Binary   Decimal   Octal   and   Hexadecimal number systems

Conversion of binary to decimal ( base 2 to base 10)

Example:  convert (1000100)\textsubscript{2} to decimal

\[
= 64 + 0 + 0 + 0 + 4 + 0 + 0
\]

\[
= (68)\text{10}
\]

Conversion of decimal to binary ( base 10 to base 2)

Example:  convert (68)\text{10} to binary

\[
\begin{align*}
68/2 & = 34 \text{ remainder is 0} \\
34/2 & = 17 \text{ remainder is 0} \\
17/2 & = 8 \text{ remainder is 1} \\
8/2 & = 4 \text{ remainder is 0} \\
4/2 & = 2 \text{ remainder is 0} \\
2/2 & = 1 \text{ remainder is 0} \\
1/2 & = 0 \text{ remainder is 1}
\end{align*}
\]

Answer = 1 0 0 0 1 0 0
Conversion of decimal fraction to binary fraction

• Instead of division, multiplication by 2 is carried out and the integer part of the result is saved and placed after the decimal point.

   The fractional part is again multiplied by 2 and the process repeated.

**Example:** convert \((0.68)_{10}\) to binary fraction.

\[
\begin{align*}
0.68 \times 2 &= 1.36 \quad \text{integer part is 1} \\
0.36 \times 2 &= 0.72 \quad \text{integer part is 0} \\
0.72 \times 2 &= 1.44 \quad \text{integer part is 1} \\
0.44 \times 2 &= 0.88 \quad \text{integer part is 0}
\end{align*}
\]

Answer = 0.1010…

**Example:** convert \((68.68)_{10}\) to binary equivalent.

Answer = 1000100.1010….
Octal Number System

• Base or radix 8 number system.

• 1 octal digit is equivalent to 3 bits.

• Octal numbers are 0 to 7. (see the chart down below)

• Numbers are expressed as powers of 8.

Conversion of octal to decimal (base 8 to base 10)

Example: convert (632)₈ to decimal

= (6 x 8²) + (3 x 8¹) + (2 x 8⁰)

= (6 x 64) + (3 x 8) + (2 x 1)

= 384 + 24 + 2

= (410)₁₀

Conversion of decimal to octal (base 10 to base 8)
*Example:* convert \((177)_10\) to octal

\[
\begin{align*}
177 / 8 &= 22 \text{ remainder is } 1 \\
22 / 8 &= 2 \text{ remainder is } 6 \\
2 / 8 &= 0 \text{ remainder is } 2
\end{align*}
\]

Answer = \(2\ 6\ 1\)

Note: the answer is read from bottom to top as \((261)_8\), the same as with the binary case.

Conversion of decimal fraction to octal fraction is carried out in the same manner as decimal to binary except that now the multiplication is carried out by 8.
## Decimal, Binary, Octal, and Hex Numbers

<table>
<thead>
<tr>
<th>Decimal</th>
<th>Binary</th>
<th>Octal</th>
<th>Hexadecimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0000</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0001</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0010</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>0011</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>0100</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>0101</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>0110</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>0111</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>1000</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>1001</td>
<td>11</td>
<td>9</td>
</tr>
<tr>
<td>10</td>
<td>1010</td>
<td>12</td>
<td>A</td>
</tr>
<tr>
<td>11</td>
<td>1011</td>
<td>13</td>
<td>B</td>
</tr>
<tr>
<td>12</td>
<td>1100</td>
<td>14</td>
<td>C</td>
</tr>
<tr>
<td>13</td>
<td>1101</td>
<td>15</td>
<td>D</td>
</tr>
<tr>
<td>14</td>
<td>1110</td>
<td>16</td>
<td>E</td>
</tr>
<tr>
<td>15</td>
<td>1111</td>
<td>17</td>
<td>F</td>
</tr>
</tbody>
</table>
**Hexadecimal Number System**

• Base or radix 16 number system.

• 1 hex digit is equivalent to 4 bits.

• Numbers are 0, 1, 2…..8, 9, A, B, C, D, E, F.

  B is 11, E is 14

• Numbers are expressed as powers of 16.

  \[16^0 = 1, \ 16^1 = 16, \ 16^2 = 256, \ 16^3 = 4096, \ 16^4 = 65536, \ldots\]

**Conversion of hex to decimal (base 16 to base 10)**

*Example:* convert \((\text{F4C})_{16}\) to decimal

\[= (\text{F} \times 16^2) + (4 \times 16^1) + (\text{C} \times 16^0)\]

\[= (15 \times 256) + (4 \times 16) + (12 \times 1)\]
Conversion of decimal to hex (base 10 to base 16)

Example: convert \((4768)_{10}\) to hex.

\[
\begin{align*}
&= 4768 / 16 = 298 \text{ remainder } 0 \\
&= 298 / 16 = 18 \text{ remainder } 10 \,(A) \\
&= 18 / 16 = 1 \text{ remainder } 2 \\
&= 1 / 16 = 0 \text{ remainder } 1
\end{align*}
\]

Answer: 1 2 A 0

Note: the answer is read from bottom to top, same as with the binary case.

\[
\begin{align*}
&= 3840 + 64 + 12 + 0 \\
&= (3916)_{10}
\end{align*}
\]
Conversion of binary to octal and hex

• Conversion of binary numbers to octal and hex simply requires grouping bits in the binary numbers into groups of three bits for conversion to octal and into groups of four bits for conversion to hex.

• Groups are formed beginning with the LSB and progressing to the MSB.

• Thus, $111001112 = 3478$

• $11100101010010012 = 30252218$

• $11100112 = E716$

• $1100101010000112 = 18A8716$