe" < "Joe"?



BBB



Pos = 2



ABBE



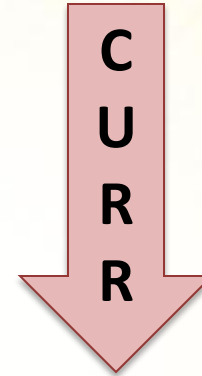
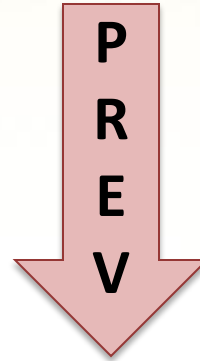
SAM



ANN

# Sorting a List

## ■ Insertion Sort



"Sam" < "Joe"?

Pos = 3



ABE



BOB



JOE



SAM

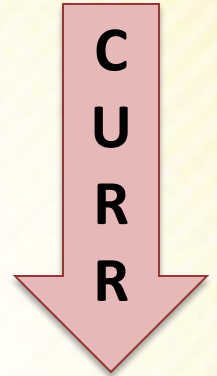
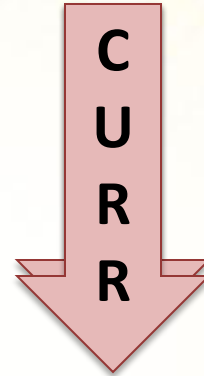
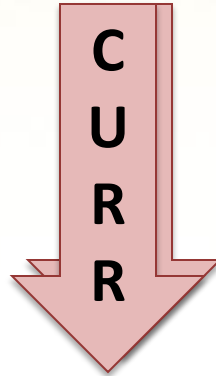
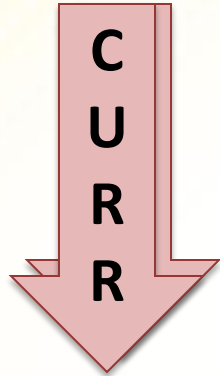
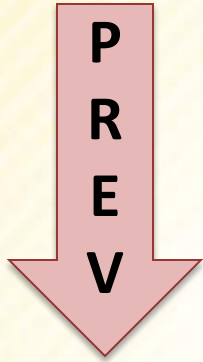


ANN



# Sorting a List

## ■ Insertion Sort



“Ann” < “Abe”? “Ann” < “Bob”? “Ann” < “Joe”? “Ann” < “Sam”?

Pos = 4



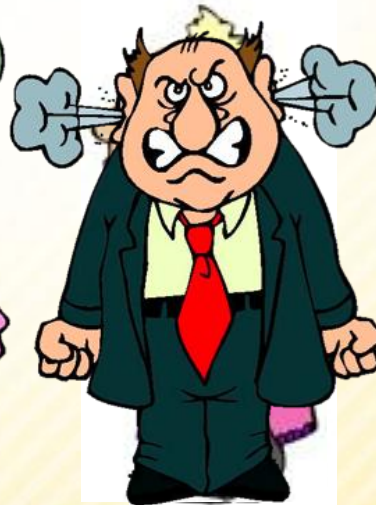
ABE



ANN



BOB



JOE



SAM

# Sorting a List

## Bubble Sort

- The basic idea behind bubble sort is that you always compare consecutive elements, going left to right.
  - Whenever two elements are out of place, swap them.
  - At the end of a single iteration, the max element will be in the last spot.
- Now, just repeat this n times
- On each pass, one more maximal element will be put in place.
- As if the maximum elements are slowly “*bubbling*” up to the top.



BOB



JOE



ABE



SAM

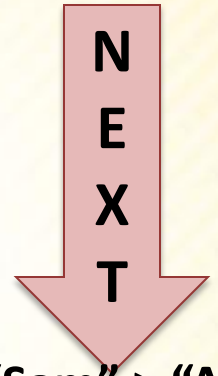
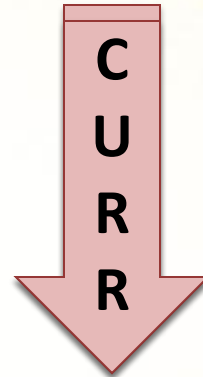
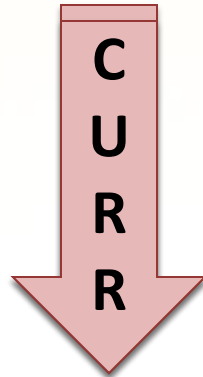
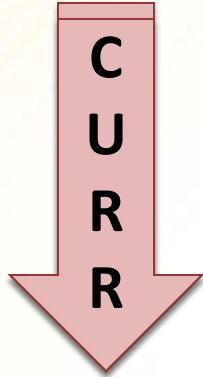
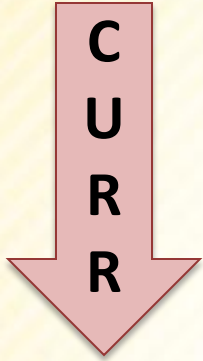


ANN



# Sorting a List

## ■ Bubble Sort



"Bob" > "Joe"?

"Joe" > "Abe"?

"Joe" > "Sam"? "Sam" > "Ann"?



**BOB**



**ABE**



**ABE**



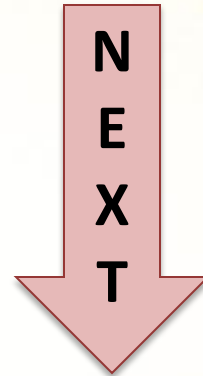
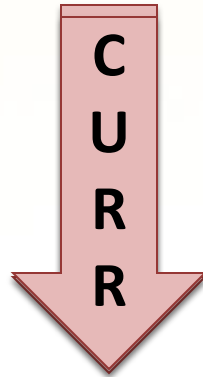
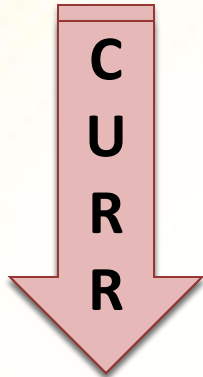
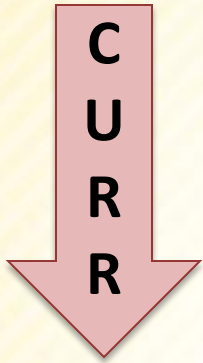
**SAM**



**SAM**

# Sorting a List

## ■ Bubble Sort



"Bob" > "Abe"?

"Rob" > "Joe"?

"Joe" > "Ann"?



BBB



BBB



ADEN



ADEN

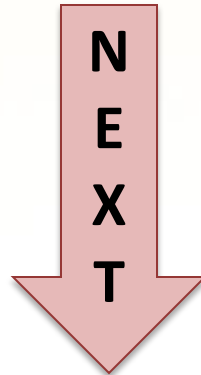
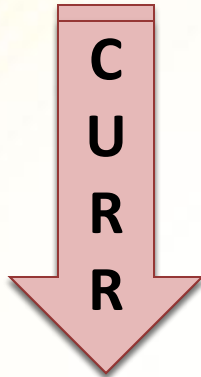
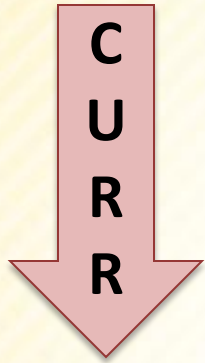


SAM



# Sorting a List

## ■ Bubble Sort



"Abe" > "Bob"?

"Bob" > "Ann"?



ABE



ANN



BOB



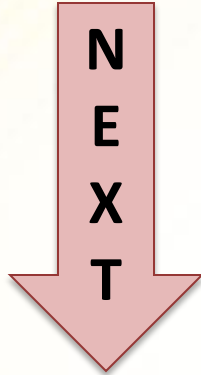
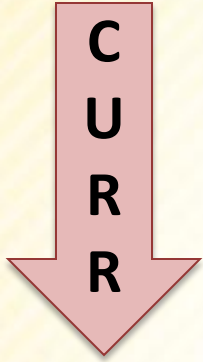
JOE



SAM

# Sorting a List

## ■ Bubble Sort



"Abe" > "Anne"?



ABE



ANN



BOB



JOE



SAM



# Limitation of Sorts that only swap adjacent elements

- A sorting algorithm that only swaps adjacent elements can only run so fast.
  - In order to see this, we must first define an inversion:
    - An inversion is a pair of numbers in a list that is out of order.
    - In the following list: 3, 1, 8, 4, 5
    - the inversions are the following pairs of numbers: (3, 1), (8, 4), and (8, 5).
  - When we swap adjacent elements in an array, we can remove at most one inversion from that array.



# Limitation of Sorts that only swap adjacent elements

- Note that if we swap non-adjacent elements in an array, we can remove multiple inversions. Consider the following:
  - 8 2 3 4 5 6 7 1
    - Swapping 1 and 8 in this situation removes every inversion in this array (there are 13 of them total).
- Thus, the run-time of an algorithm that swaps adjacent elements only is constrained by the total number of inversions in an array.





# Limitation of Sorts that only swap adjacent elements

- Let's consider the average case.
  - There are  $\binom{n}{2} = \frac{(n-1)n}{2}$  pairs of numbers in a list of  $n$  numbers.
    - Of these pairs, on average, half of them will be inverted.
  - Thus, on average, an unsorted array will have  $\frac{(n-1)n}{4} = \Omega(n^2)$  number of inversions,
    - and any sorting algorithm that swaps adjacent elements only will have a  $\Omega(n^2)$  run-time.

