Sorting:
Merge Sort

Computer Science Department
University of Central Florida

COP 3502 – Computer Science I

Sorting: Merge Sort

- Problem with Bubble/Insertion/Selection Sorts:
  - All of these sorts make a large number of comparisons and swaps between elements
  - As mentioned last class (while covering n^2 sorts):
    - Any algorithm that swaps adjacent elements can only run so fast
  - So one might ask is there a more clever way to sort numbers
    - A way that does not require looking at all these pairs
  - Indeed, there are several ways to do this
  - And one of them is Merge Sort

Sorting: Merge Sort

- Merge Sort
  - Conceptually, Merge Sort works as follows:
    - If the "list" is of length 0 or 1, then it is already sorted!
    - Otherwise:

1. Divide the unsorted list into two sub-lists of about half the size
   - So if your list has n elements, you will divide that list into two sub-lists, each having approximately n/2 elements:
2. Recursively sort each sub-list by calling recursively calling Merge Sort on the two smaller lists
3. **Merge** the two sub-lists back into one sorted list
   - This Merge is a function that we study on its own
     - In a bit…

Sorting: Merge Sort

- Merge Sort
  - Basically, given a list:
    - You will split this list into two lists of about half the size
    - Then you recursively call Merge Sort on each list
    - What does that do?
      - Each of these new lists will, individually, be split into two lists of about half the size.
      - So now we have four lists, each about ¼ the size of the original list
    - This keeps happening…the lists keep getting split into smaller and smaller lists
      - <u>Until you get to a list of size 1 or size 0</u>…which is sorted!
    - Then we Merge them into a larger, sorted list

Sorting: Merge Sort

- Merge Sort
  - Incorporates two main ideas to improve its runtime:
    1) A small list will take fewer steps to sort than a large list
    2) Fewer steps are required to construct a sorted list from two sorted lists than two unsorted lists
       - For example:
         - You only have to traverse each list once if they're already sorted

Sorting: Merge Sort

- **Merge** function
  - The key to Merge Sort: the **Merge** function
  - Given two sorted lists, **Merge** them into one sorted list
  - Problem:
    - You are given two arrays, each of which is already sorted
    - Your job is to efficiently combine the two arrays into one larger array
    - The larger array should contain all the values of the two smaller arrays
    - Finally, the larger array should be in sorted order

Sorting: Merge Sort

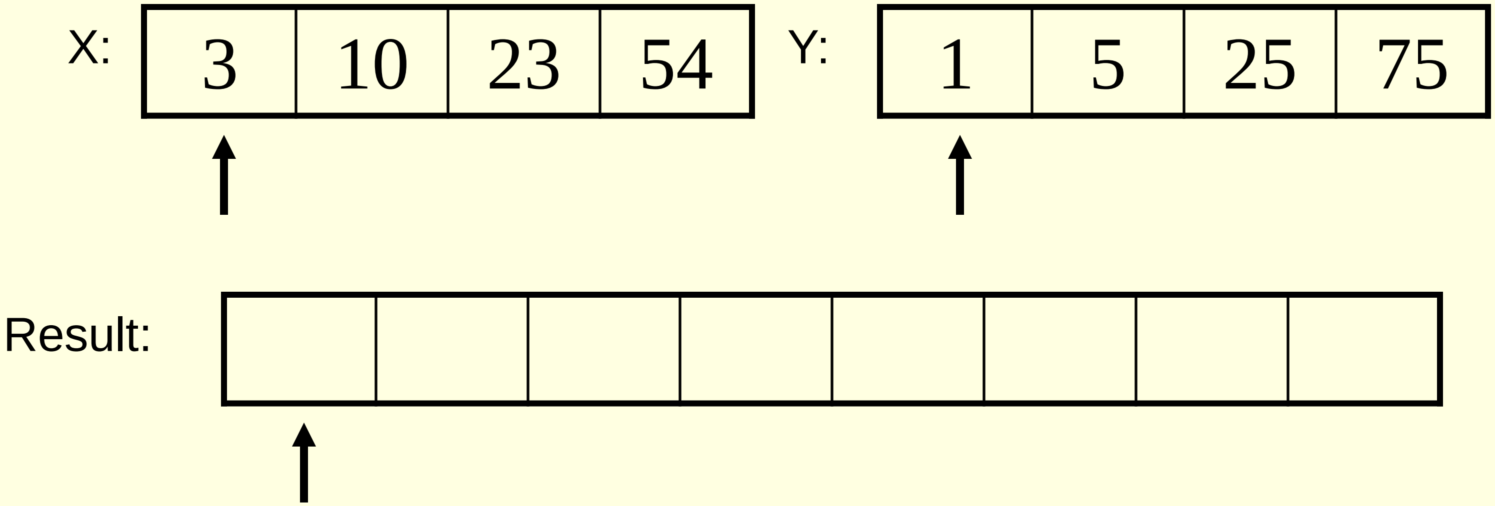
- **Merge** function
  - The key to Merge Sort: the **Merge** function
  - Given two sorted lists, **Merge** them into one sorted list
  - If you have two lists:
    - X ($x_1 < x_2 < \ldots < x_m$) and Y ($y_1 < y_2 < \ldots < y_n$)
    - Merge these into one list: Z ($z_1 < z_2 < \ldots < z_{m+n}$)
  - Example:
    - List 1 = {3, 8, 9} and List 2 = {1, 5, 7}
    - Merge(List 1, List 2) = {1, 3, 5, 7, 8, 9}

Sorting: Merge Sort

- **<u>Merge</u>** function
  - Solution:
    - Keep track of the smallest value in each array that hasn't been placed, in order, in the larger array yet
    - Compare these two smallest values from each array
      - One of these MUST be the smallest of all the values in both arrays that are left
      - Place the smallest of the two values in the next location in the larger array
    - Adjust the smallest value for the appropriate array
    - Continue this process until all values have been placed in the large array

Sorting: Merge Sort

- Example of **Merge** function:



Sorting: Merge Sort

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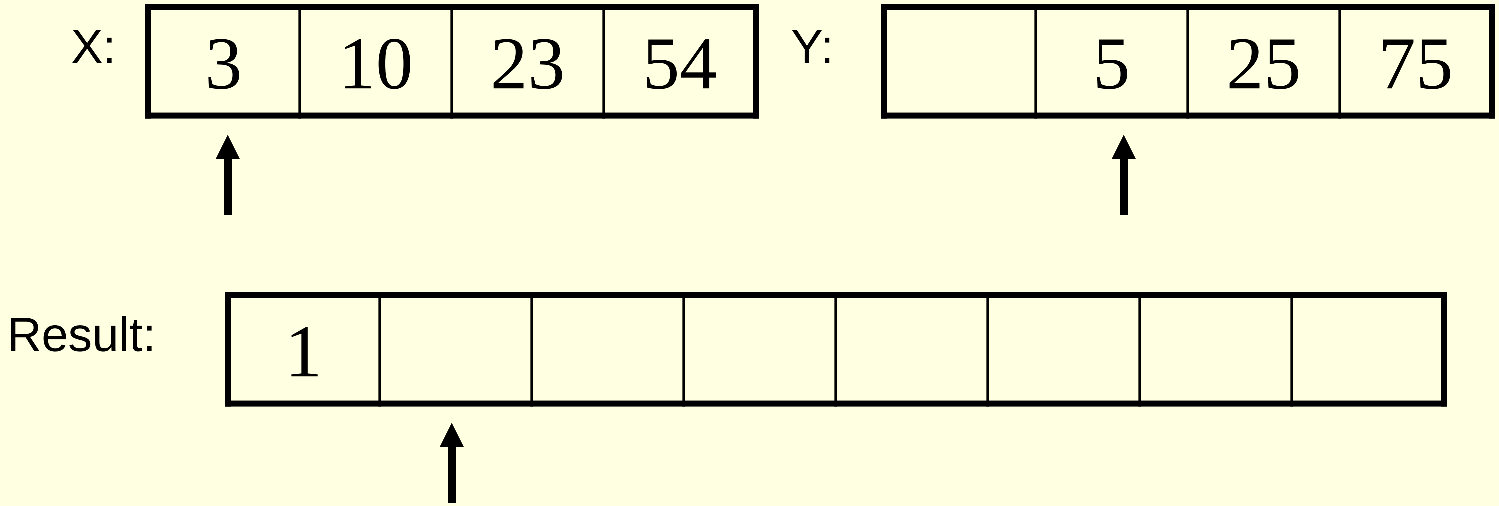

Sorting: Merge Sort

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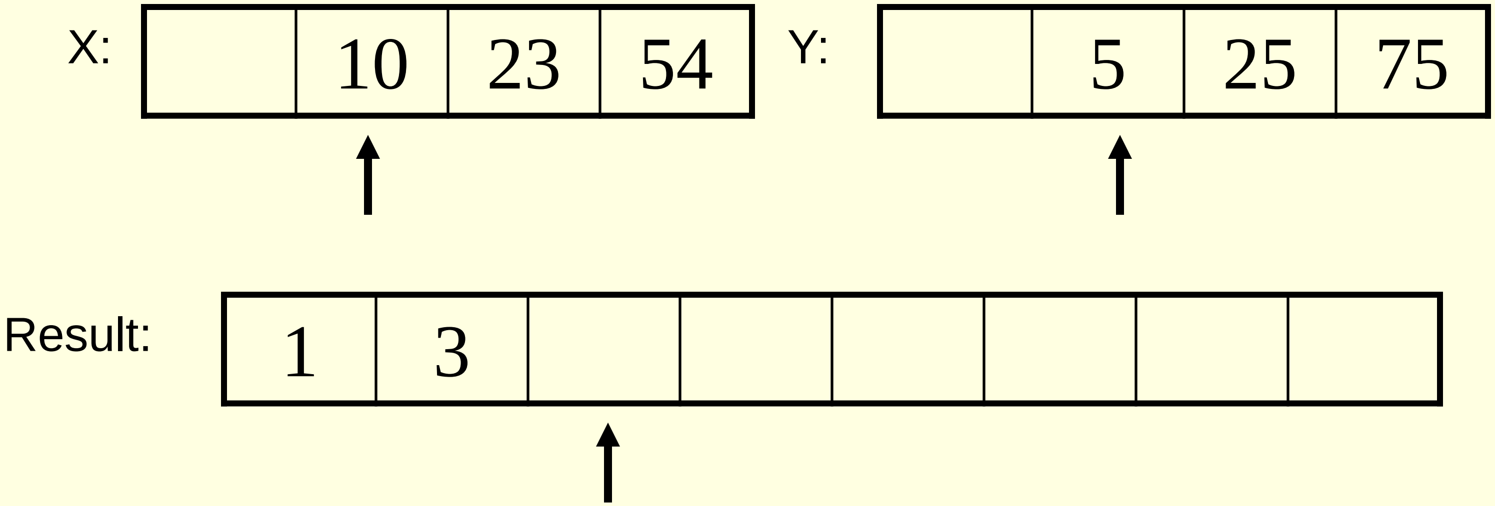

Sorting: Merge Sort

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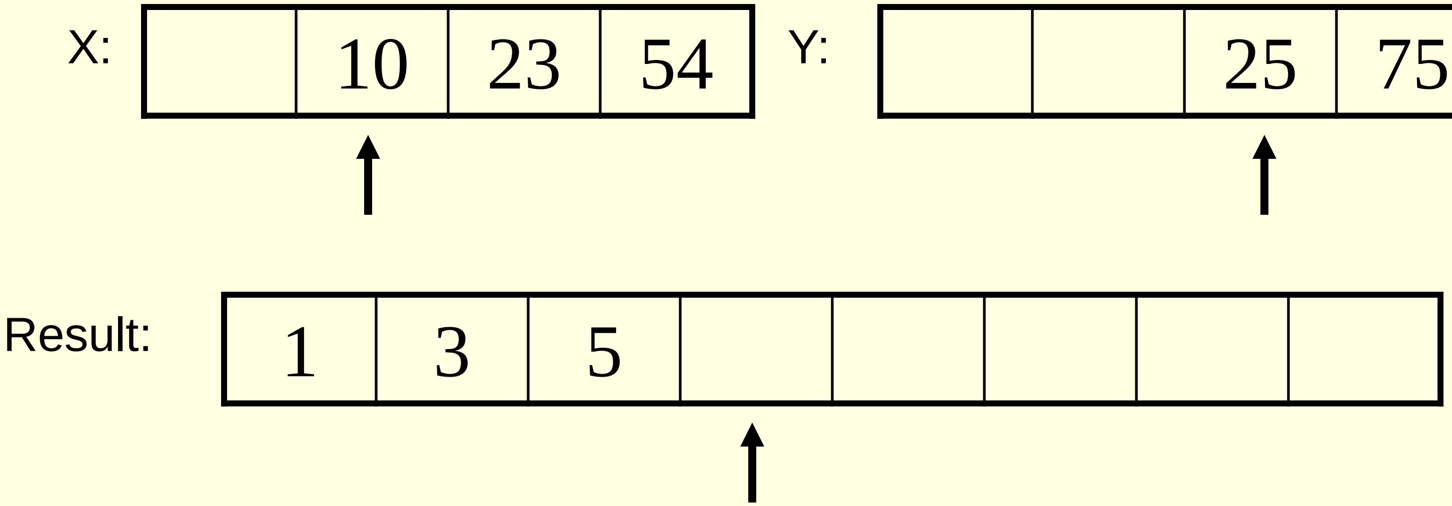

Sorting: Merge Sort

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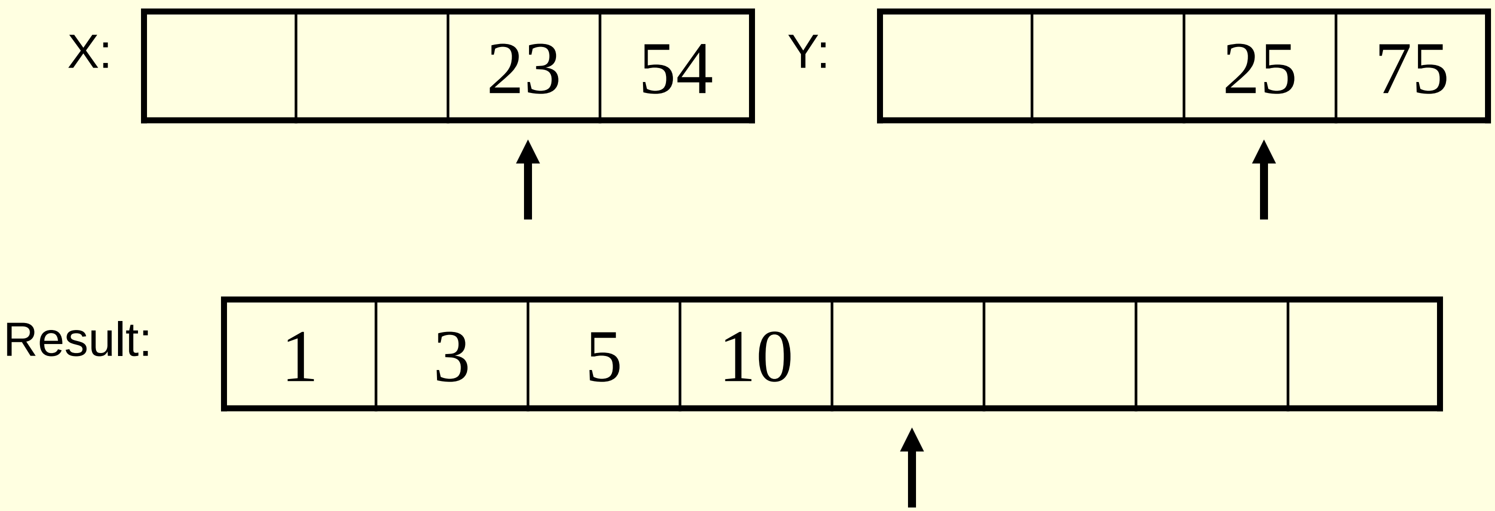

Sorting: Merge Sort

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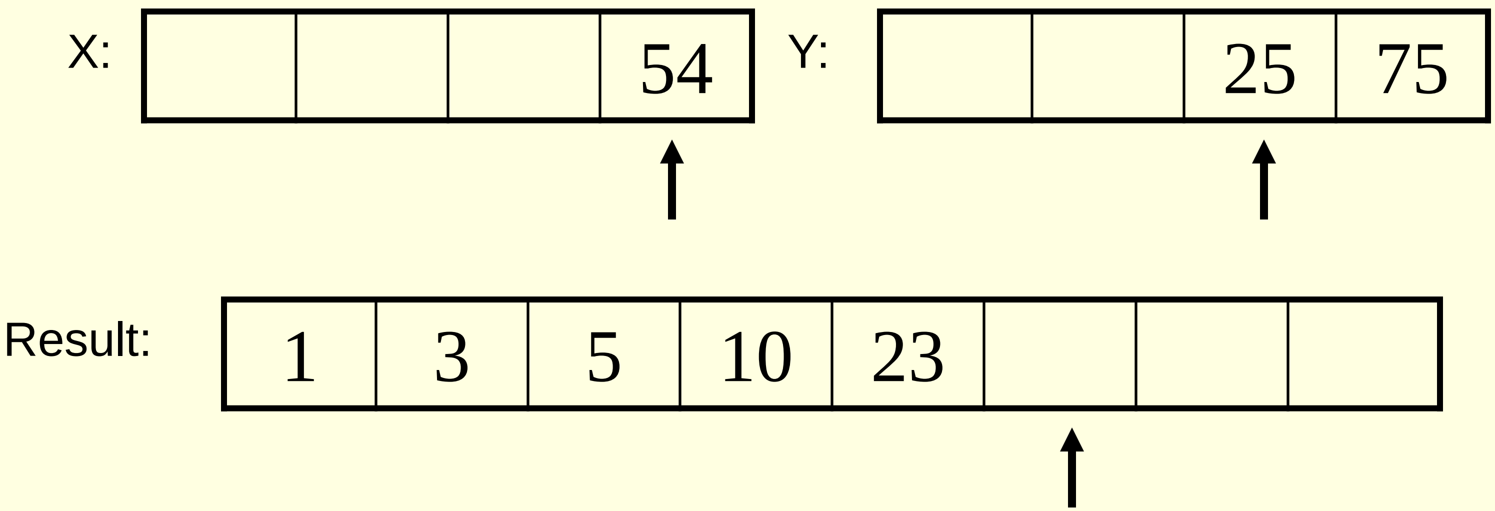

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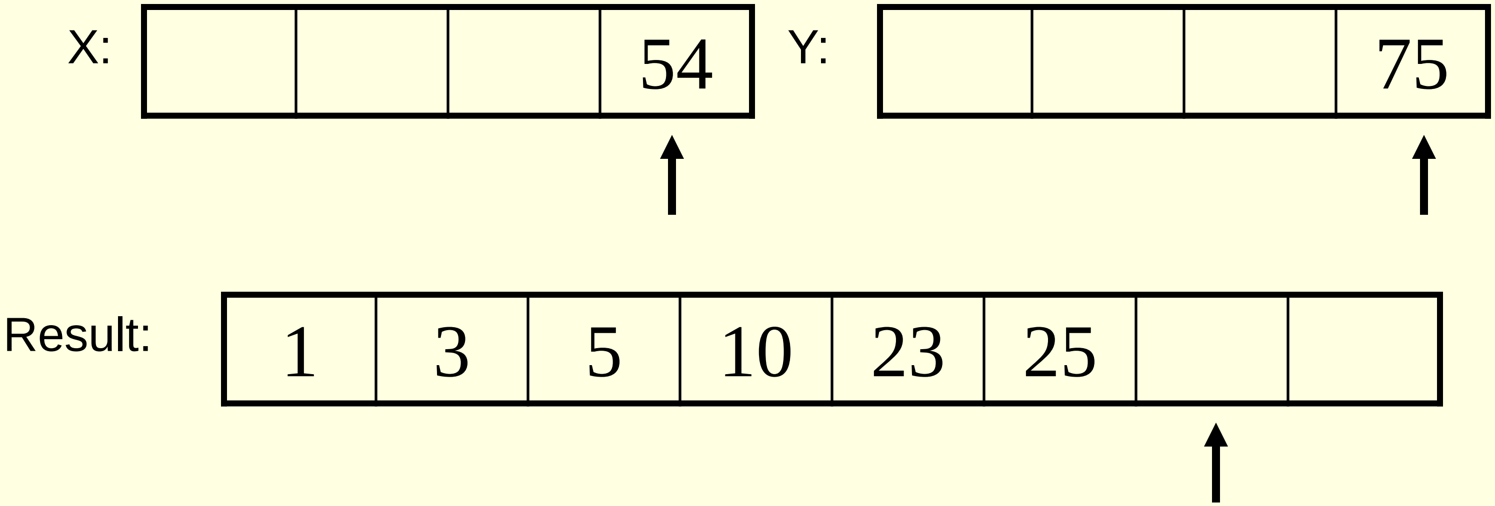

Sorting: Merge Sort

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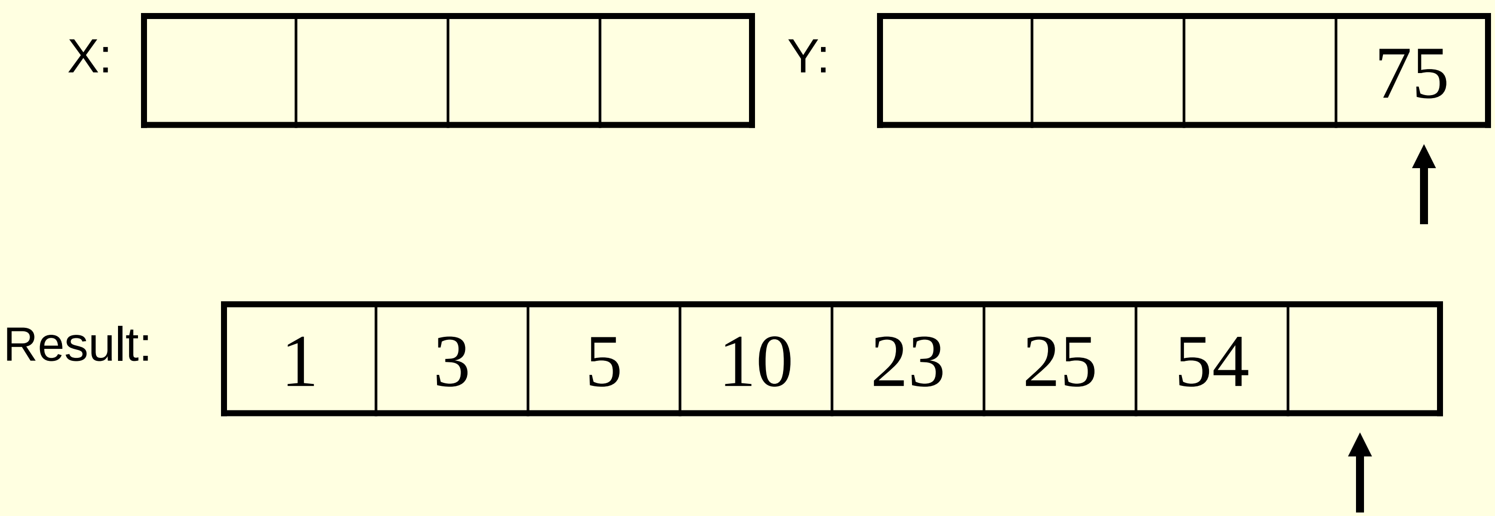

Sorting: Merge Sort

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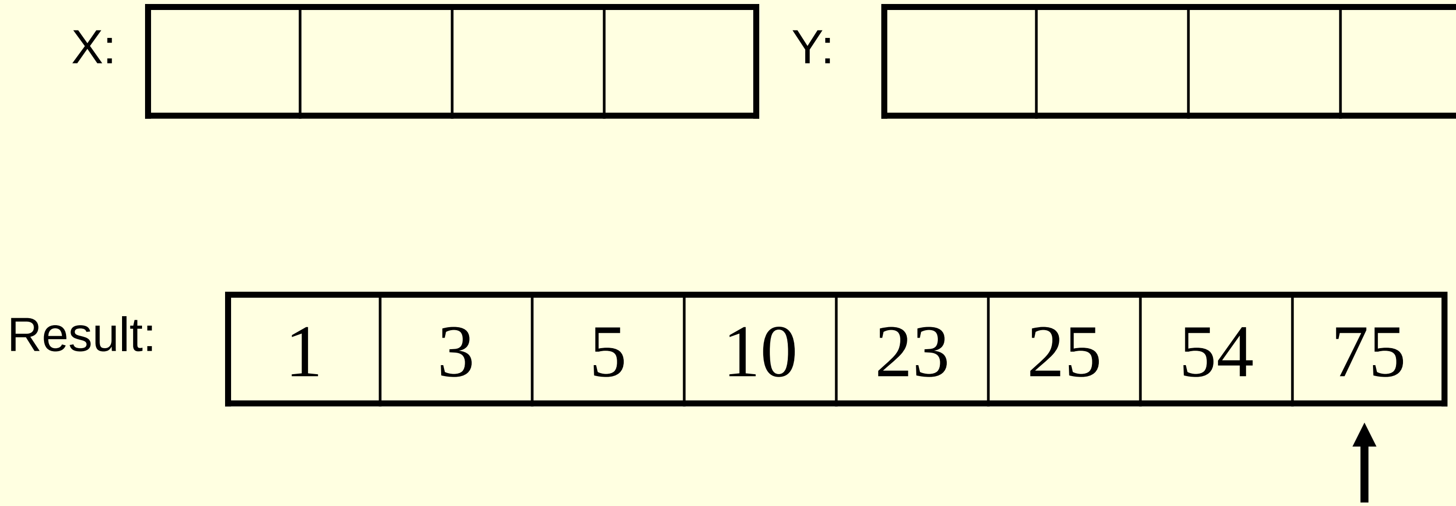

Sorting: Merge Sort

- **<u>Merge</u>** function
  - The big question:
    - How can we use this Merge function to sort an entire, unsorted array?
    - This function only "sorts" a specific scenario:
      - You have to have two, **<u>already sorted</u>**, arrays
    - **<u>Merge</u>** can then "sort" (merge) them into one larger array
    - So can we use this Merge function to somehow sort a large, unsorted array???

  - This brings us back to Merge Sort

Sorting: Merge Sort

- Merge Sort
  - Again, here is the main idea for Merge Sort:
  1) Sort the first half of the array, using Merge Sort
  2) Sort the second half of the array, using Merge Sort
     - Now, we do indeed have a situation where we can use the Merge function!
     - Each half is already sorted!
  3) So simply merge the first half of the array with the second half.
  - And this points to a recursive solution…

Sorting: Merge Sort

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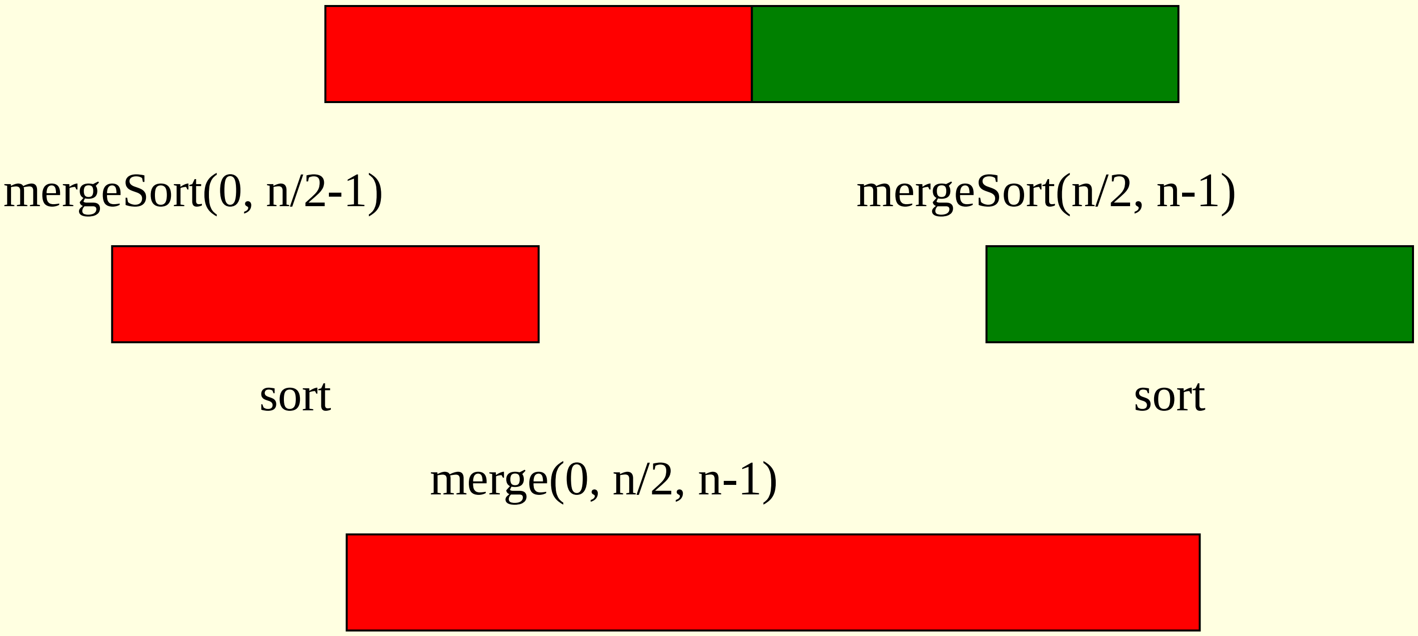
    - This keeps happening…the lists keep getting split into smaller and smaller lists
      - Until you get to a list of size 1 or size 0
    - Then we Merge them into a larger, sorted list

Sorting: Merge Sort

- Merge sort idea:
  - Divide the array into two halves.
  - Recursively sort the two halves (using merge sort).
  - Use **<u>Merge</u>** to combine the two arrays.



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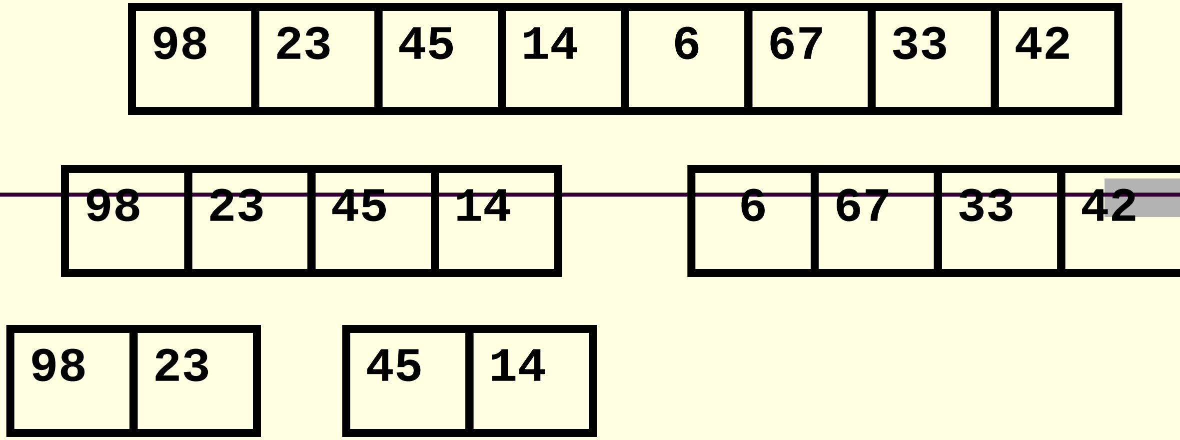

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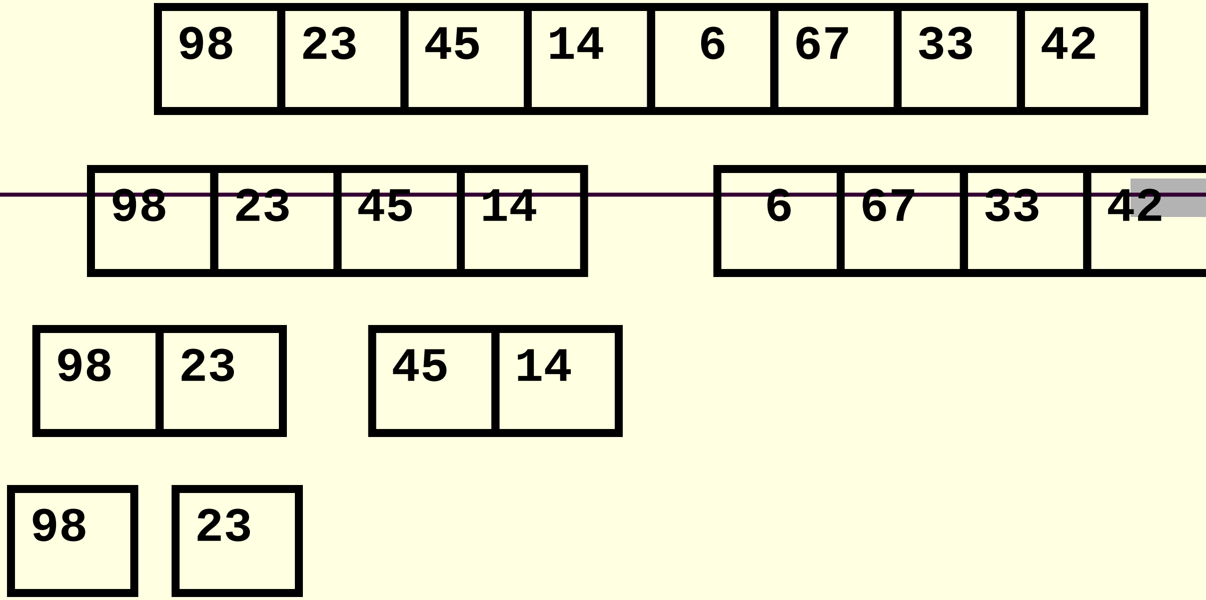
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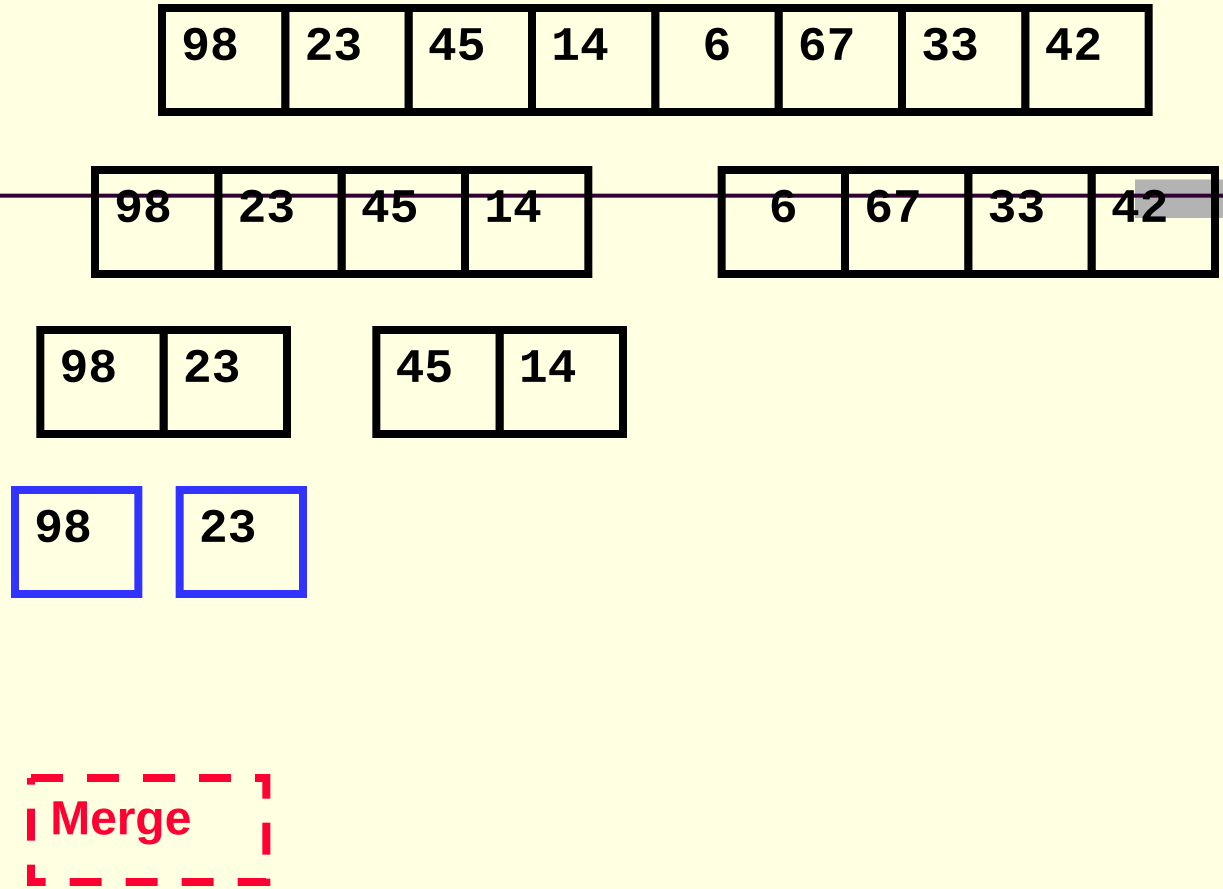
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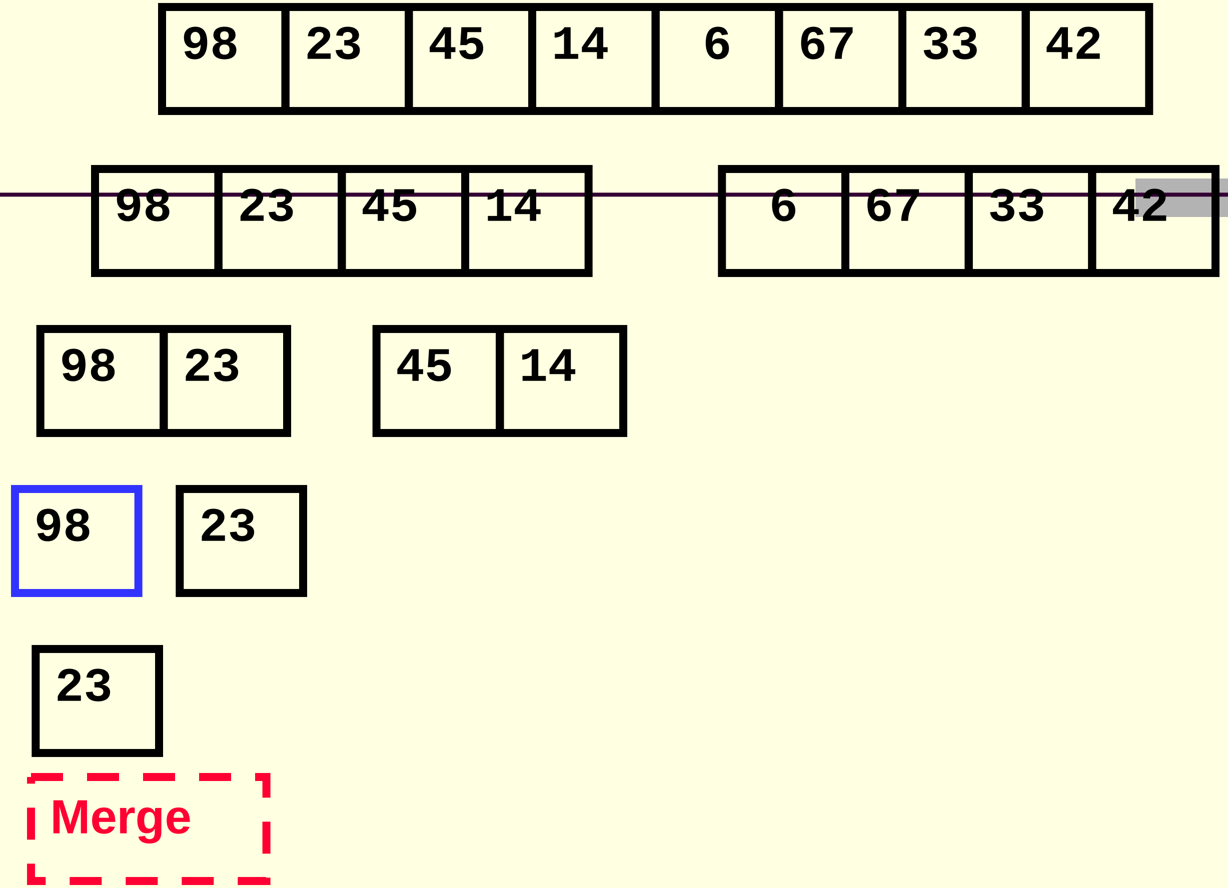
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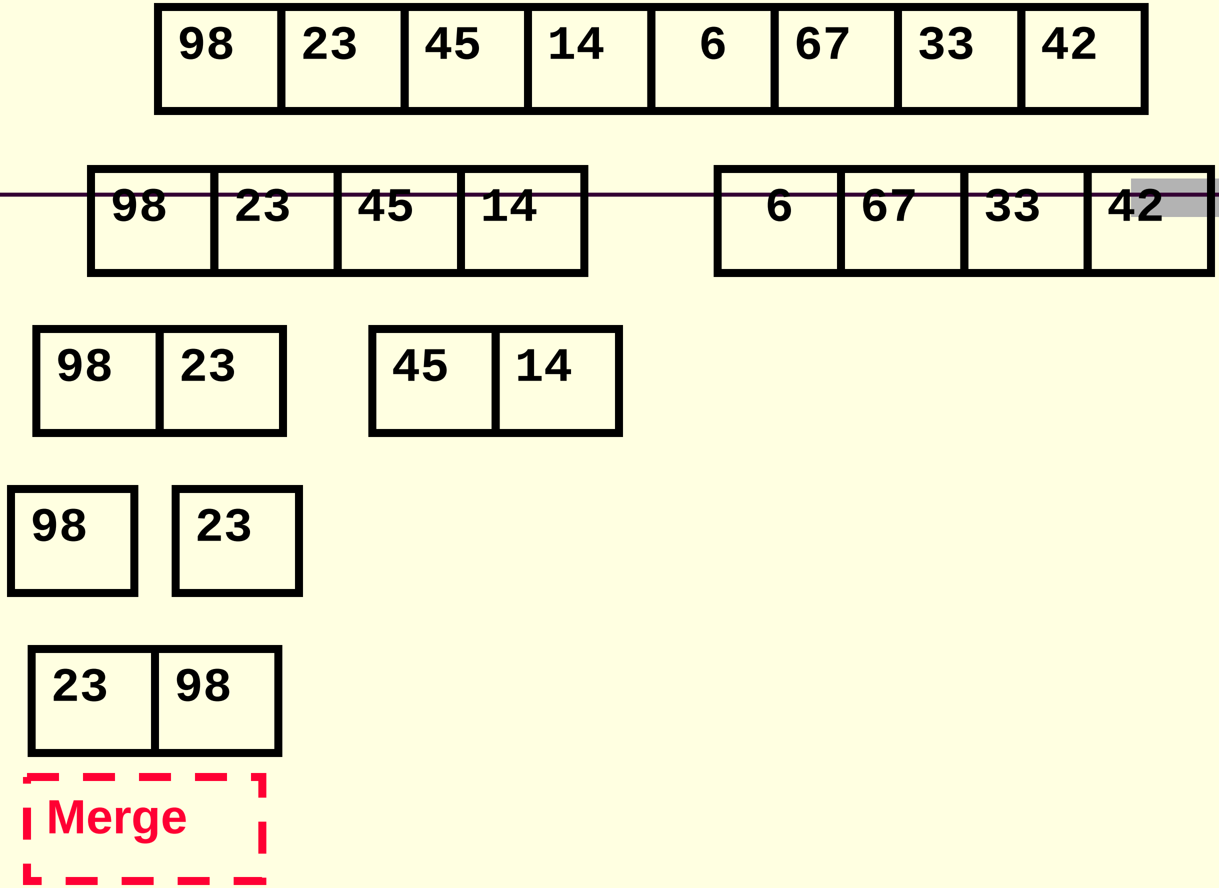

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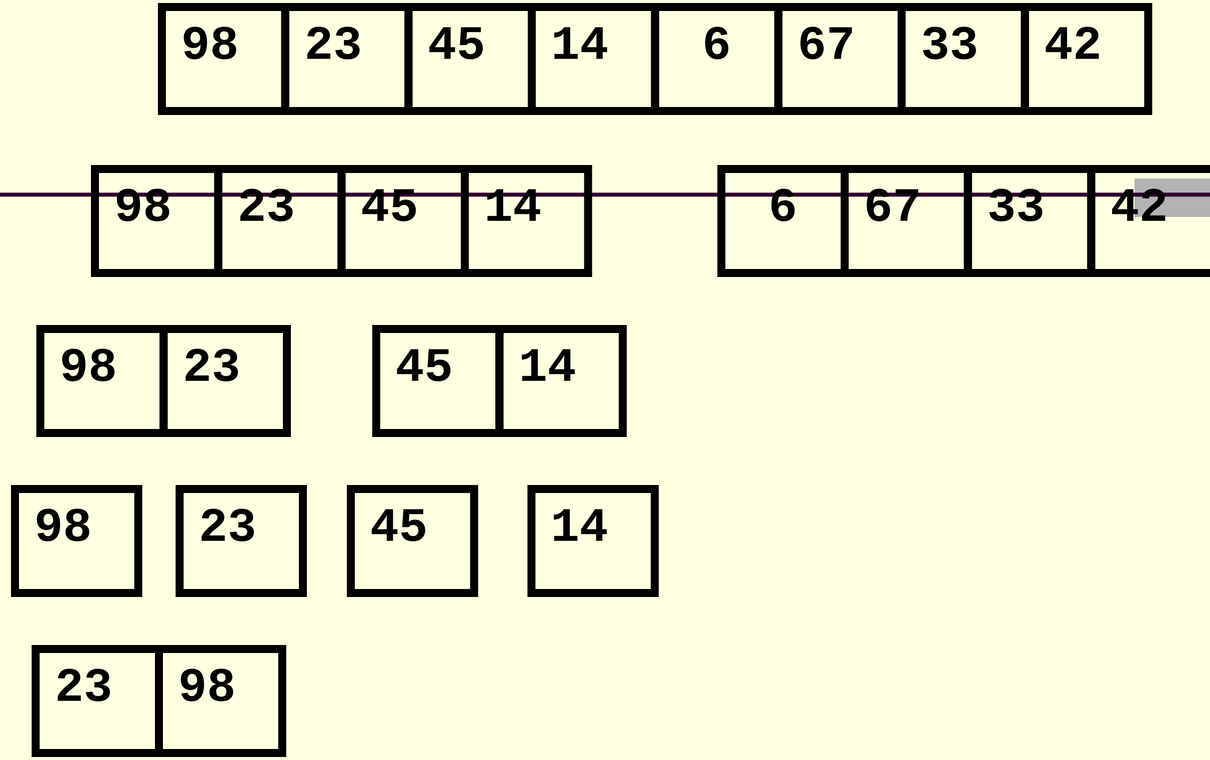
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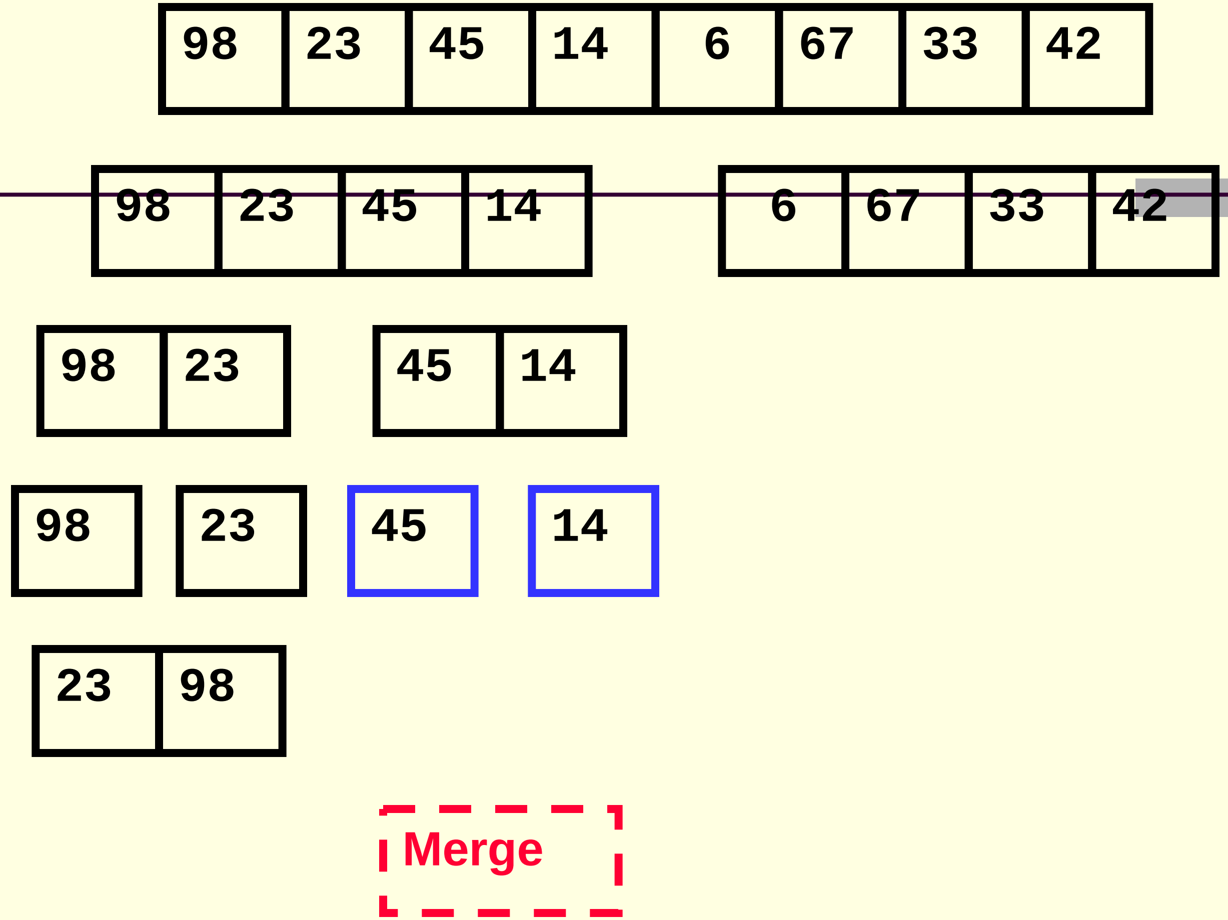
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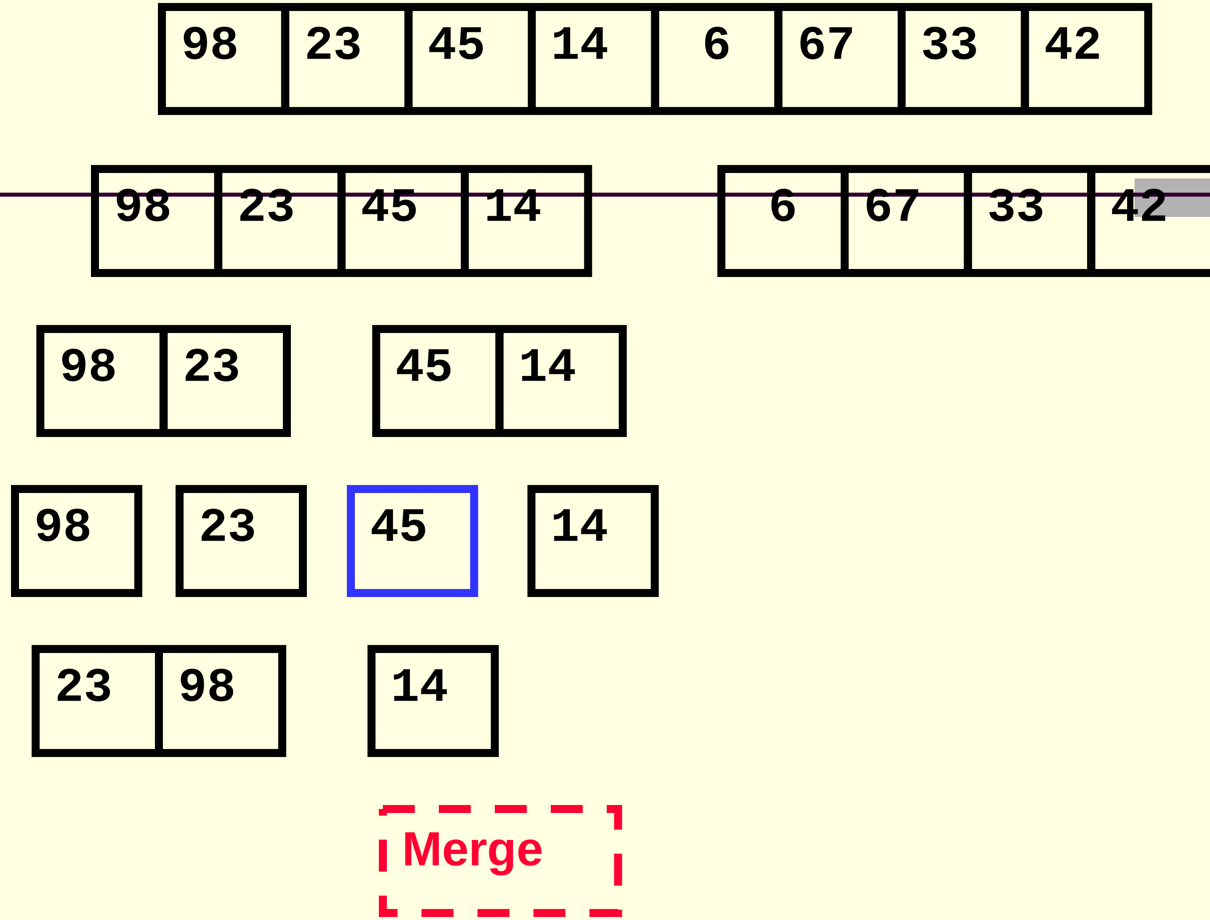
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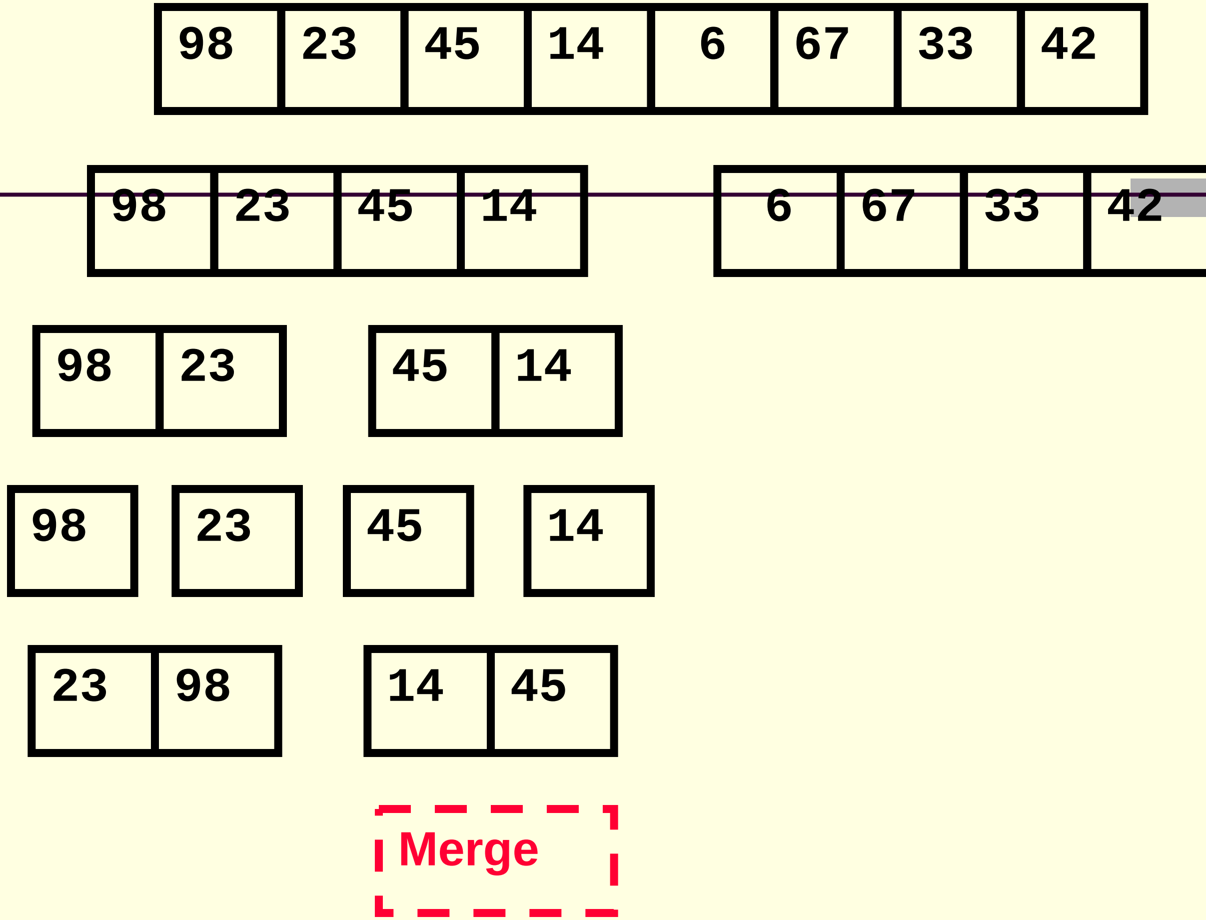
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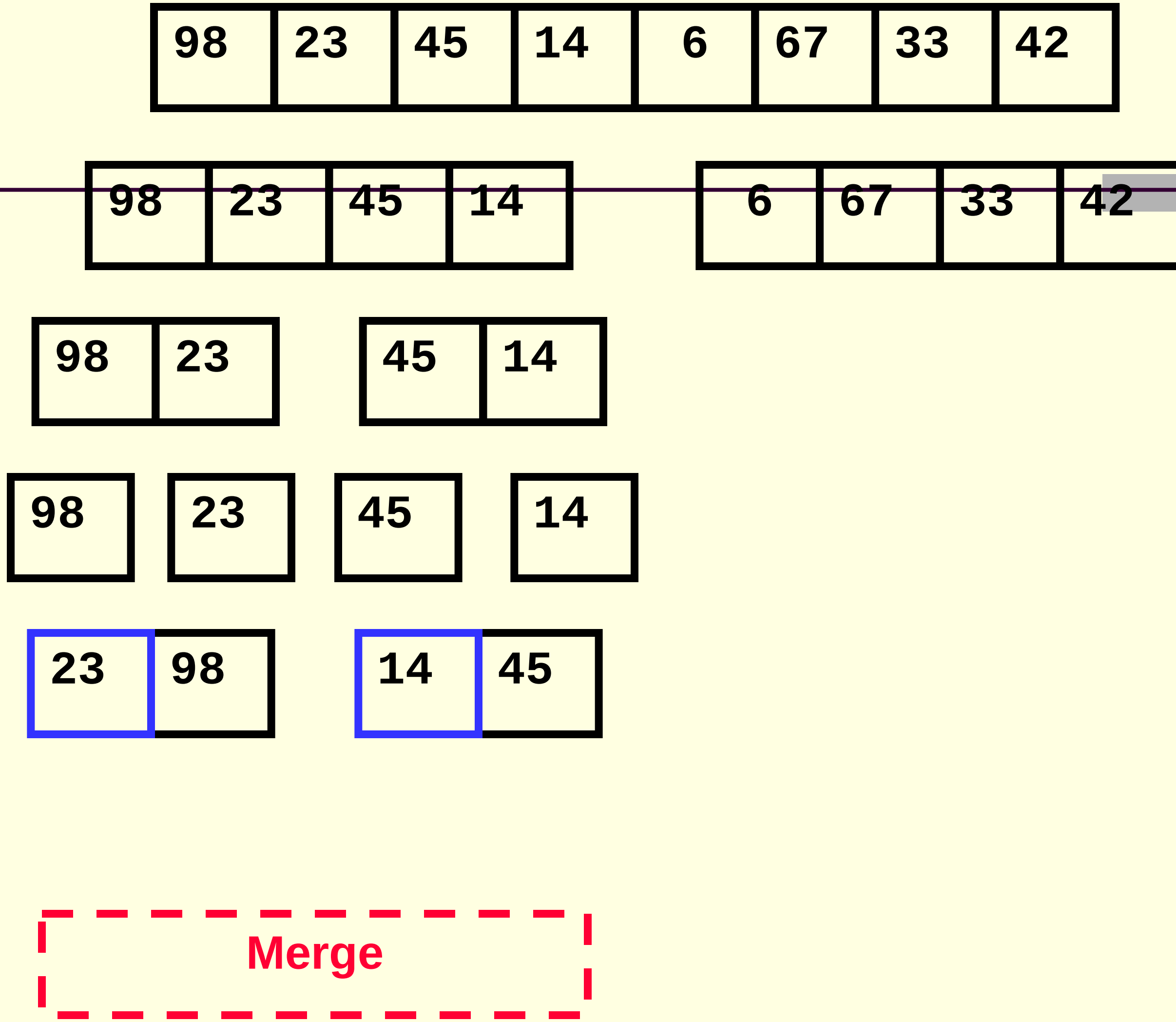
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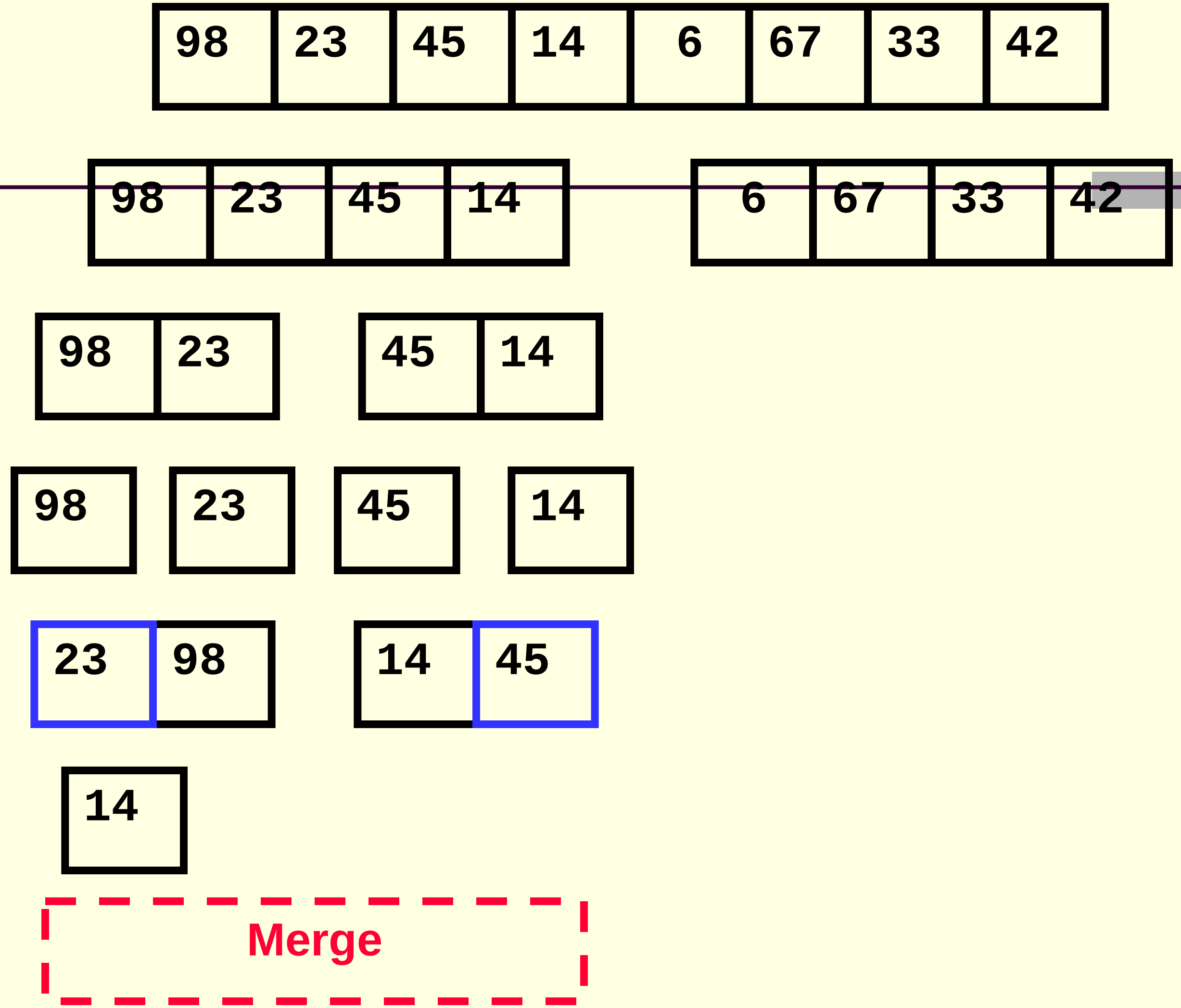
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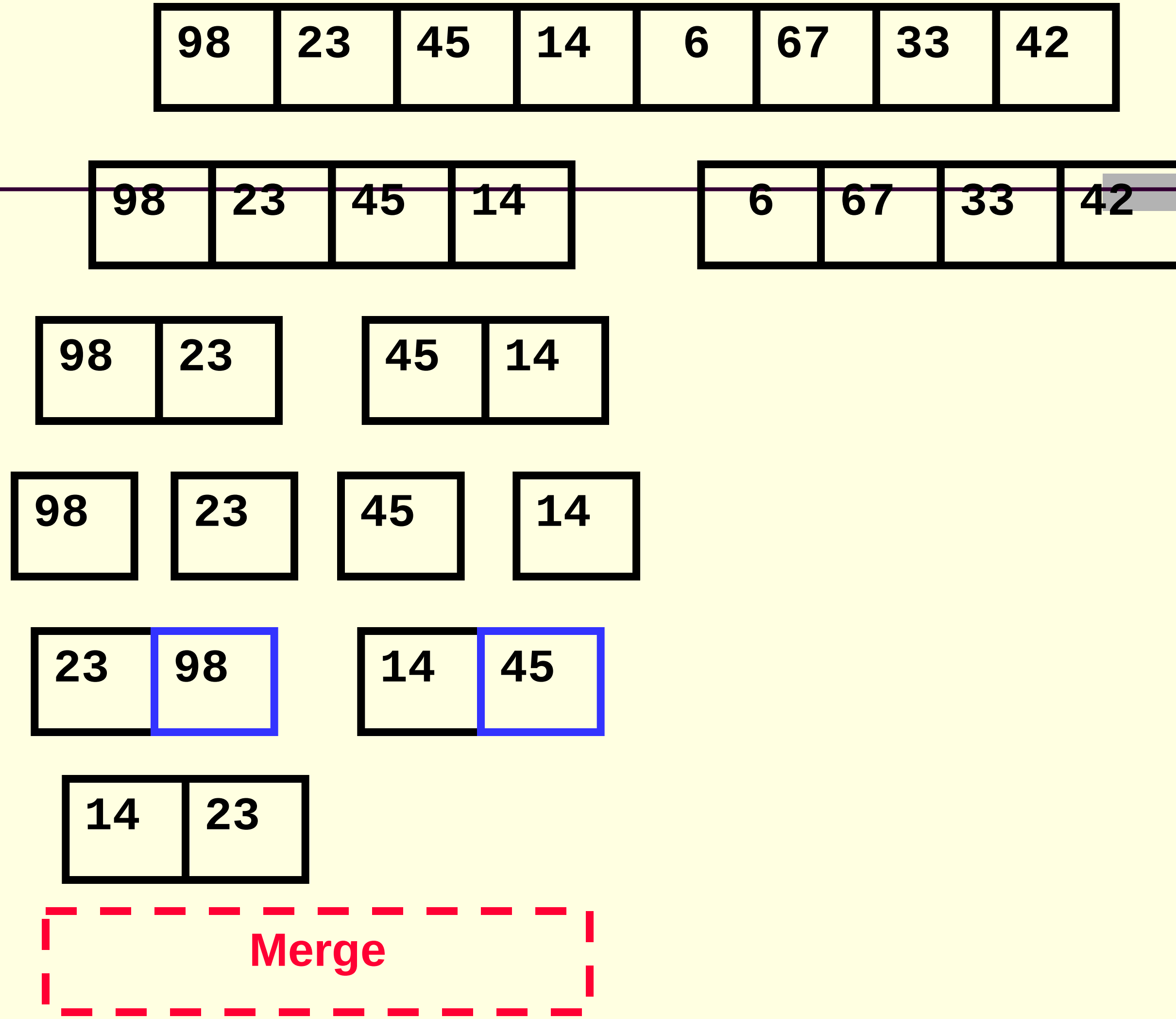
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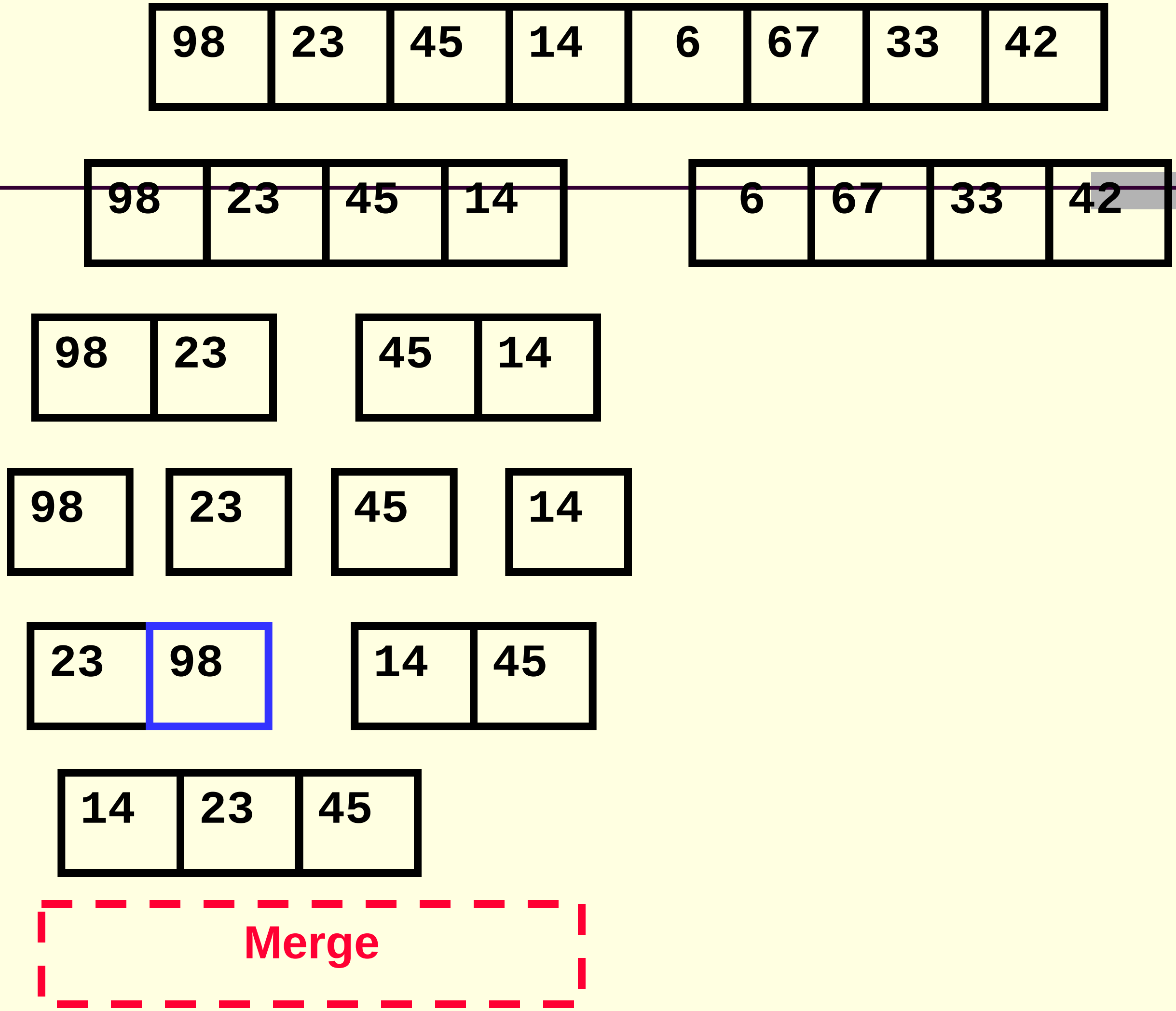
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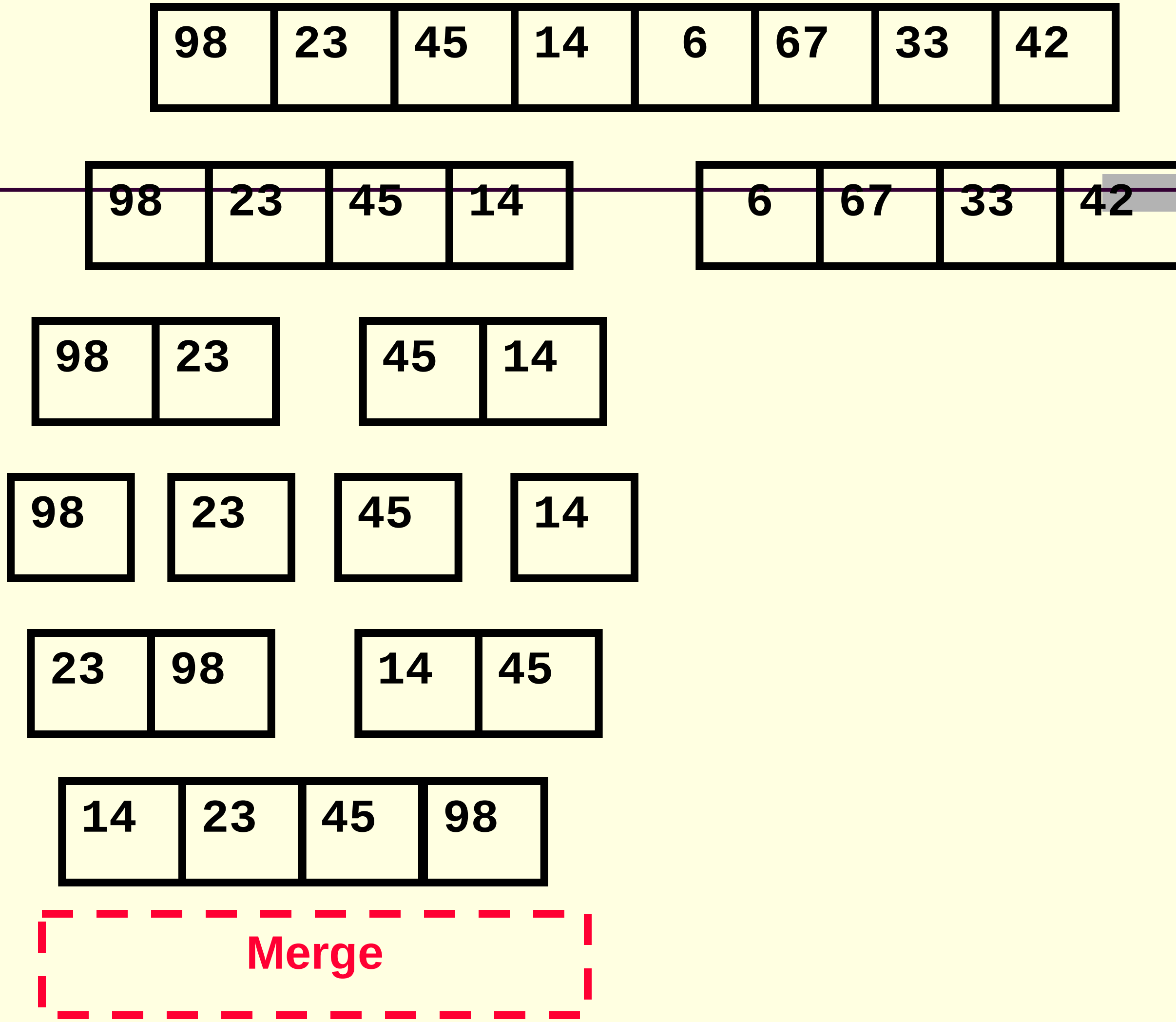
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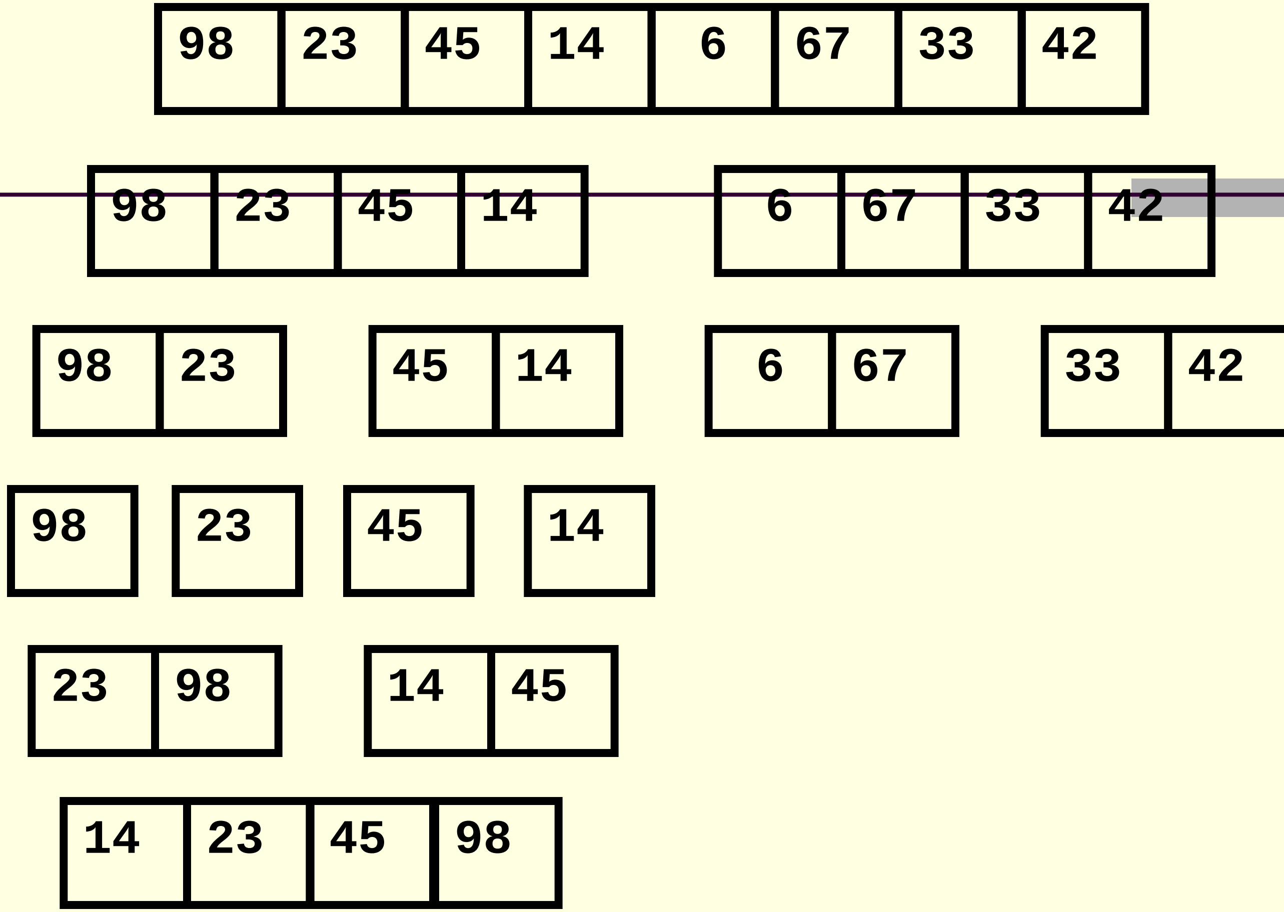
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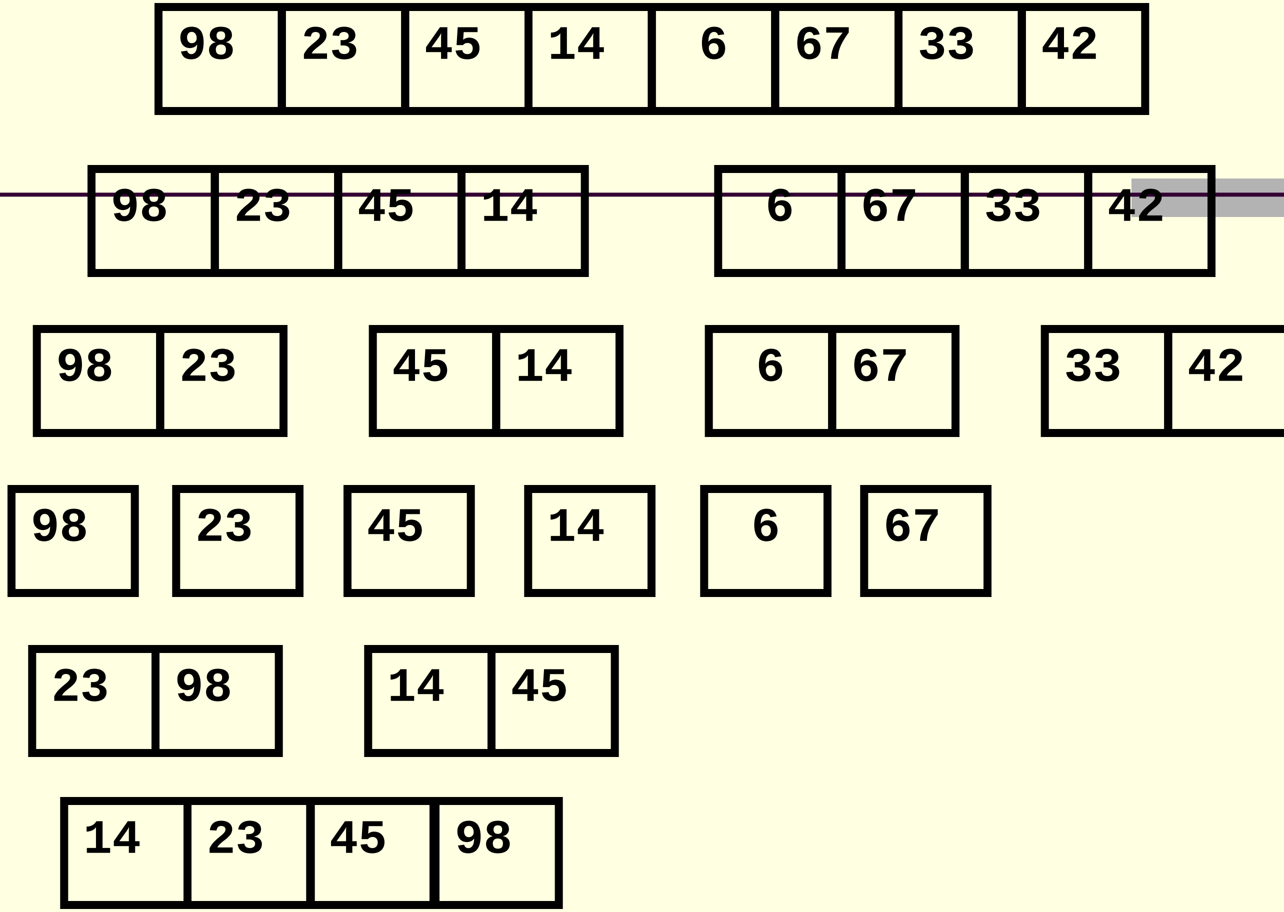
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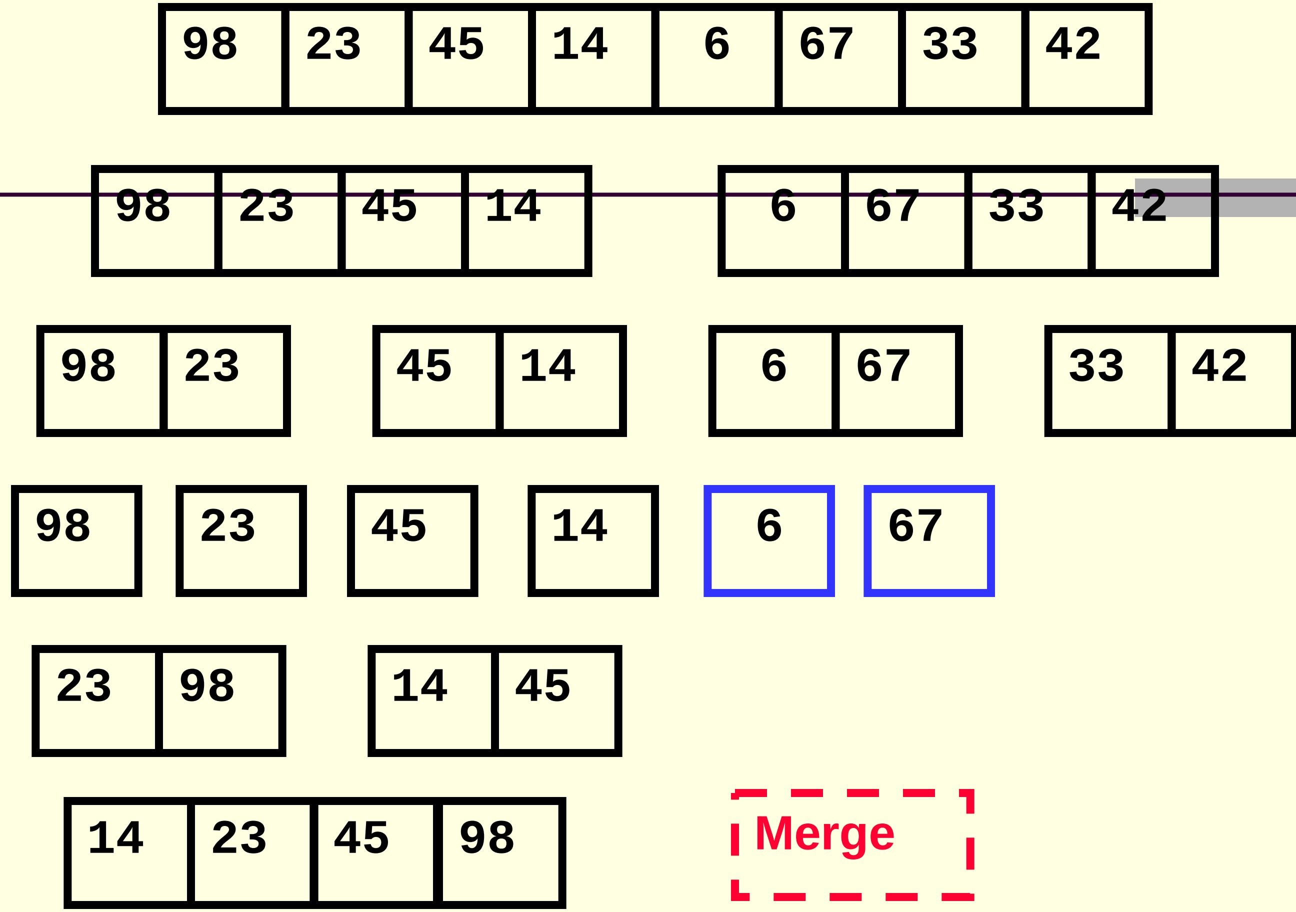
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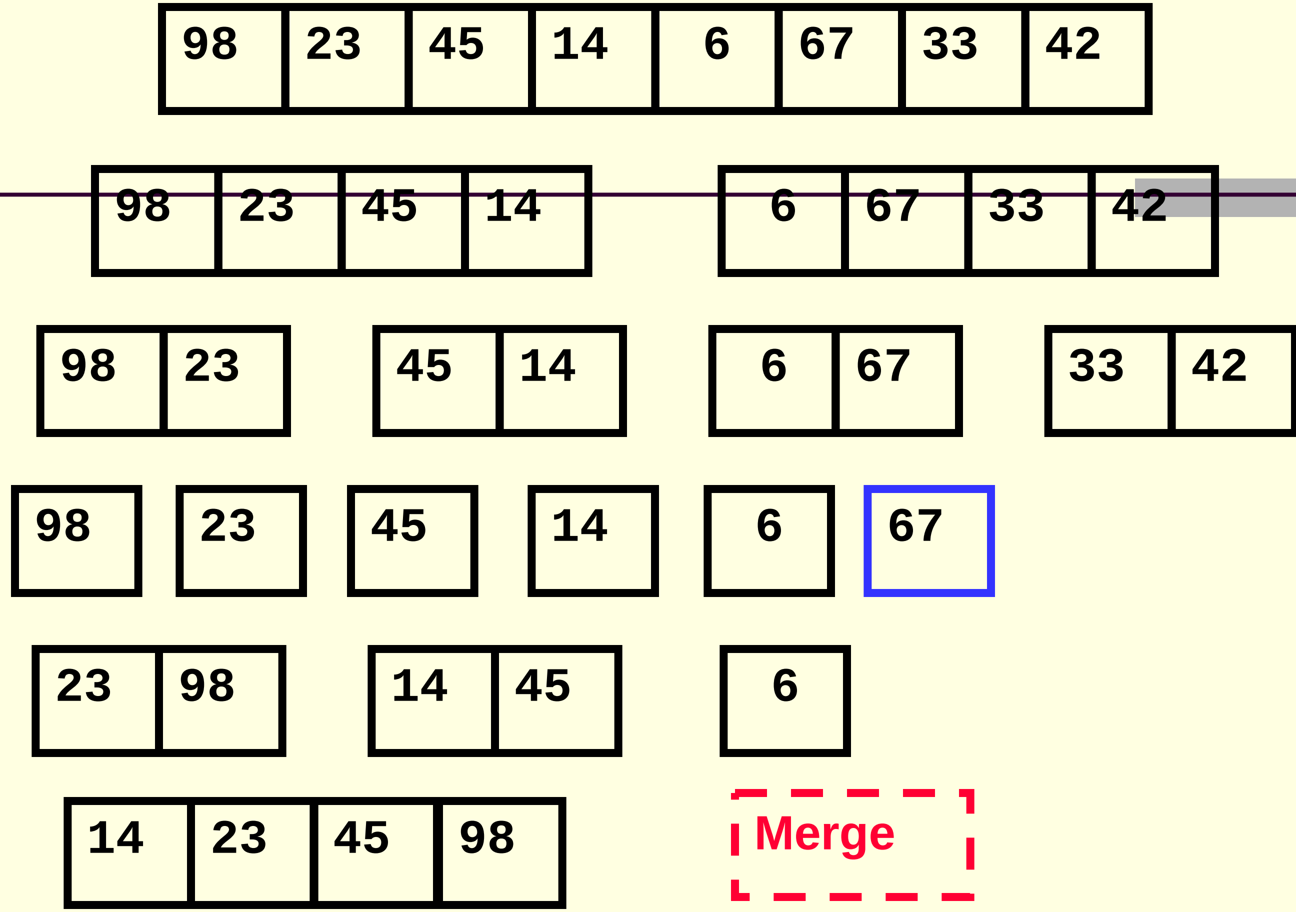
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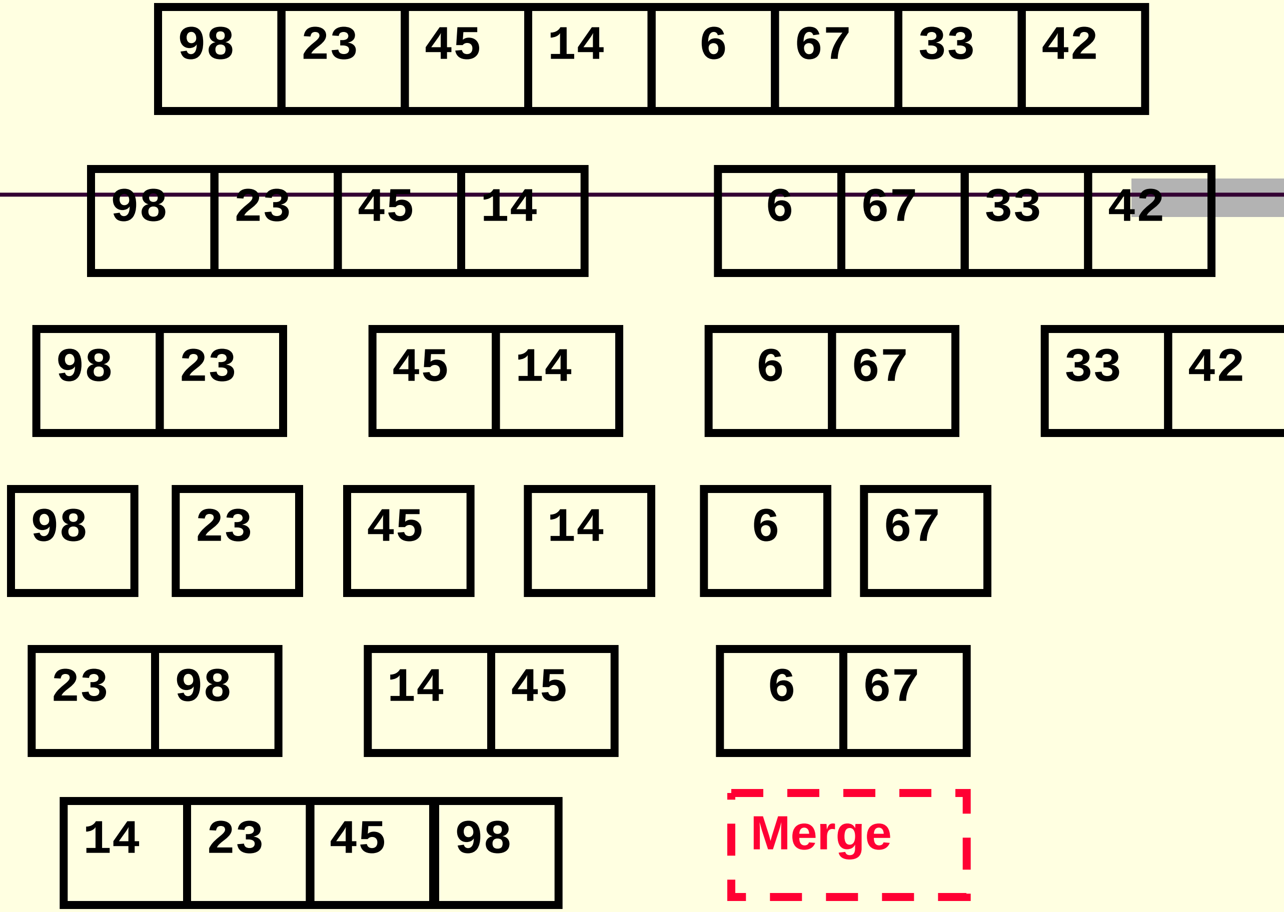
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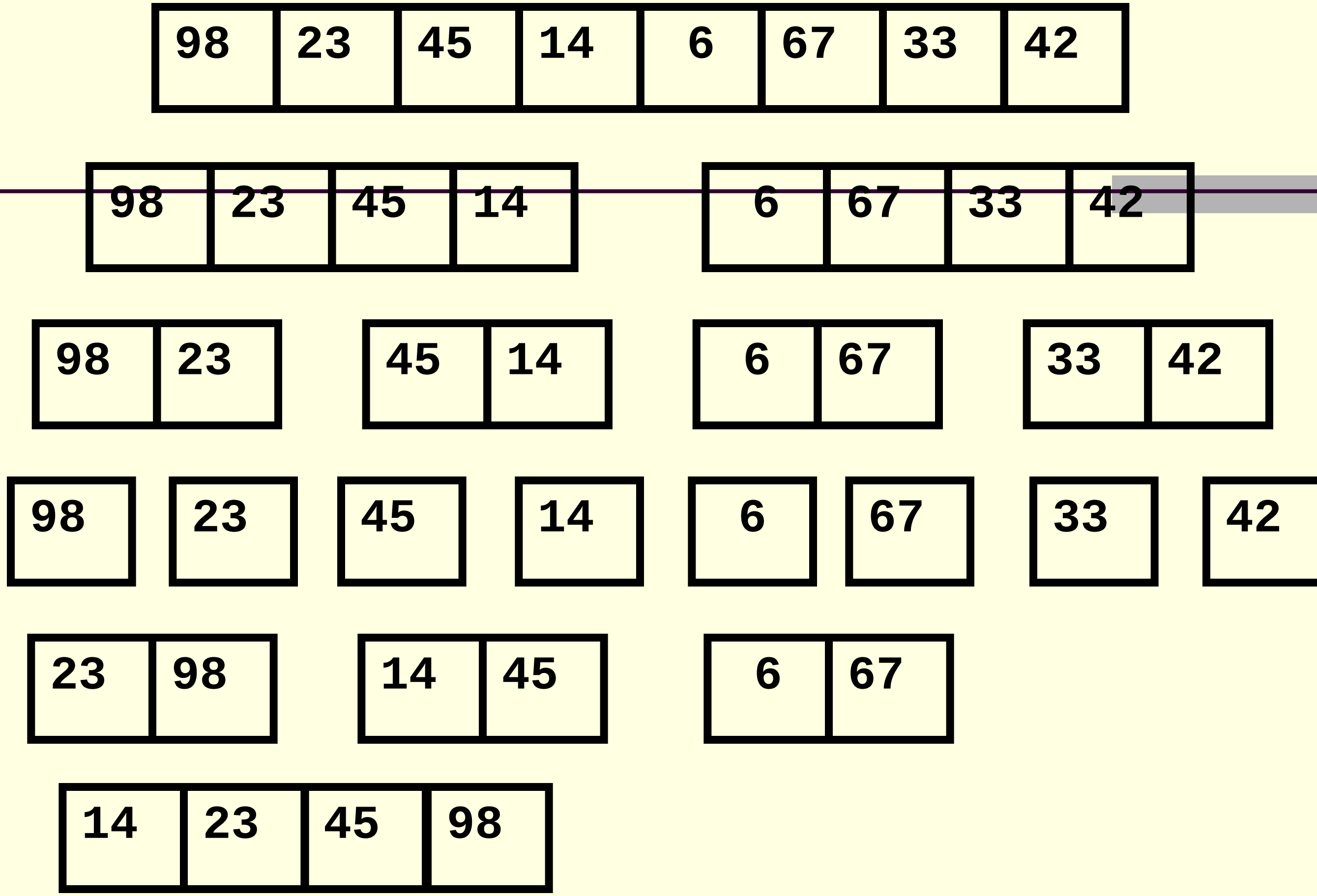
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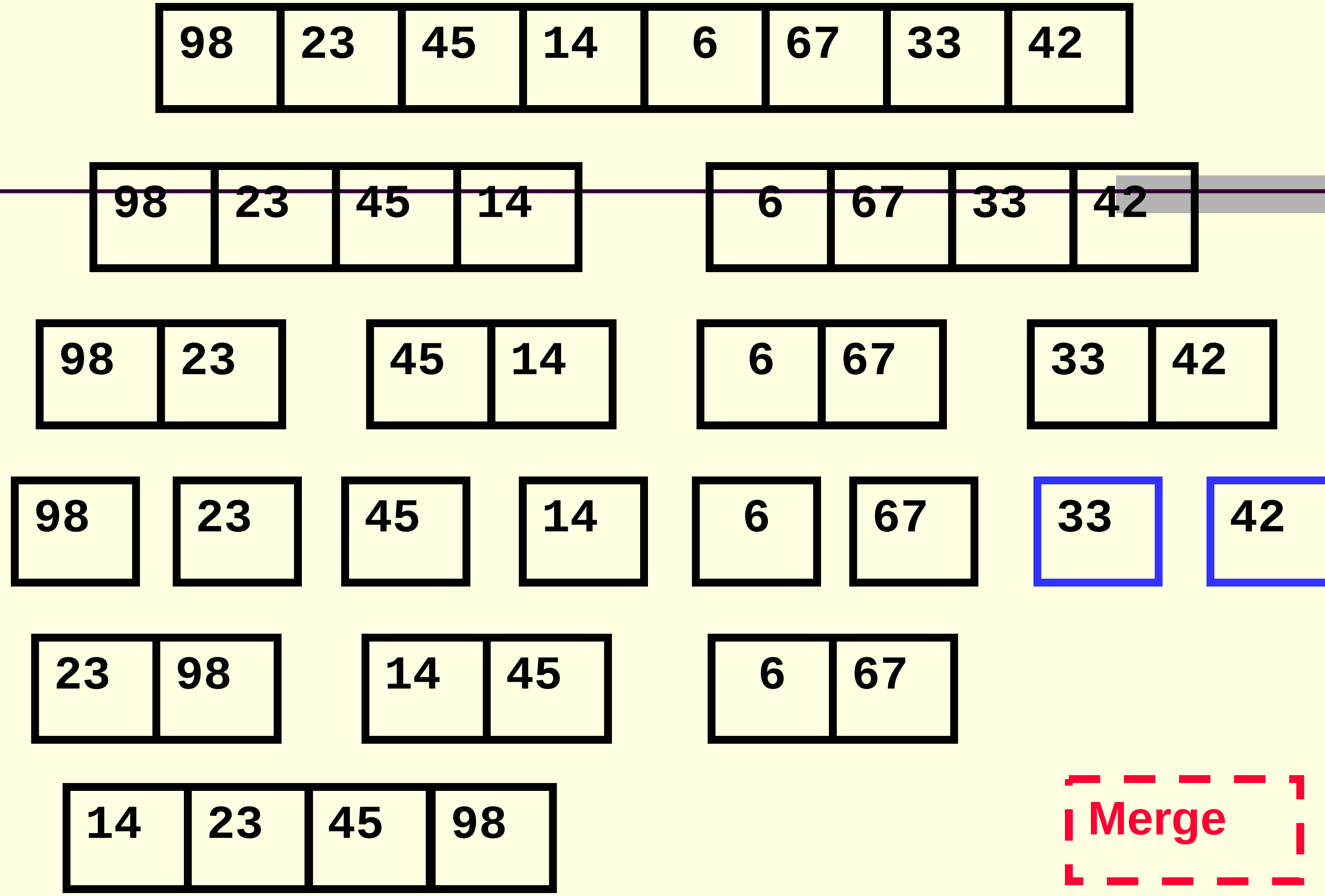
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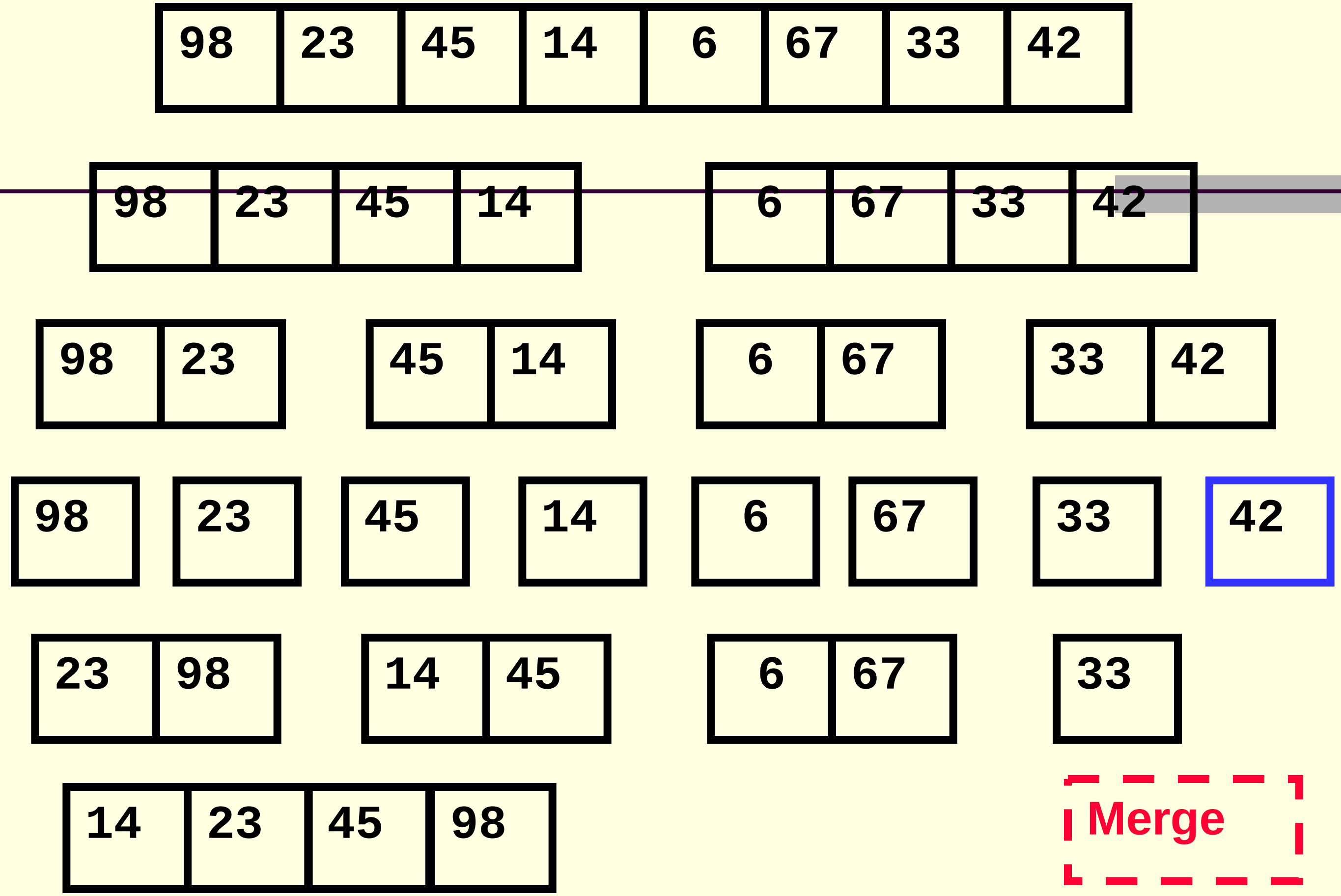
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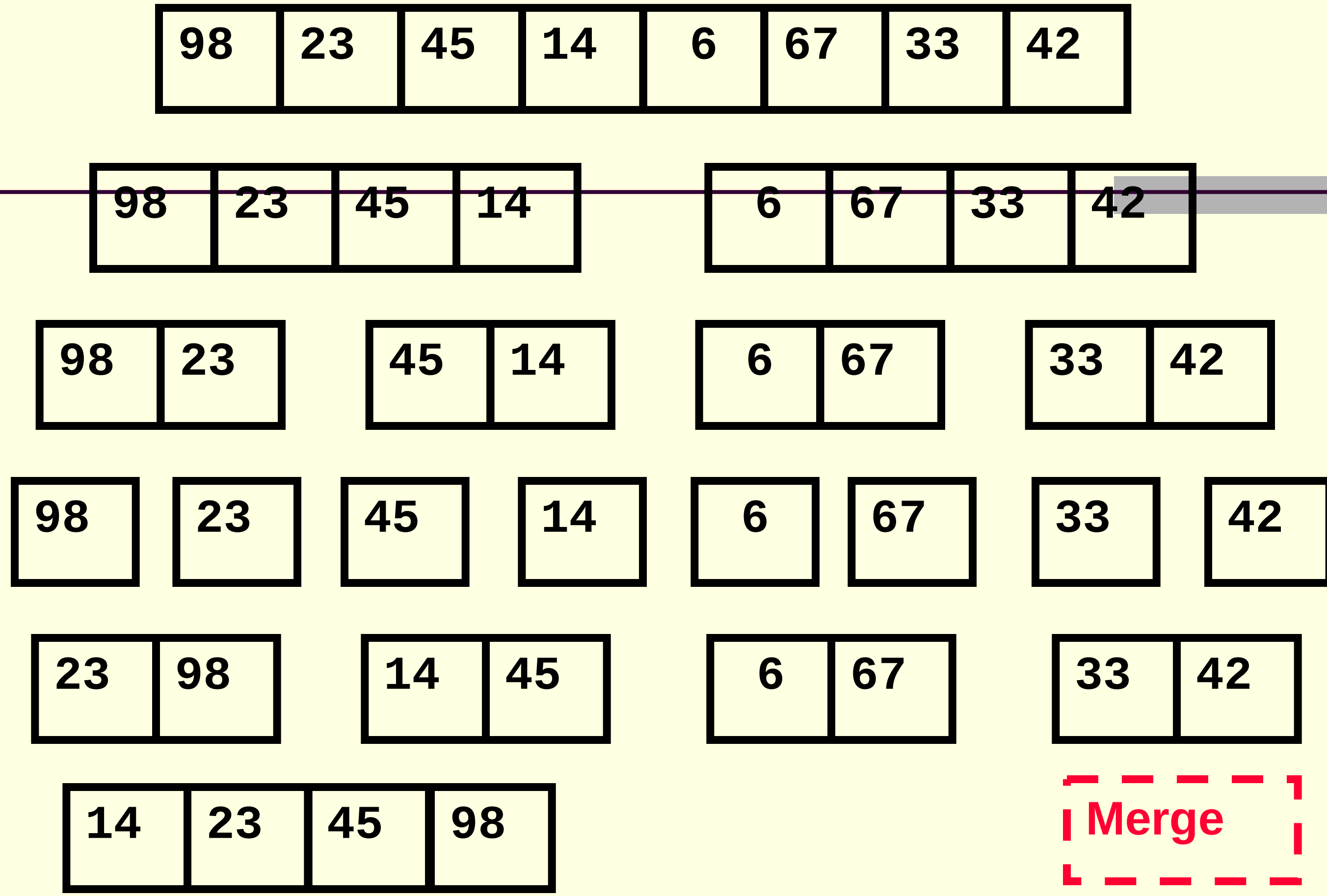
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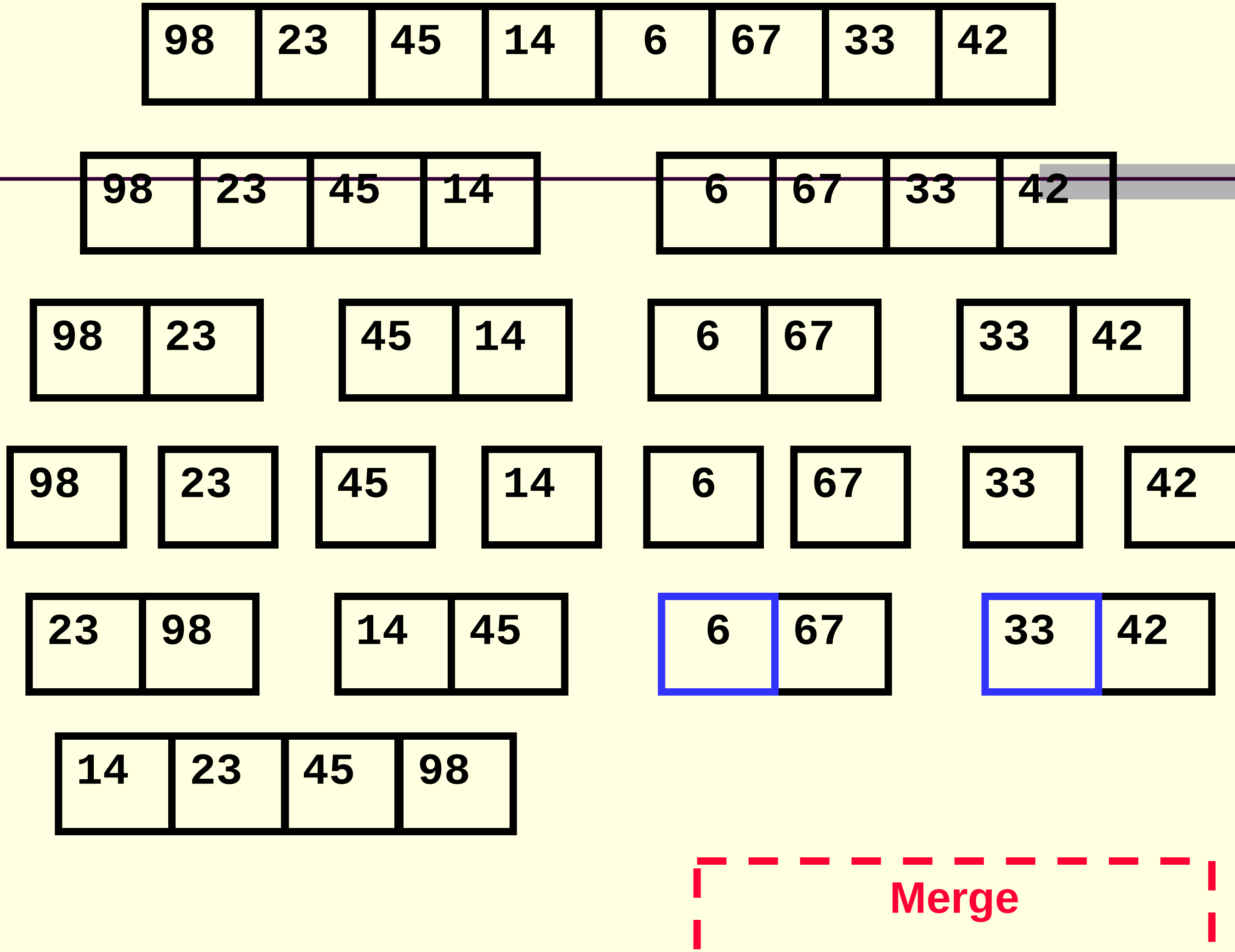
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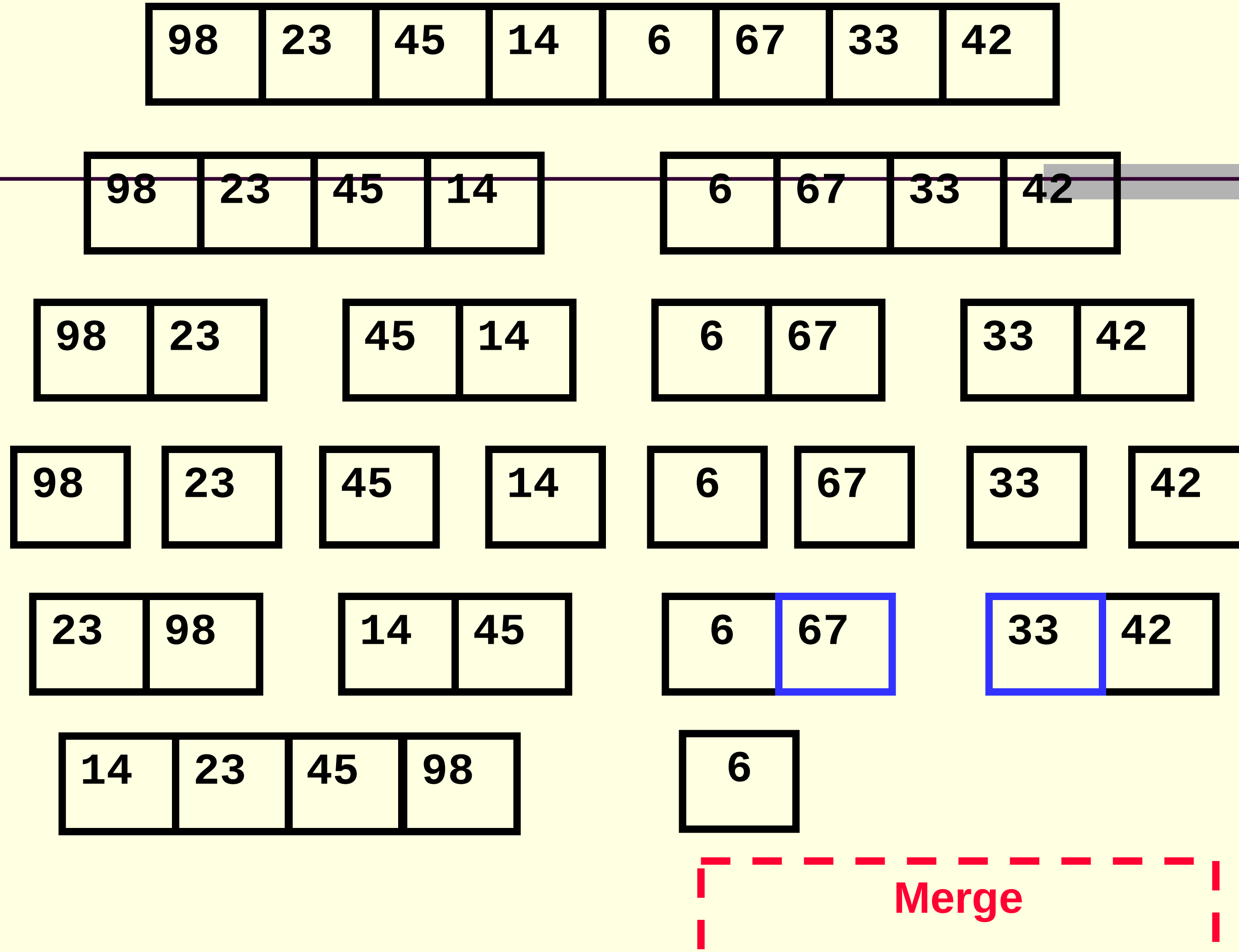
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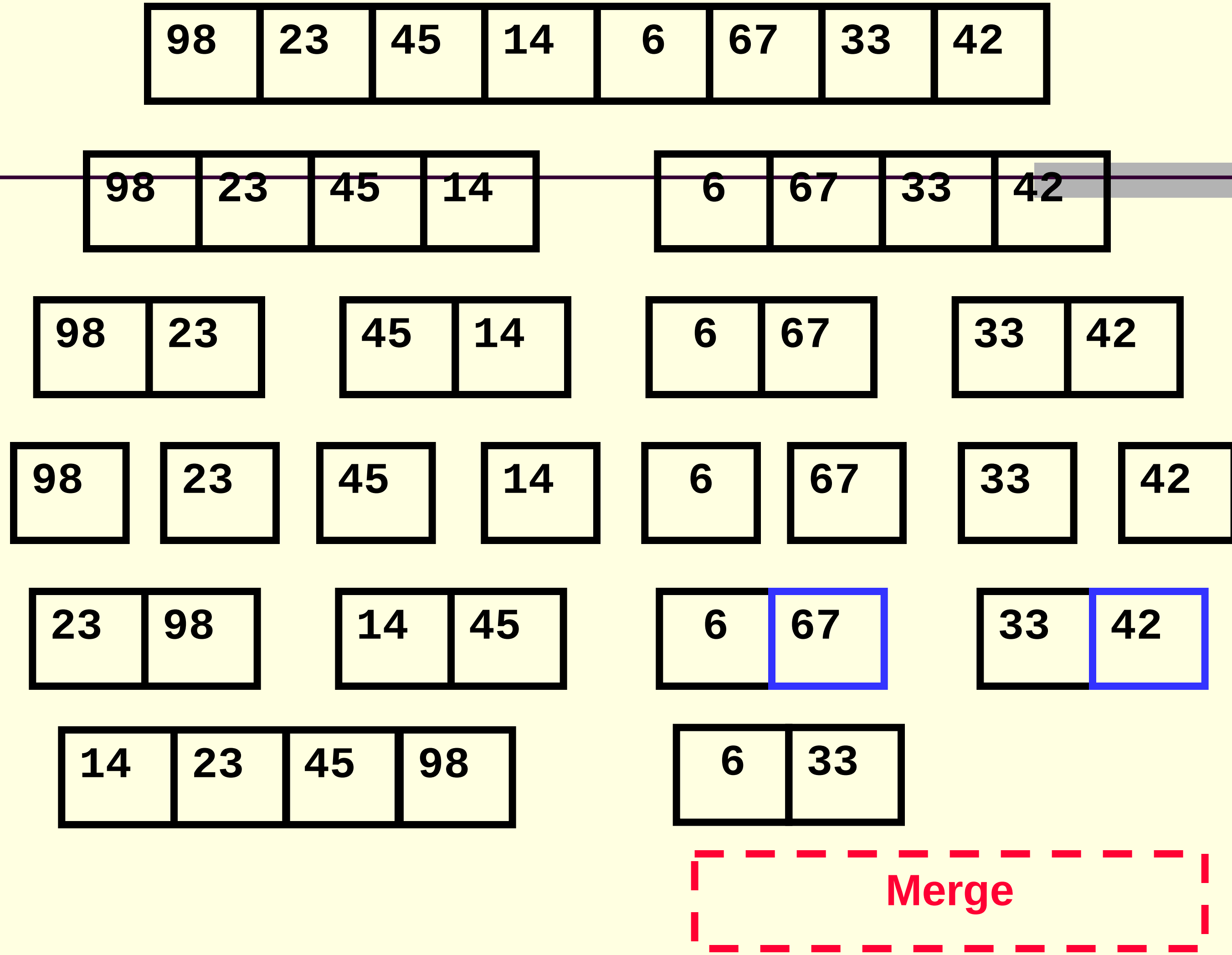
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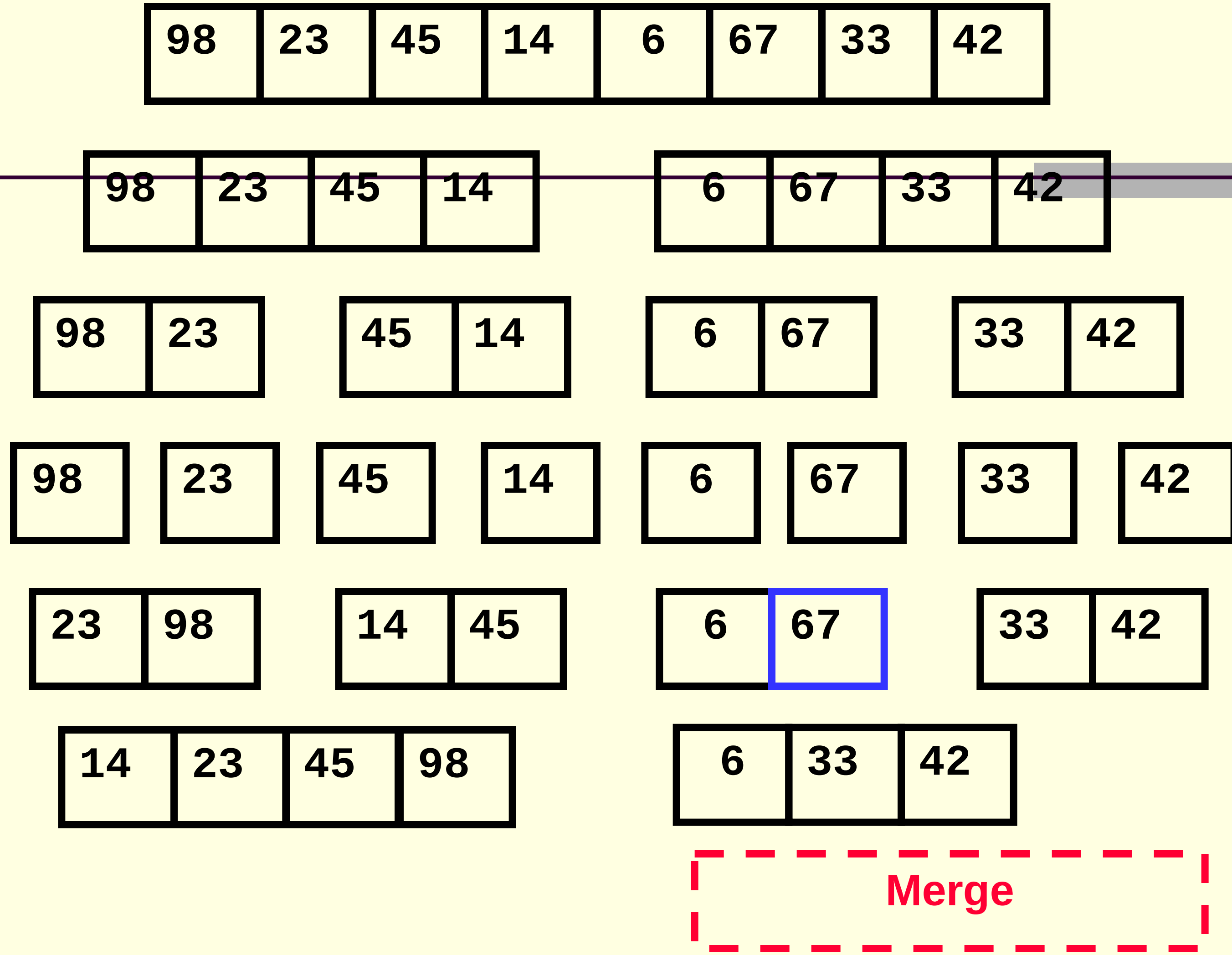
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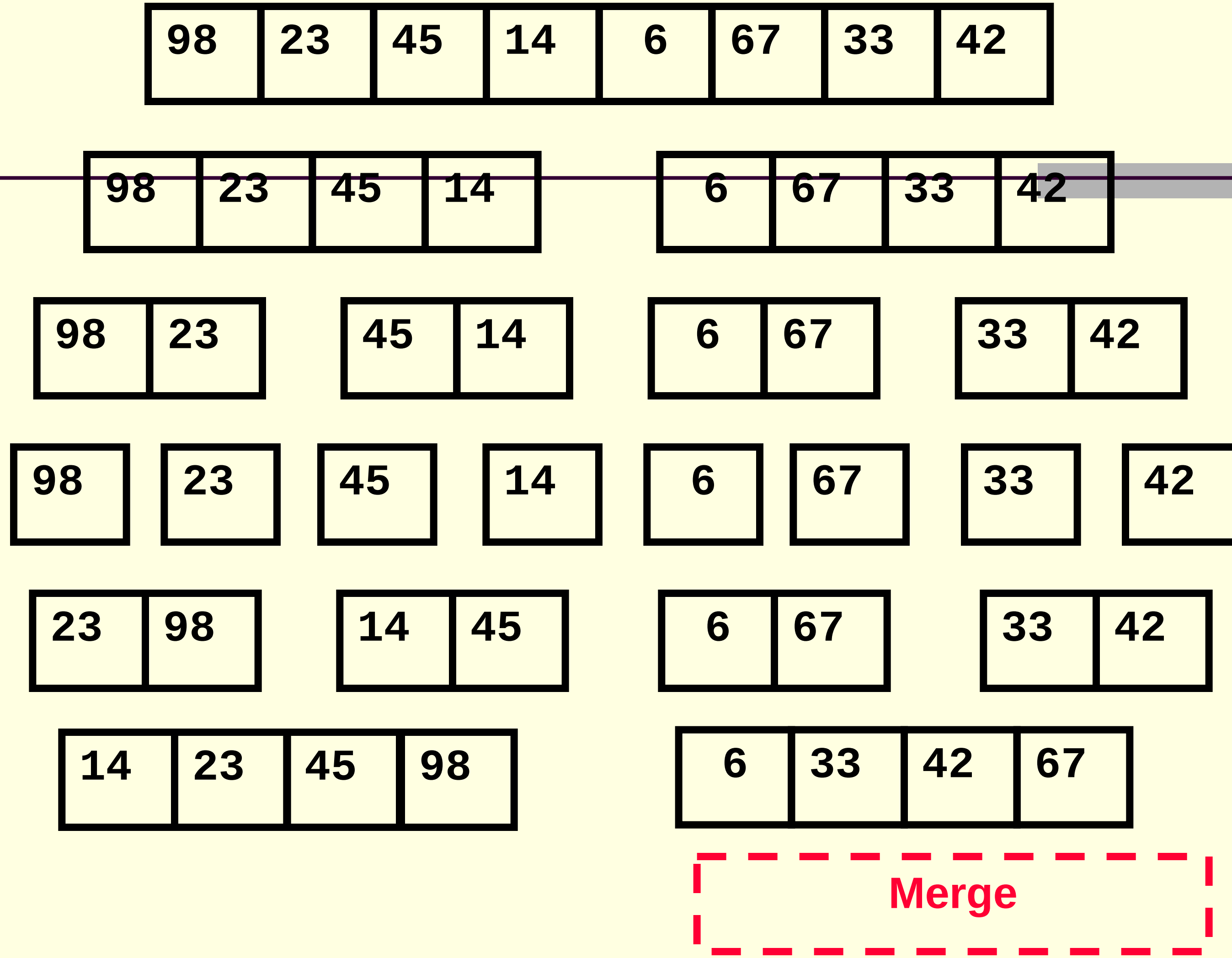
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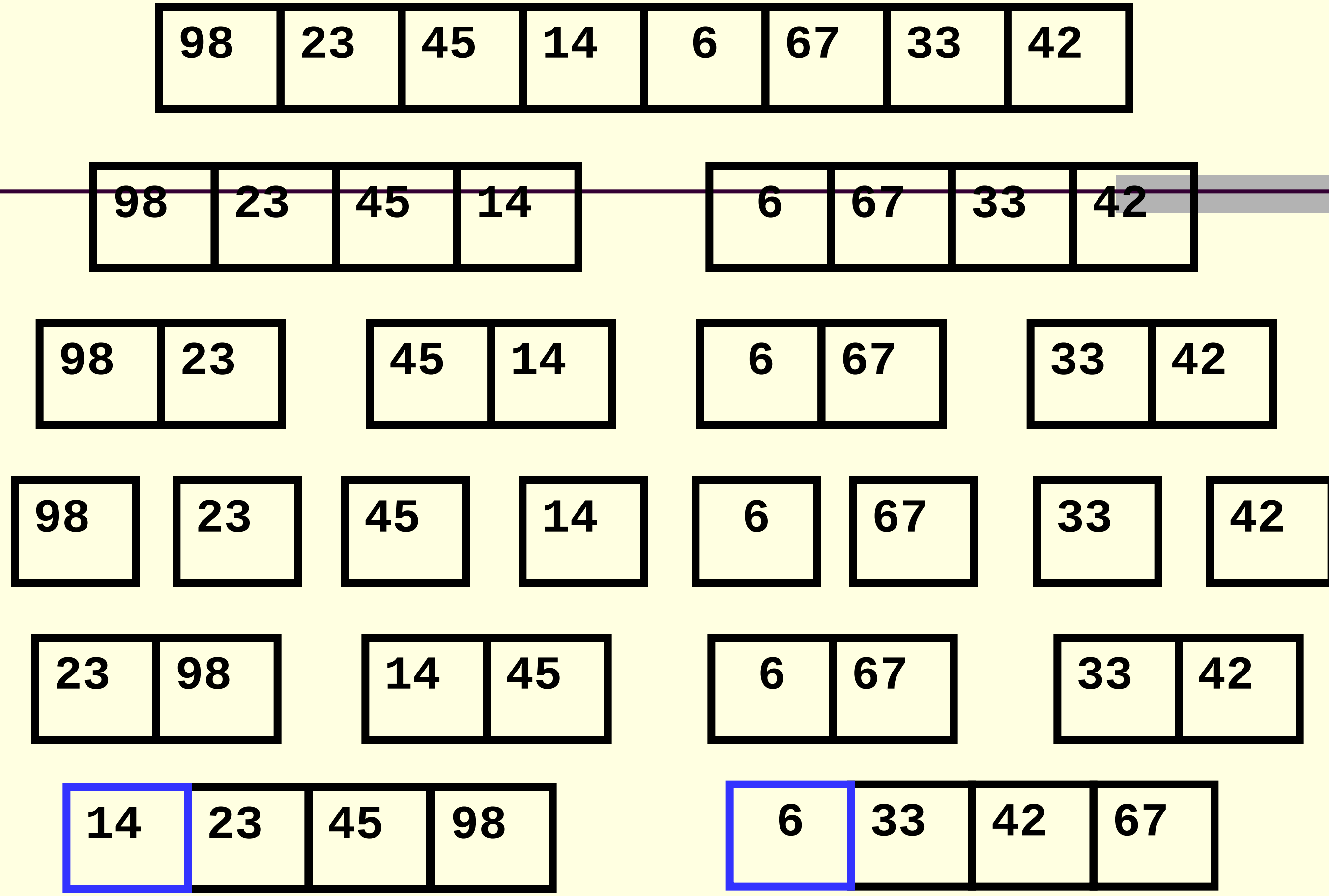
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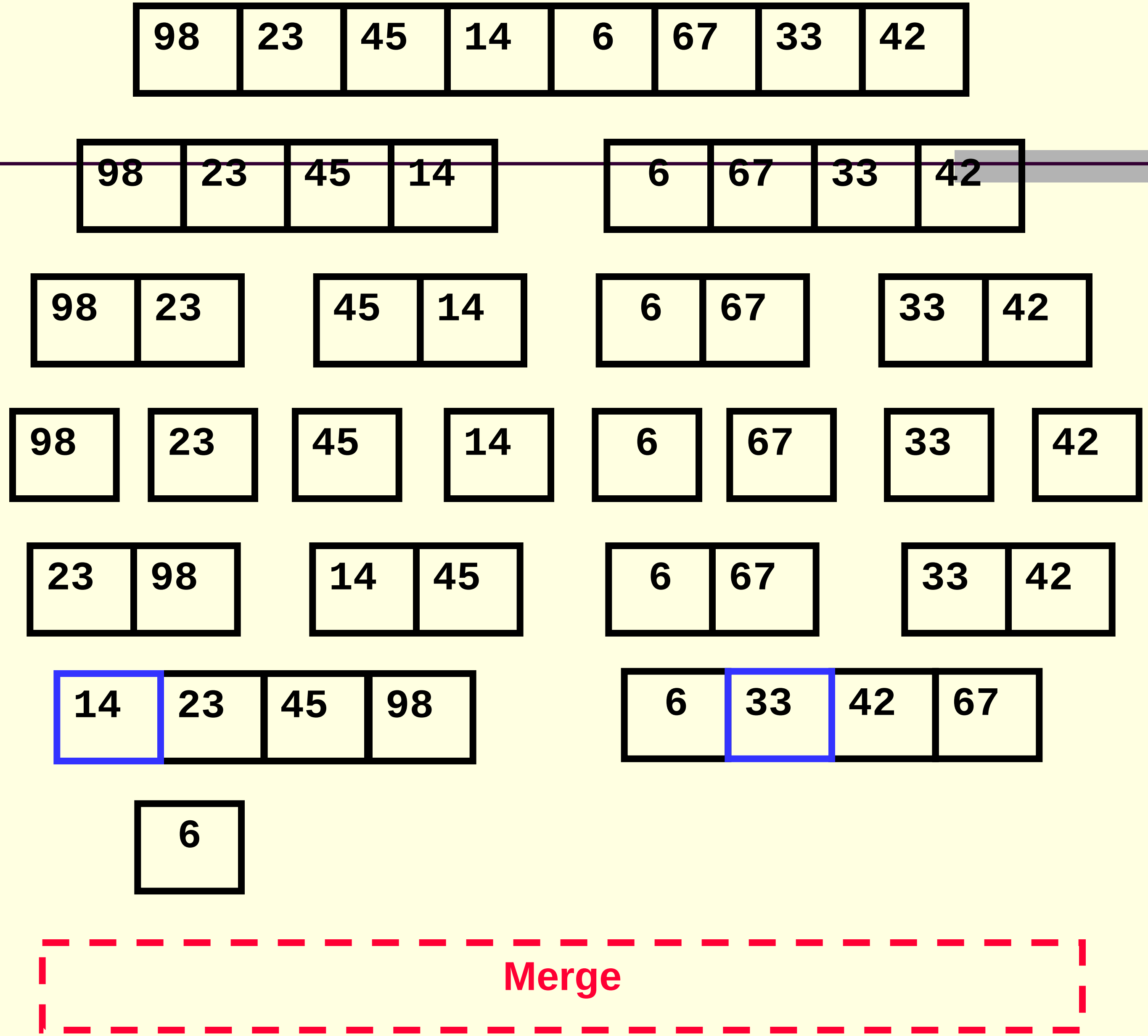
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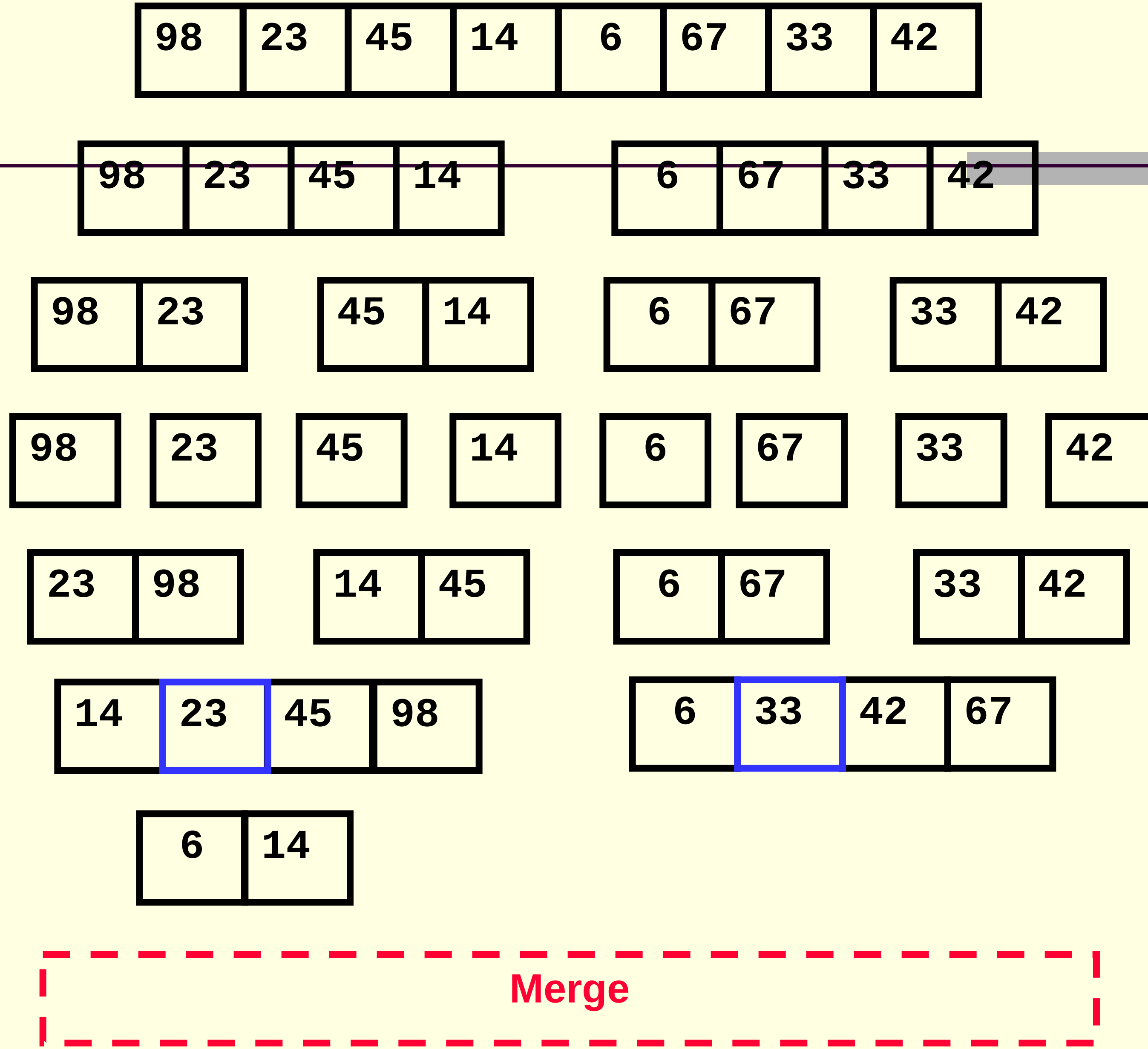

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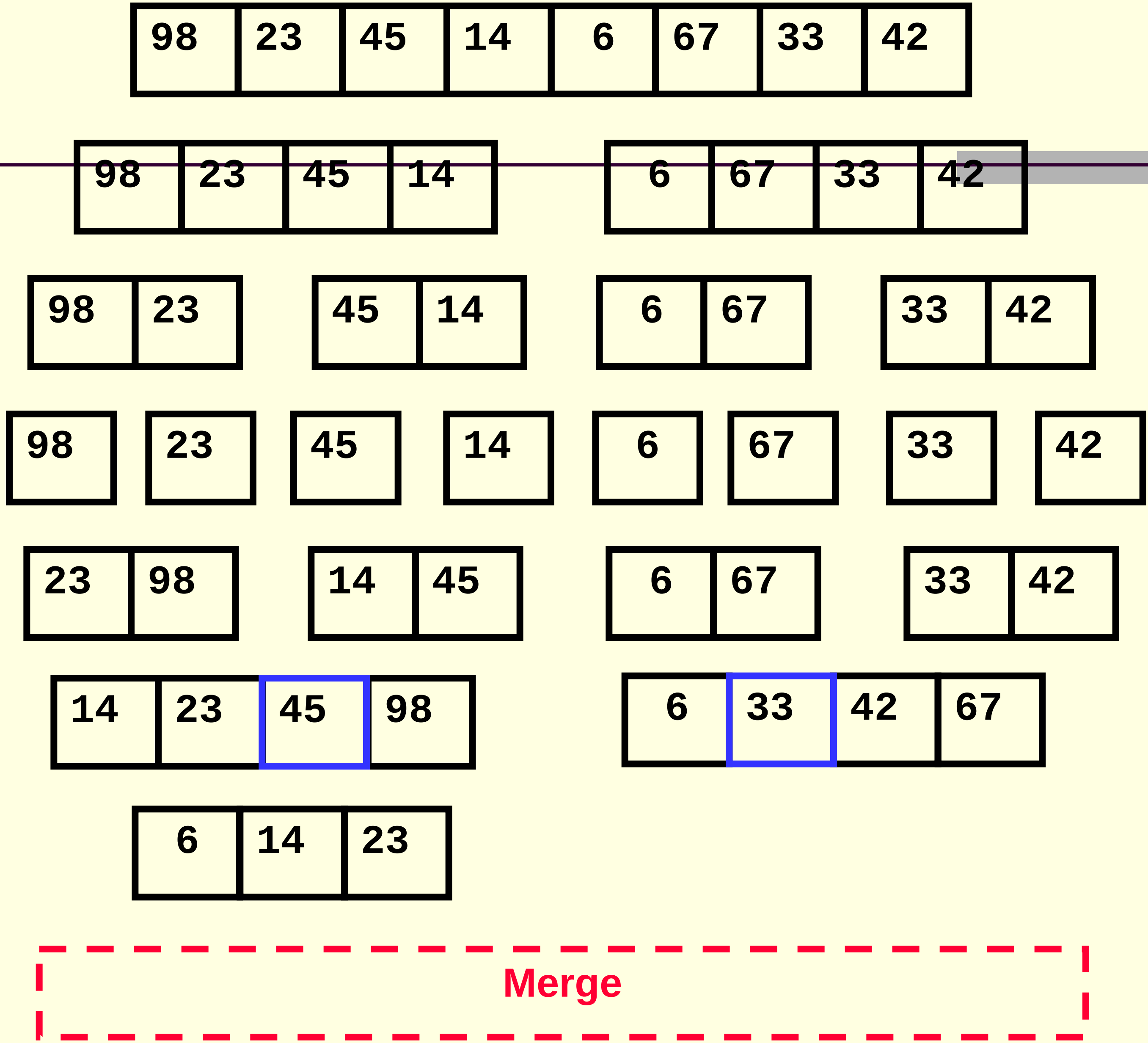
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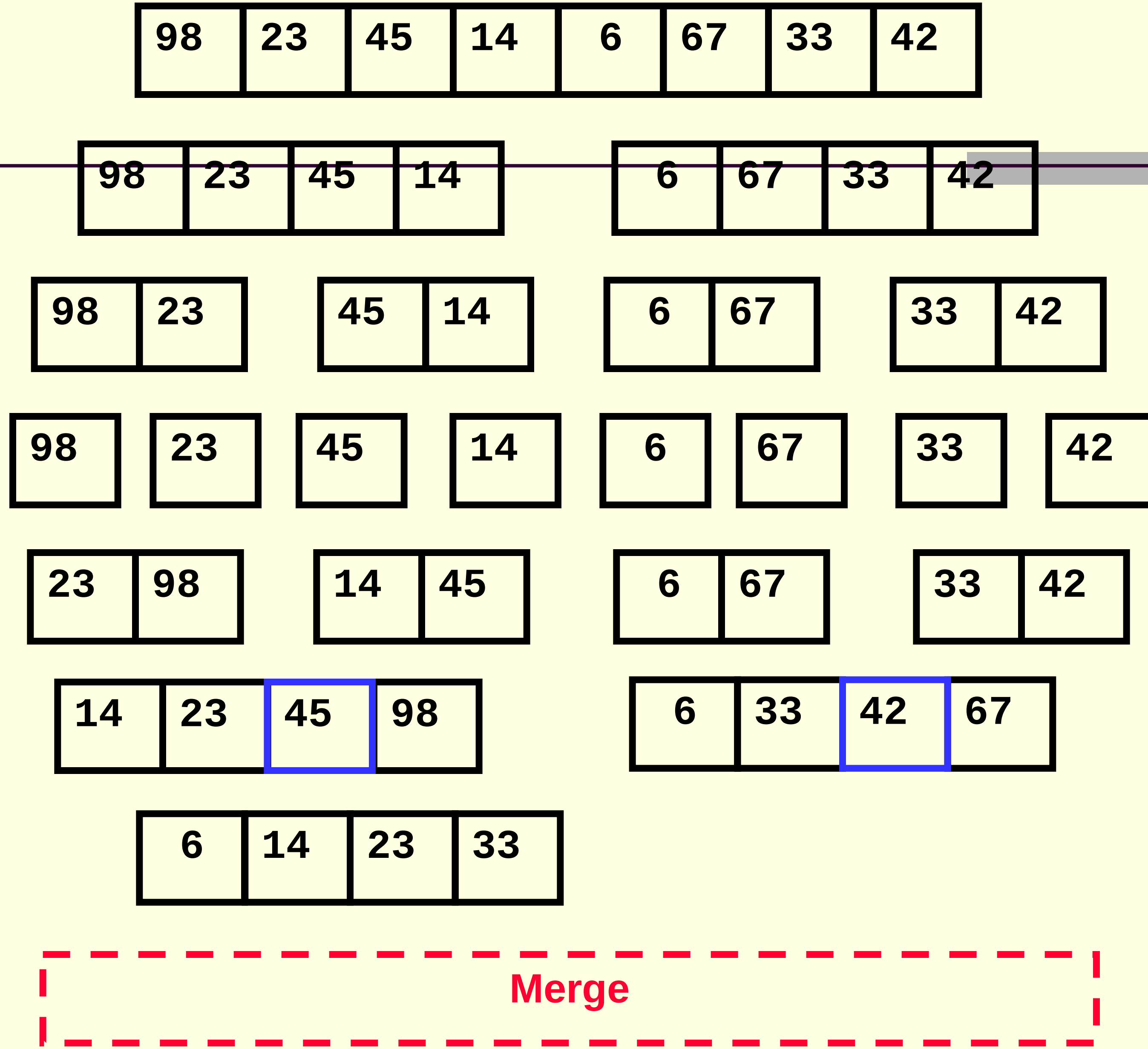
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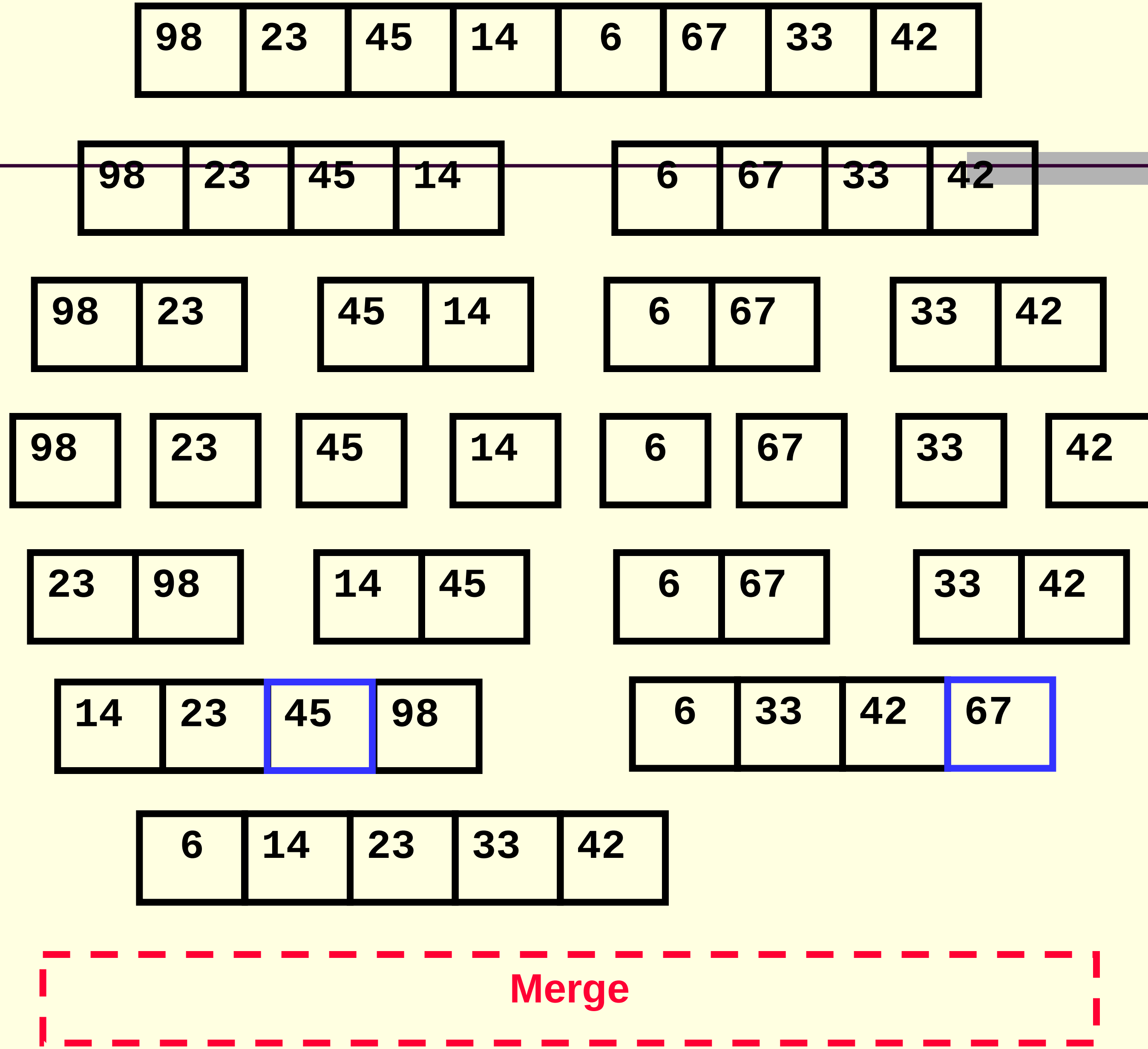
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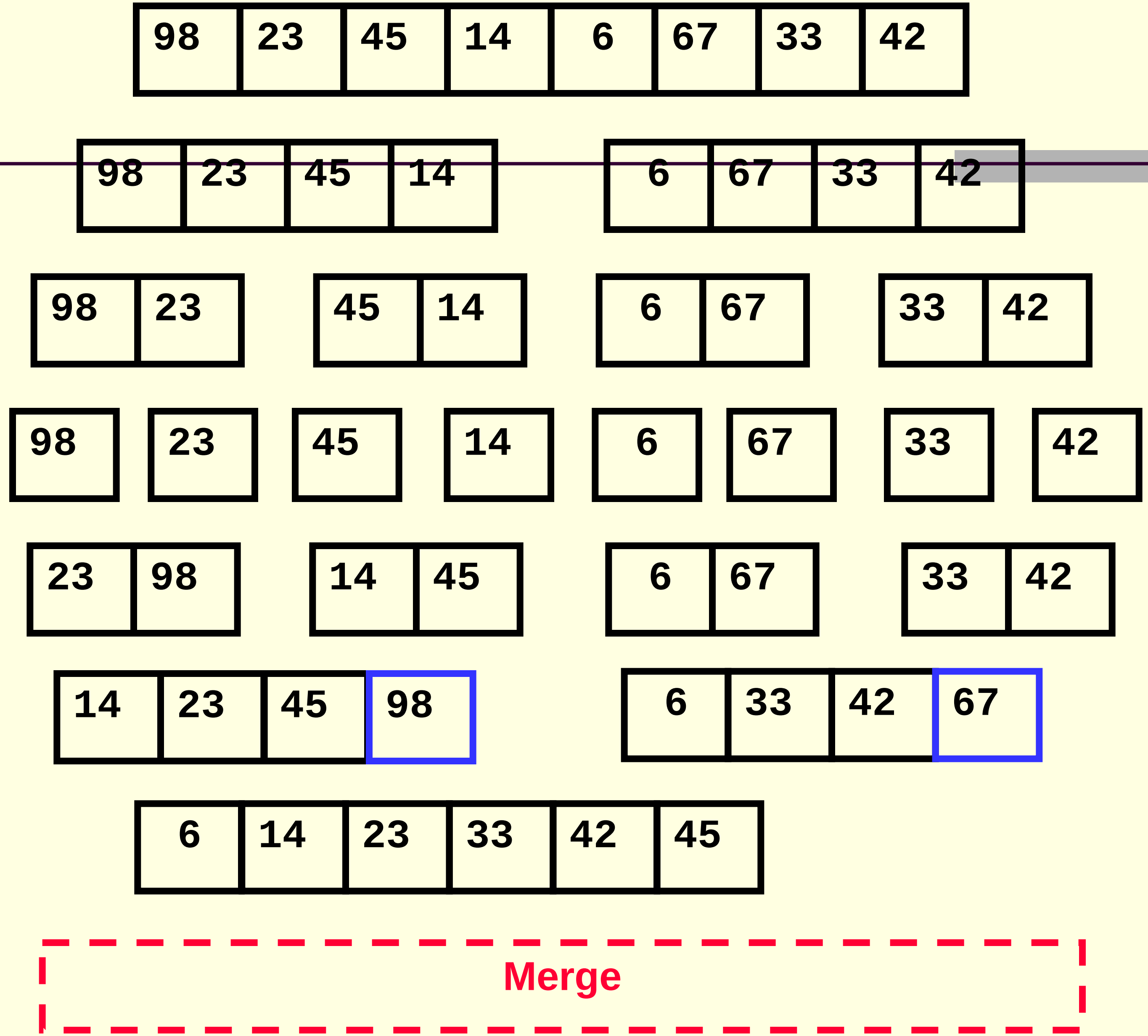
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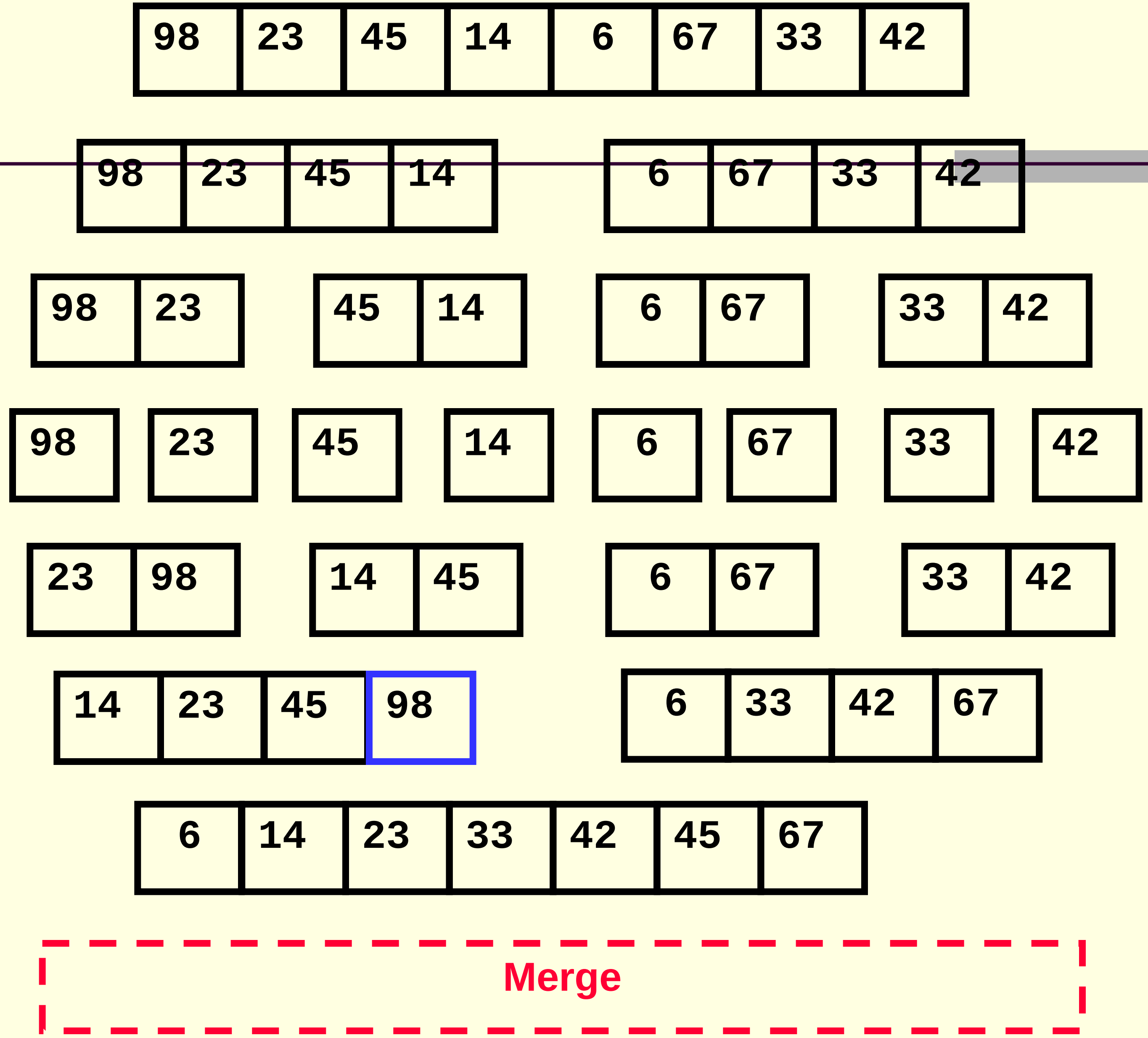
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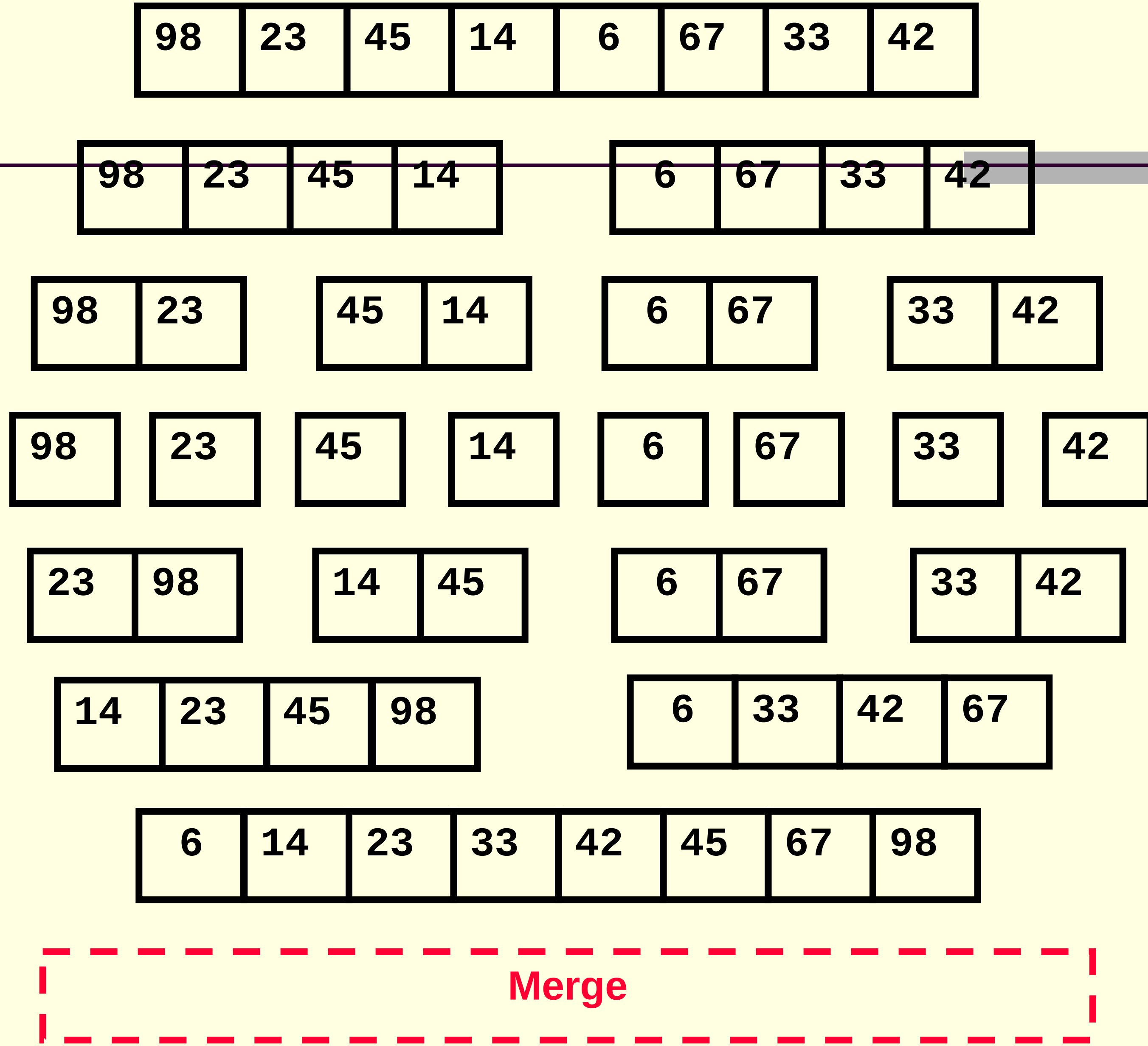
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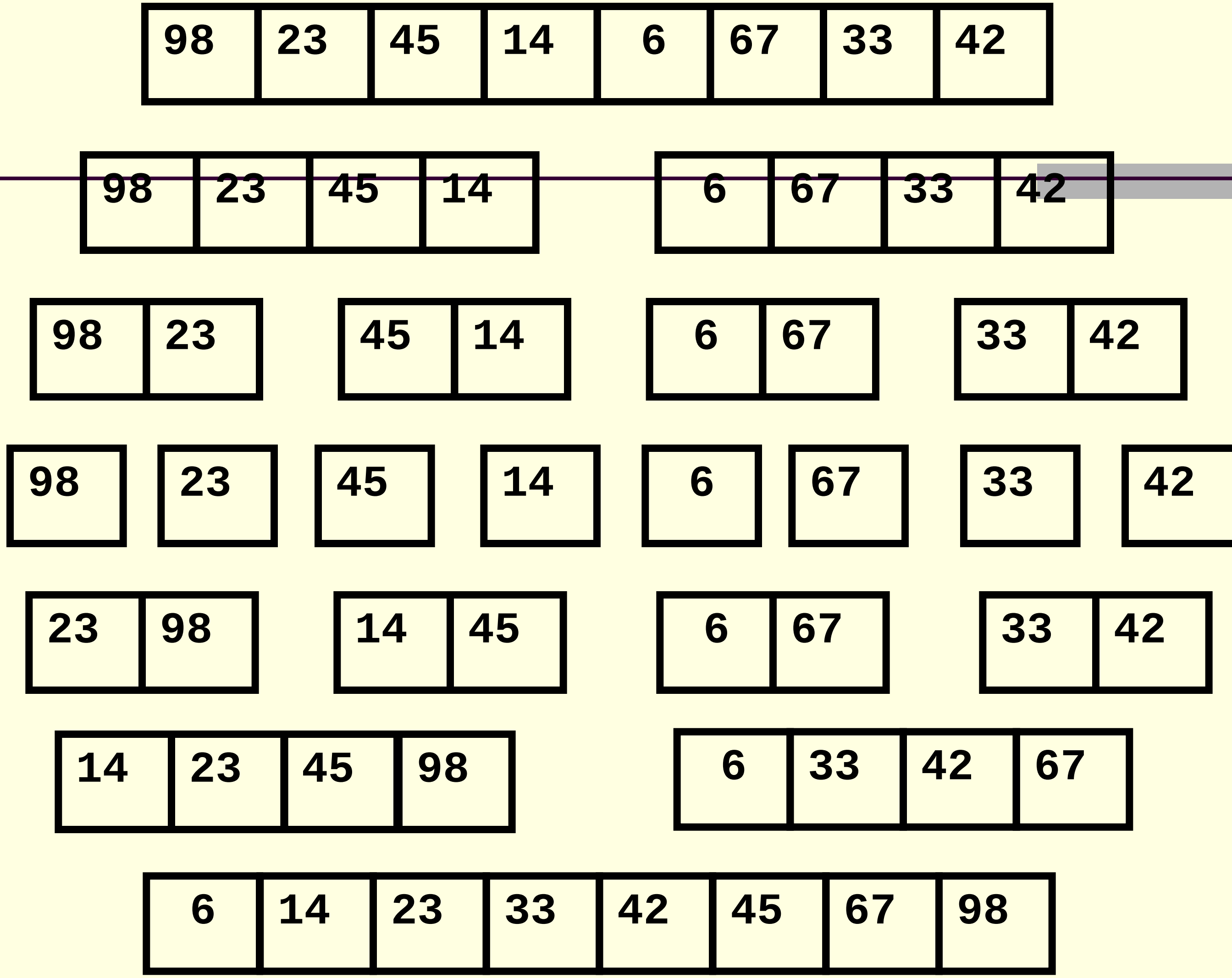
Merge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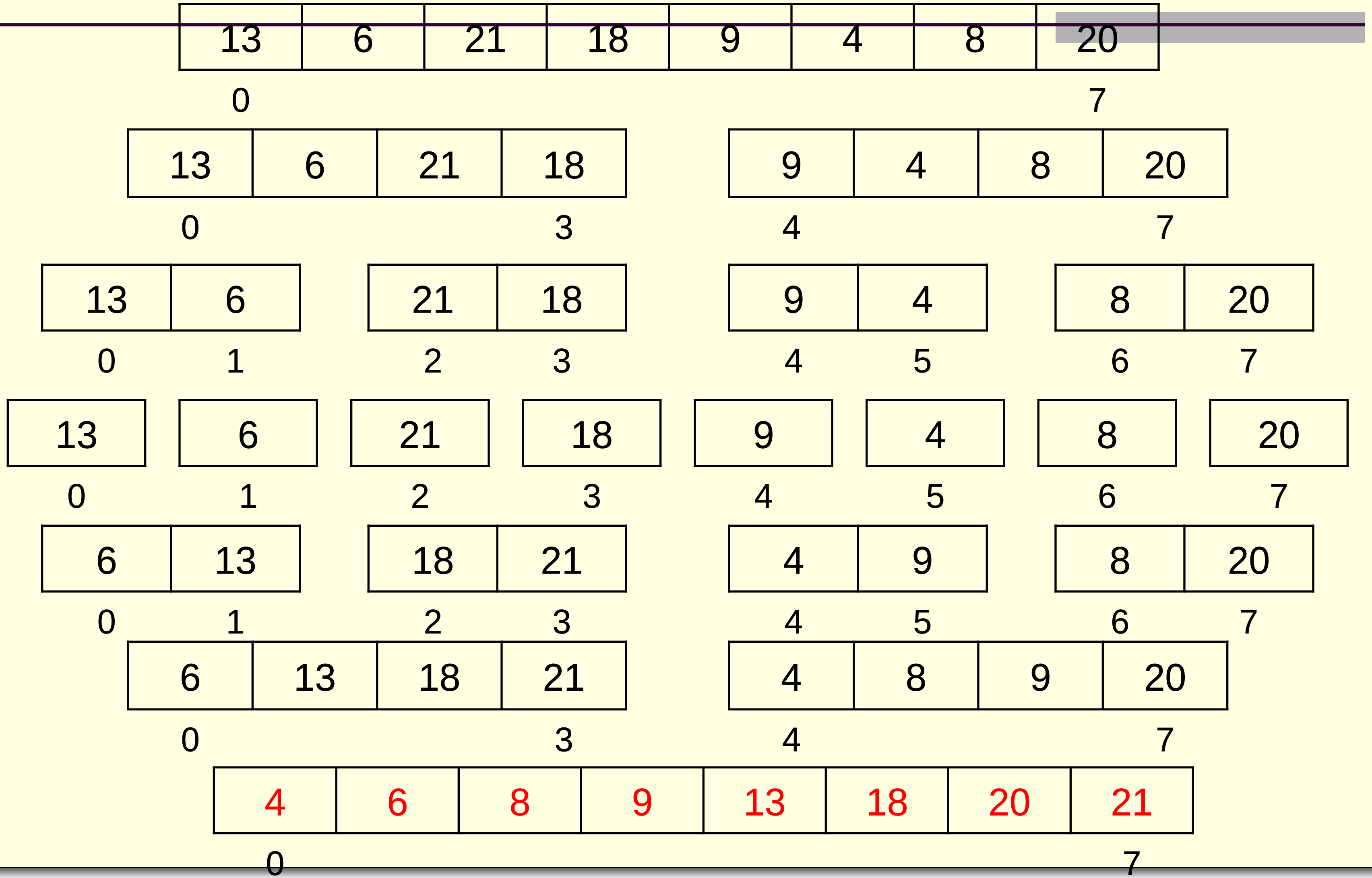
98	23	45	14	6	67	33	42

6	14	23	33	42	45	67	98

Sorting: Merge Sort Example #2

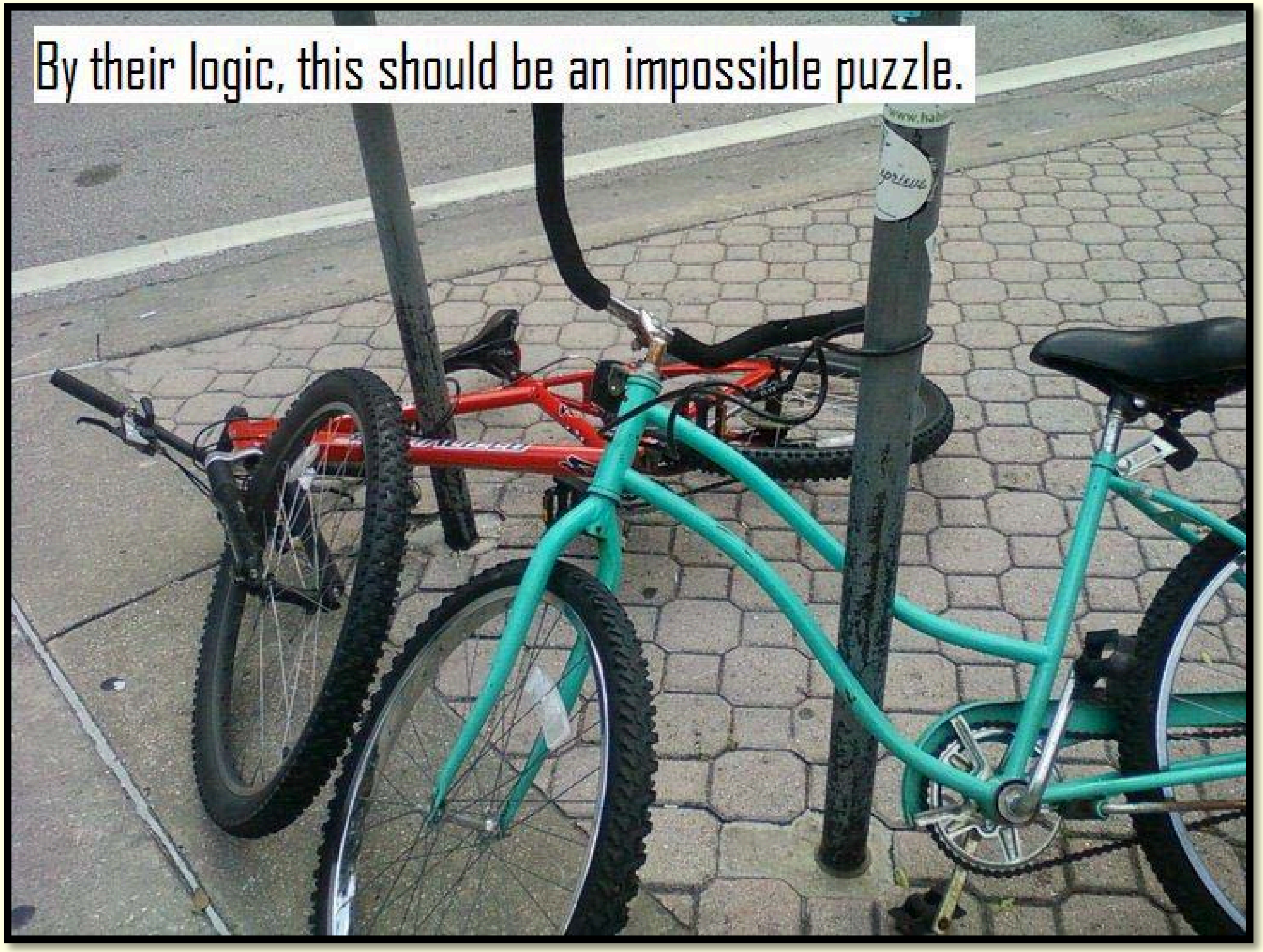


Brief Interlude: FAIL Picture

UCF Daily Bike Fail

Courtesy of
Sean Lunceford

UCF Weekly Bike Fail



Courtesy of
Sean Lunceford

Sorting: Merge Sort

- Merge Sort Code

```
void MergeSort(int values[], int start, int end) {
        int mid;
        // Check if our sorting range is more than one element.
        if (start < end) {

                mid = (start+end)/2;

                // Sort the first half of the values.
                MergeSort(values, start, mid);

                // Sort the last half of the values.
                MergeSort(values, mid+1, end);

                // Put it all together.
                Merge(values, start, mid+1, end);
        }
}
```

Sorting: Merge Sort

- Merge Code
  - This code is longer
  - And a bit convoluted
    - But all it does it Merge the values from two arrays into one larger array
    - Of course, keeping the items in order
    - Just like the example shown earlier in the slides
  - Code can be found here on the website:
    - http://www.cs.ucf.edu/courses/cop3502/sum2011/programs/sorting/mergesort.c
    - You need to fully understand how this code works
      - Including the Merge function!

Sorting: Merge Sort

- Merge Sort **Analysis**
  - Again, here are the steps of Merge Sort:
  1) Merge Sort the first half of the list
  2) Merge Sort the second half of the list
  3) Merge both halves together

  - Let T(n) be the running time of Merge Sort on an input size n
  - Then we have:
    - T(n) = (Time in step 1) + (Time in step 2) + (Time in step 3)

Sorting: Merge Sort

- Merge Sort Analysis
  - T(n): running time of Merge Sort on input size n
  - Therefore, we have:
    - T(n) = (Time in step 1) + (Time in step 2) + (Time in step 3)
  - Notice that Step 1 and Step 2 are sorting problems also
    - **<u>But they are of size n/2…we are halving the input</u>**
  - And the Merge function runs in O(n) time
  - Thus, we get the following equation for T(n)
  - $T(n) = T(n/2) + T(n/2) + O(n)$
  - $T(n) = 2T(n/2) + O(n)$

Sorting: Merge Sort

- Merge Sort Analysis
  - T(n) = 2T(n/2) + O(n)
  - For the time being, let's simplify O(n) to just n
  - T(n) = 2T(n/2) + n
  - and we know that T(1) = 1
  - So we now have a Recurrence Relation
  - Is it solved?
    - NO!
  - Why?
    - Damn T's!

Sorting: Merge Sort

- Merge Sort Analysis
  - $T(n) = 2T(n/2) + n$ and $T(1) = 1$
  - So we need to solve this, by removing the T(…)'s from the right hand side
  - Then T(n) will be in its closed form
  - And we can state its Big-O running time
  - We do this in steps
    - We replace n with **n/2** on both sides of the equation
    - We plug the result back in
    - And then we do it again…till a "light goes off" and we see something

Sorting: Merge Sort

- Merge Sort Analysis
  - T(n) = 2T(n/2) + n and T(1) = 1
  - Do you know what T(n/2) equals
    - Does it equal 2,125 operations? We don't know!
  - So we need to develop an equation for T(n/2)
  - How?
  - Take the original equation shown above
  - **Wherever you see an 'n', substitute with 'n/2'**
  - T(n/2) = 2T(n/4) + n/2
  - So now we have an equation for T(n/2)

Sorting: Merge Sort

- Merge Sort Analysis
  - $T(n) = 2T(n/2) + n$ and $T(1) = 1$
  - $T(n/2) = 2T(n/4) + n/2$
  - So now we have an equation for $T(n/2)$
    - We can take this equation and substitute it back into the original equation
  - $T(n) = 2T(n/2) + n = 2[2T(n/4) + n/2] + n$
    - now simplify
  - $T(n) = 4T(n/4) + 2n$
    - Same thing here: do you know what $T(n/4)$ equals?
    - No we don't! So we need to develop an eqn for $T(n/4)$

Sorting: Merge Sort

- Merge Sort Analysis
  - $T(n) = 2T(n/2) + n$ and $T(1) = 1$
  - $T(n/2) = 2T(n/4) + n/2$
  - $T(n) = 4T(n/4) + 2n$
    - Same thing here: do you know what $T(n/4)$ equals?
    - No we don't! So we need to develop an eqn for $T(n/4)$
    - Take the eqn above and again substitute 'n/2' for 'n'
  - $T(n/4) = 2T(n/8) + n/4$
  - So now we have an equation for $T(n/4)$
    - We can take this equation and substitute it back the equation that we currently have in terms of $T(n/4)$

Sorting: Merge Sort

- Merge Sort Analysis
  - $T(n) = 2T(n/2) + n$ and $T(1) = 1$
  - $T(n/2) = 2T(n/4) + n/2$
  - $T(n) = 4T(n/4) + 2n$
  - $T(n/4) = 2T(n/8) + n/4$
  - So now we have an equation for $T(n/4)$
    - We can take this equation and substitute it back the equation that we currently have in terms of $T(n/4)$
  - $T(n) = 4T(n/4) + 2n = 4[2T(n/8) + n/4] + 2n$
    - Simplify a bit
  - $T(n) = 8T(n/8) + 3n$

Sorting: Merge Sort

- Merge Sort Analysis
  - So now we have three equations for T(n):
  - $T(n) = 2T(n/2) + n$ ← 1st step of recursion
  - $T(n) = 4T(n/4) + 2n$ ← 2nd step of recursion
  - $T(n) = 8T(n/8) + 3n$ ← 3rd step of recursion
  - So on the kth step/stage of the recursion, we get a generalized recurrence relation:
  - $T(n) = 2^kT(n/2^k) + kn$ ← k^{th} step of recursion
  - Whew! So now we're done right? Wrong!

Sorting: Merge Sort

- Merge Sort Analysis
  - So on the kth step/stage of the recursion, we get a generalized recurrence relation:
  - $T(n) = 2^k T(n/2^k) + kn$
  - We need to get rid of the T(…)'s on the right side
  - Remember, we know T(1) = 1
  - So we make a substitution:
    - Let $n = 2^k$
    - and also solve for k
    - $k = \log_2 n$
  - Plug these back in…

Sorting: Merge Sort

- Merge Sort Analysis
  - So on the kth step/stage of the recursion, we get a generalized recurrence relation:
  - $T(n) = 2^k T(n/2^k) + kn$
    - Let $n = 2^k$
    - and also solve for k
    - $k = \log_2 n$
  - Plug these back in…
  - $T(n) = 2^{\log_2 n} T(n/n) + (\log_2 n)n$
  - $T(n) = n*T(1) + n\log n = n + n*\log n$
  - So Merge Sort runs in O(n*logn) time

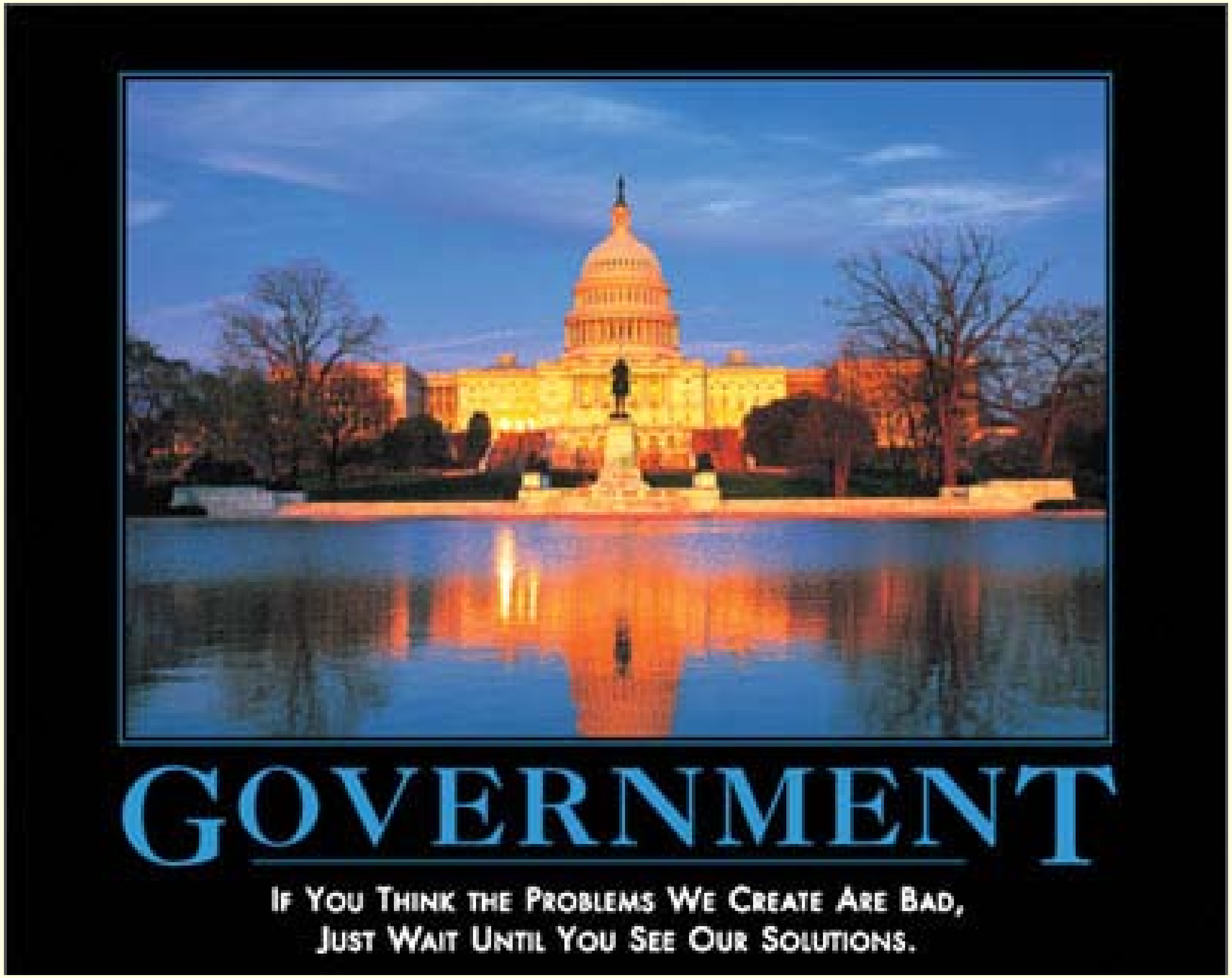
Sorting: Merge Sort

- Merge Sort Summary
  - Avoids all the unnecessary swaps of n^2 sorts
  - Uses recursion to split up a list until we get to "lists" of 1 or 0 elements
  - Uses a Merge function to merge ("sort") these smaller lists into larger lists
  - Is MUCH faster than n^2 sorts
  - Merge Sort runs in O(nlogn) time

Sorting: Merge Sort

WASN'T THAT THE COOLEST!

Daily Demotivator



Sorting: Merge Sort

Computer Science Department
University of Central Florida

COP 3502 – Computer Science I