

Summer 2020 COP 3502 Section 2 Final Exam - Part C Solutions

1) (10 pts) What are the **worst case run times** of each of the following operations? Make sure to list your answer in terms of the appropriate variables in the prompt. Note that on occasion, some of the run times won't be dependent on some of the variables listed in the prompt.

- (a) Inserting an item to the front of a linked list of n elements. $O(1)$
- (b) Sorting n integers using Quick Sort. $O(n^2)$
- (c) Merging a sorted list of a elements with a sorted list of b elements. $O(a+b)$, $O(\max(a,b))$
- (d) Inserting an item into a binary heap of n elements. $O(\lg n)$
- (e) Deleting an item from a binary search tree of n elements. $O(n)$
- (f) Deleting an item from an AVL tree of n elements. $O(\lg n)$
- (g) Printing out each permutation of n elements, where printing one value takes $O(1)$ time. $O(n*n!)$, $O((n+1)!)$
- (h) Calculating $a^b \bmod c$ where individual multiplications and mods take $O(1)$ time, using fast modular exponentiation. $O(\lg b)$
- (i) A floodfill on a grid with r rows and c columns. $O(rc)$
- (j) Dequeuing an item from a queue of n elements. $O(1)$

Grading: 1 pt per item, all or nothing for each.

2) (5 pts) An algorithm answers a query on a database of n elements in $O(\sqrt{n})$ time. For a database of size $n = 10^6$, it takes 2 seconds to perform 5,000 queries. How many seconds should it take to answer 3,000 queries on a database of size $n = 10^8$? (Note: all credit is based on the work and not the answer.)

Solution

Let $T(n)$ be the run time of the algorithm for a single query. Then $T(n) = c\sqrt{n}$, for some constant c . Solve for c using the given information:

$$\begin{aligned} 5000T(1000000) &= 5000c\sqrt{1000000} = 2 \text{ sec} \\ (5000)(1000)c &= 2 \text{ sec} \\ c &= \frac{2}{5} \times 10^{-6} \text{ sec} \end{aligned}$$

Now, let's solve for how long 3,000 queries will take on a database with size 10^8 :

$$3000T(100000000) = 3000c\sqrt{100000000} = (3000)c(10000) = 3 \times 10^7 \times \frac{2}{5} \times 10^{-6} \text{sec}$$

$$= \frac{6}{5} \times 10 \text{ sec} = \mathbf{12 \text{ sec}}$$

Grading: 2 pts to find c, 1 pt set up to answer question, 1 pt for answer unsimplified, 1 pt to simplify down to 12 sec.

3) (10 pts) Determine the following summation in terms of n. Please express your answer in the form, $an^2 + bn + c$, for constants a, b and c.

$$\sum_{i=n}^{3n-1} (5i + 7)$$

Solution

$$\begin{aligned} \sum_{i=n}^{3n-1} (5i + 7) &= 5 \sum_{i=n}^{3n-1} i + \left(\sum_{i=n}^{3n-1} 7 \right) \\ &= 5 \left(\sum_{i=1}^{3n-1} i - \sum_{i=1}^{n-1} i \right) + 7(3n - 1 - (n - 1)) \\ &= 5 \left(\frac{(3n-1)3n}{2} - \frac{(n-1)n}{2} \right) + 7(2n) \\ &= \frac{5}{2} (9n^2 - 3n - (n^2 - n)) + 14n \\ &= \frac{5}{2} (9n^2 - 3n - n^2 + n) + 14n \\ &= \frac{5}{2} (8n^2 - 2n) + 14n \\ &= 20n^2 - 5n + 14n \\ &= 20n^2 + 9n \end{aligned}$$

Grading: For this method, 1 pt sum split, 1 pt left sum split, 2 pt evals right sum, 2 pts plug in formulas left sums, 4 pts algebra