Computer Science I – Summer 2011
Recitation #5: Summation Practice (<u>will be tested AGAIN on Final Exam</u>)

1. Determine a closed-form solution for the following sum in terms of n: $\sum_{i=n+1}^{2n}\left(2i+3n^2\right)$

$$\sum_{i=n+1}^{2n}\left(2i+3n^2\right)=\sum_{i=n+1}^{2n}2i+\sum_{i=n+1}^{2n}3n^2=\sum_{i=1}^{2n}2i-\sum_{i=1}^{n}2i+\sum_{i=1}^{2n}3n^2-\sum_{i=1}^{n}3n^2$$
$$=2\frac{2n(2n+1)}{2}-2\frac{n(n+1)}{2}+6n^3-3n^3=2n(2n+1)-n(n+1)+3n^3$$
$$=n\left(2(2n+1)-(n+1)+3n^2\right)=n\left(4n+2-n-1+3n^2\right)=n\left(3n^2+3n+1\right)$$

2. Determine a closed-form solution for the following sum in terms of n: $\sum_{i=0}^{n}\left(2\sum_{j=n+1}^{3n}(i+j)\right)$

$$\sum_{i=0}^{n}\left(2\sum_{j=n+1}^{3n}(i+j)\right)=\sum_{i=0}^{n}\left(2(\sum_{j=n+1}^{3n}i)+2(\sum_{j=n+1}^{3n}j)\right)$$
$$=\sum_{i=0}^{n}\left(2i(2n)+2(\sum_{j=1}^{3n}j-\sum_{j=1}^{n}j)\right)$$
$$=\sum_{i=0}^{n}\left(4in+2(\frac{3n(3n+1)}{2}-\frac{n(n+1)}{2})\right)$$
$$=\sum_{i=0}^{n}\left(4in+3n(3n+1)-n(n+1)\right)$$
$$=\sum_{i=0}^{n}\left(4in+n[3(3n+1)-(n+1)]\right)$$
$$=\sum_{i=0}^{n}\left(4in+n[9n+3-n-1]\right)$$
$$=\sum_{i=0}^{n}\left(4in+n[9n+3-n-1]\right)$$
$$=\sum_{i=0}^{n}\left(4in+n(8n+2)\right)$$
$$=\frac{4n(n+1)n}{2}+n(8n+2)(n+1)$$
$$=2n^2(n+1)+n(8n+2)(n+1)$$
$$=2n(n+1)[n+(4n+1)]$$
$$=2n(n+1)(5n+1)=10n^3+12n^2+2n$$