Towers(n)

Towers(n-1)
Move disk #n
Towers(n-1)

Towers(n, start, end)

Towers(n-1, start, mid)
Move disk n from start to end
Towers(n-1, mid, end)

1, 2, 1, 3, 1, 2, 1, 4, 1, 2, 1, 3, 1, 2, 1, 5

1, 2, 1, 3, 1, 2, 1 4, 1, 2, 1, 3, 1, 2, 1 6

Look at # MOVES

Let $T(n)$ = # of moves with n disks.

move $n-1$ s→m; move bot s→d; move $n-1$ m→e

$$T(n) = T(n-1) + 1 + T(n-1)$$

$$T(n) = 2T(n-1) + 1, \quad T(1) = 1$$

recurrence relation

Iteration Technique

$T(n) = 2T(n-1) + 1$,

$T(n-1) = 2T(n-2) + 1$

$T(n-2) = 2T(n-3) + 1$

$= 2[2T(n-2) + 1] + 1$

$= 4T(n-2) + 2 + 1$

$= 4T(n-2) + 3$

$= 4[2T(n-3) + 1] + 3$

$= 8T(n-3) + 4 + 3$

$= 8T(n-3) + 7$

→ after k steps, $= 2^k T(n-k) + (2^k - 1)$

→ I only know $T(1)$. For what k does $n-k = 1$?

$k = n-1$

Let $k = n-1$.

$$T(n) = 2^{n-1} T(n-(n-1)) + 2^{n-1} - 1$$

$$= 2^{n-1} T(1) + 2^{n-1} - 1$$

$$= 2^{n-1} + 2^{n-1} - 1 = 2^{n-1}(1+1) - 1$$

$$= \boxed{2^n - 1} \qquad = 2^{n-1}(2) - 1$$

$O(2^n)$

for 5 disks 31 moves

64 disks $2^{64} - 1$ = big #

10^{18} or 10^{19}

$T(n) = 4T(\frac{n}{2}) + n$, $T(1) = 1$

$= 4\left[4T(\frac{n}{4}) + \frac{n}{2}\right] + n$

$= 16T(\frac{n}{4}) + 2n + n$

$= 16T(\frac{n}{4}) + 3n$

$= 16\left[4T(\frac{n}{8}) + \frac{n}{4}\right] + 3n$

$= 64T(\frac{n}{8}) + 4n + 3n$

$= 64T(\frac{n}{8}) + 7n$

$T(\frac{n}{2}) = 4T(\frac{n}{4}) + \frac{n}{2}$

$T(\frac{n}{4}) = 4T(\frac{n}{8}) + \frac{n}{4}$

After k iterations $= 4^k T(\frac{n}{2^k}) + (2^k - 1)n$

Set $\frac{n}{2^k} = 1 \rightarrow 2^k = n \rightarrow k = \log_2 n$.

Let $k = \log_2 n$, $2^k = n$, $4^k = (2^2)^k = (2^k)^2 = n^2$

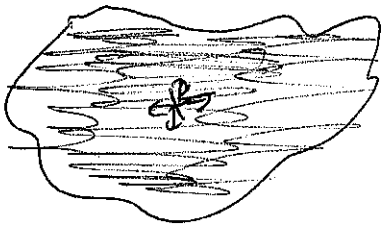
$= n^2 T(1) + (n-1)n$

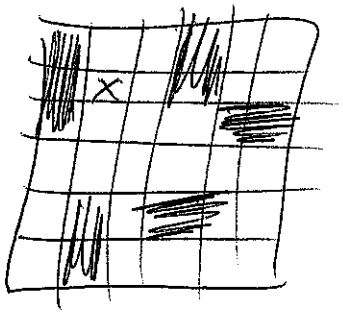
$= n^2 + n^2 - n$

$= 2n^2 - n$

$= O(n^2)$

Flood fill





fill (x, y)

1) fill location (x, y) / Marking that I've ~~xx~~ been there,

2) for each unvisited neighbor (u, v)
fill (u, v)

```
for (i=0 ; i < NUMDIR; i++) {
    inbounds (x+DX[i], y+DY[i]) &&
  if (!visited[x+DX[i]][y+DY[i]])
      fill (x+DX[i], y+DY[i]);
}
```

Minesweeper