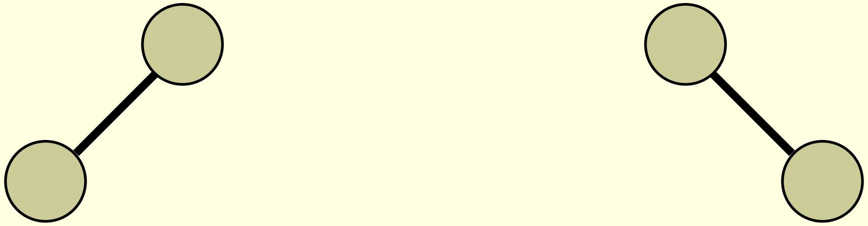
AVL Trees: Insertion Revisited

Computer Science Department
University of Central Florida

COP 3502 – Computer Science I

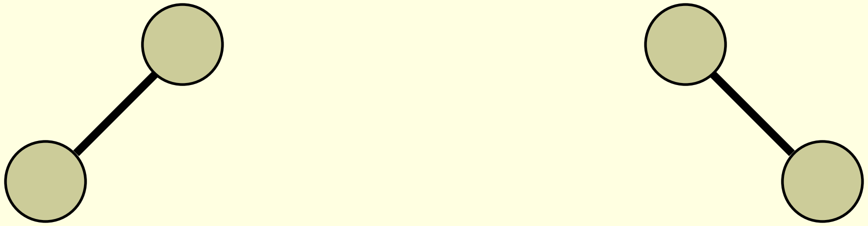
Insertion Revisited

- AVL Trees: Insertion
  - Let’s take another look at insertion into AVL Trees
  - Hopefully this will be a bit easier than previous slides
  - Assuming you only have two nodes in your tree,
  - what are the two possible trees you may have?



AVL Trees: Insertion Revisited

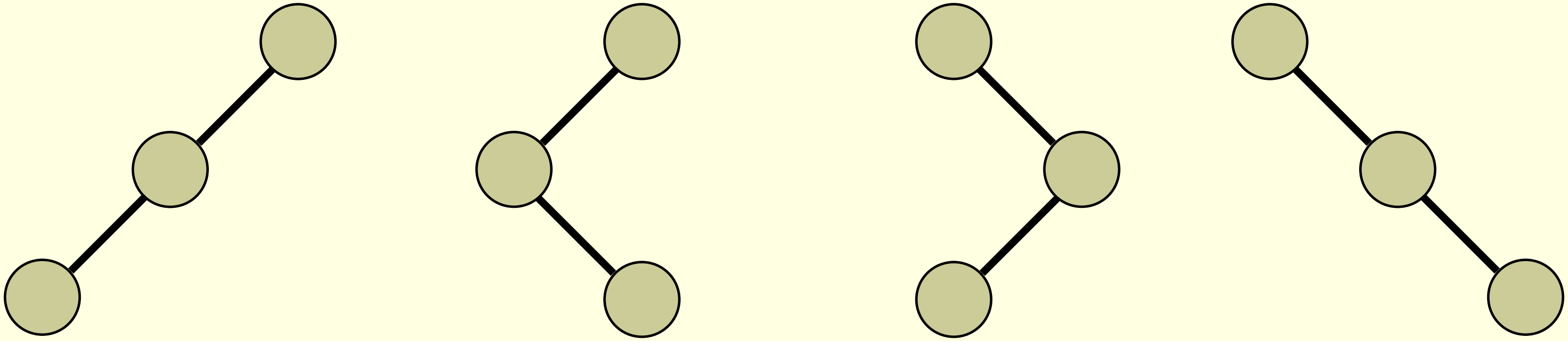
- AVL Trees: Insertion
  - Given these two trees, if we want to create an imbalance, where must we insert?



    - Clearly, we must insert at the lower of the 2 nodes
    - This will create a scenario where the left subtree has a height that is 2 greater than the right subtree
      - Or the opposite for the other tree depicted
  - Now, from these two trees, draw all FOUR possible trees that can be created by inserting a new node

AVL Trees: Insertion Revisited

- AVL Trees: Insertion
  - Here are all four unbalanced trees that we can make from three nodes:



  - Now, label these nodes with the labels A, B, and C
    - Where A is the smallest of the three nodes, B is the middle node, and C is the largest.
    - The inorder traversal of each tree should be A, B, C

AVL Trees: Insertion Revisited

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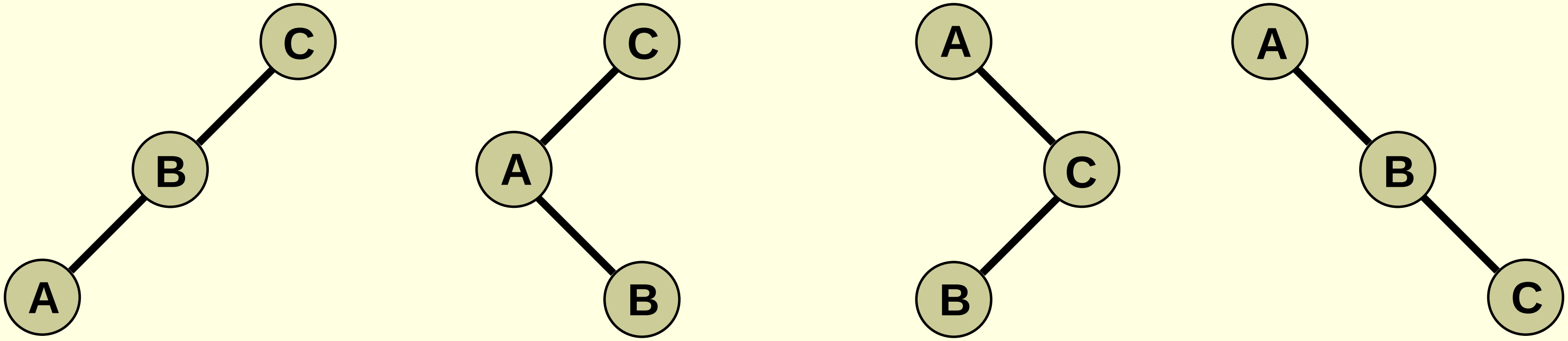
  - Here are all four trees with the node lables in their inorder listing:



- Any time an imbalance occurs, it is localized to three nodes and their four subtrees
  - These are the four possibilities
  - Now we add in the depiction of the four subtrees of A, B, and C

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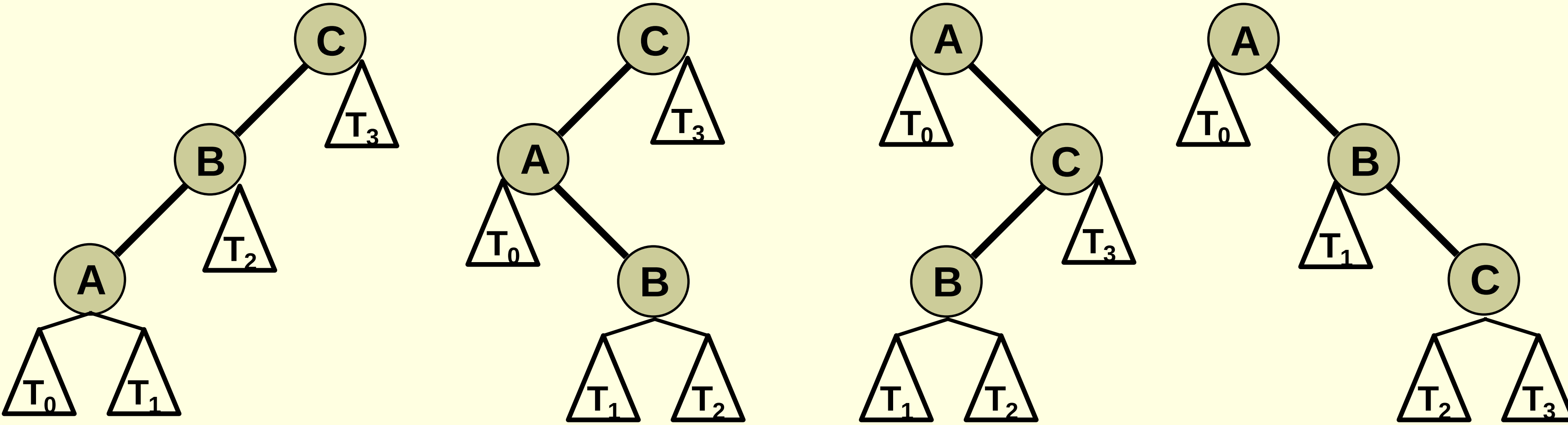
  - Here are all four trees with the node lables in their **inorder** listing with subtrees in their **inorder** listing:



  - We denote the four subtrees as T_0, T_1, T_2, and T_3
  - And they are listed in their inorder listing

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  - So what is the purpose of all this?
  - We said this method is supposedly MUCH easier than dealing with the various rotations of the tree
  - So we’ve done all this labeling
    - Finding nodes ‘A’, ‘B’, and ‘C’ and labeling them as such
  - How they heck does this help us???
  - Here ya go…

AVL Trees: Insertion Revisited

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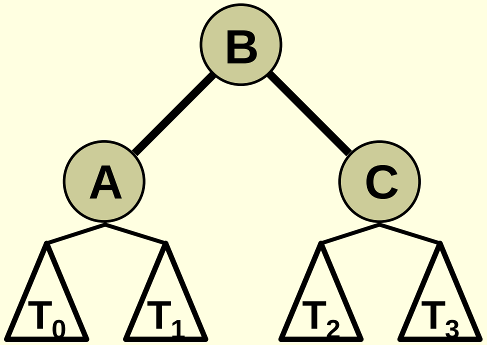
  - Part 1: Once an insertion causes an imbalance, find and label the nodes ‘A’, ‘B’, and ‘C’
  - Part 2: Once the nodes are labeled, no matter what structural imbalance occurred, they can all be fixed the same way:



    - Simply restructure those three nodes, and their four respective subtrees, as shown above, and the imbalance will be corrected!

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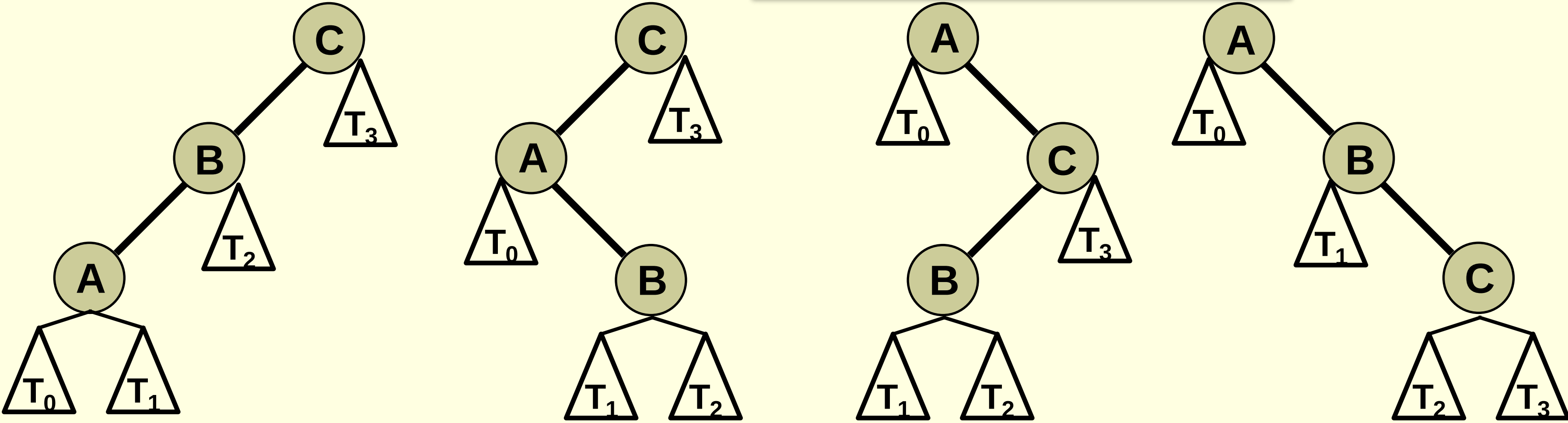
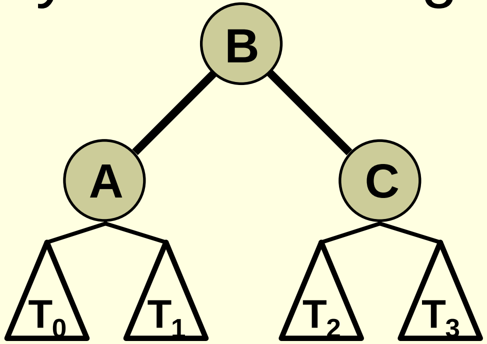
All 4 of these trees:



  - Can be fixed by restructuring into this:



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- AVL Trees: Insertion
  - Here are the basic steps:

1. Do a NORMAL binary search tree insert
   - following the ordering property of a BST
2. Restore the balance of the tree (if needed) based off of this newly inserted leaf node

- The restoration step requires a bit of clarification…

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  - Restoration of a node:
    1. Calculate the heights of the left and right subtrees. Use this to set the (potentially) new height of the node
    2. IF they are within one of each other, recursively restore the parent node.
    3. IF NOT, then perform the appropriate restructuring, described previously, on that particular node.
    4. THEN, recursively call the restore function on the appropriate parent node.
  - Note: one rebalancing will always do the trick, though we must make the recursive calls to move up the tree so that the heights stored at each node are properly recalculated.

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- AVL Trees: Insertion
  - More **Practical Rules**:
    - Insert a node following rules of BST insertion
    - Once you insert a new node, perform the following:

1. Start finding the balance factors of ALL nodes along the path from the insertion point to the root
2. As soon as you find the first node out of balance, mark that node as one of your three “restructuring nodes”
3. Then, take two steps, back down, towards the insertion point and mark those two nodes as well.
4. Label those ‘A, B, C’ nodes appropriately (and subtrees)
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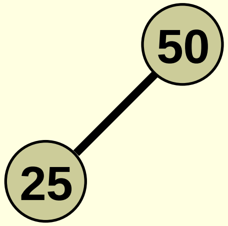
  - Example 1:
    - The most simple insert into an AVL tree, which dictates a rebalance, is inserting a third node in an AVL tree that only has two nodes.
    - Given this tree:



      - We insert a node with the value 10
      - This node clearly goes to the left of 25

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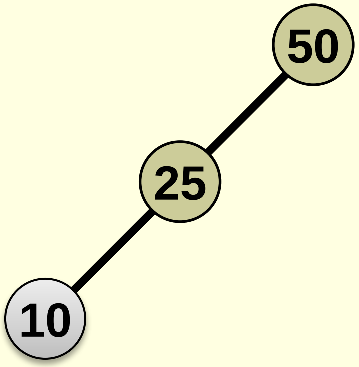

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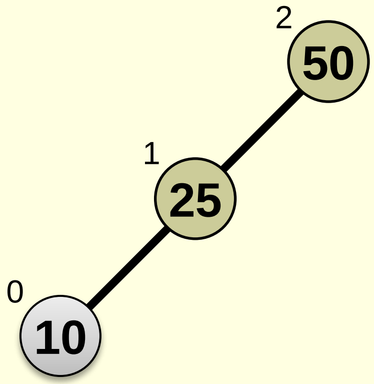

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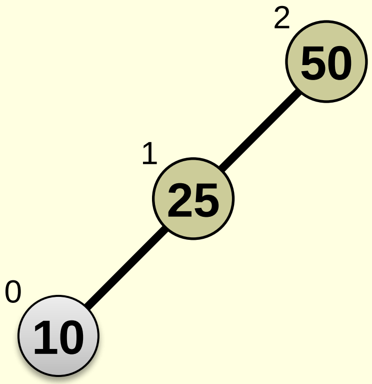

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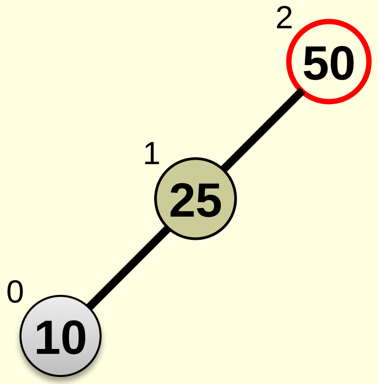

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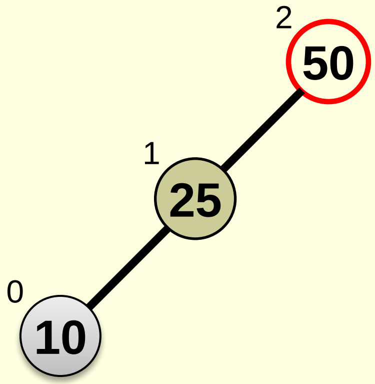

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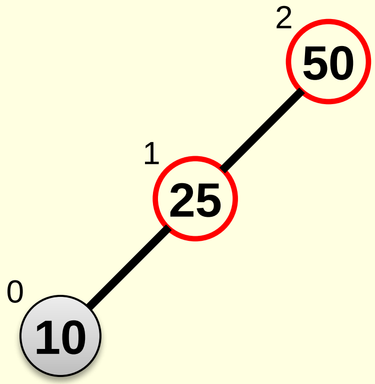

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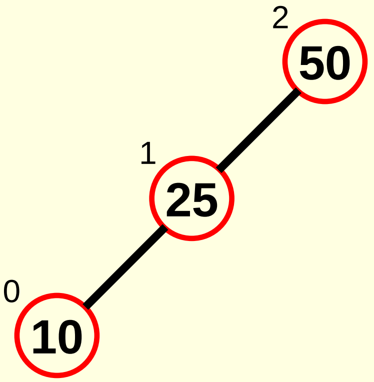

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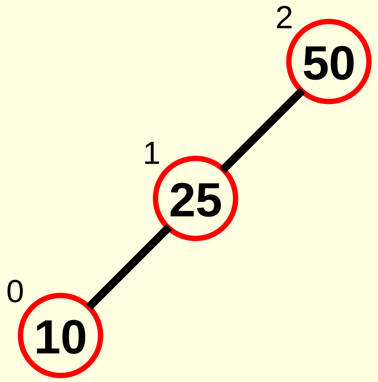

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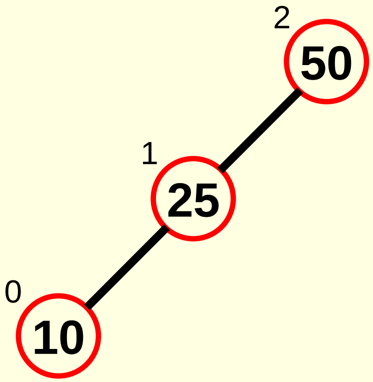

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4. Label those ‘A, B, C’ nodes appropriately (and subtrees)
   - Remember, of the three nodes:
   - ➢ The smallest node should be labeled ‘A’
   - ➢ The middle node should be labeled ‘B’
   - ➢ The largest node should be labeled ‘C’

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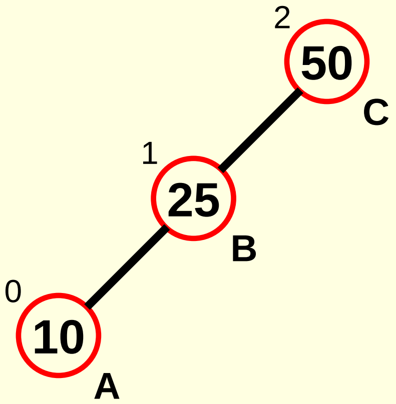

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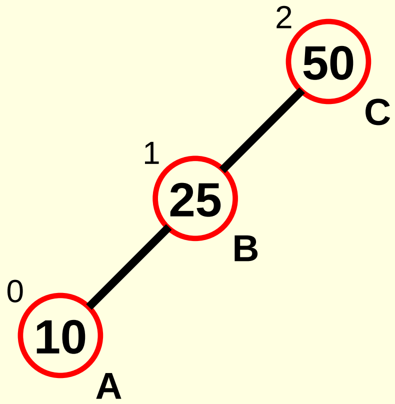

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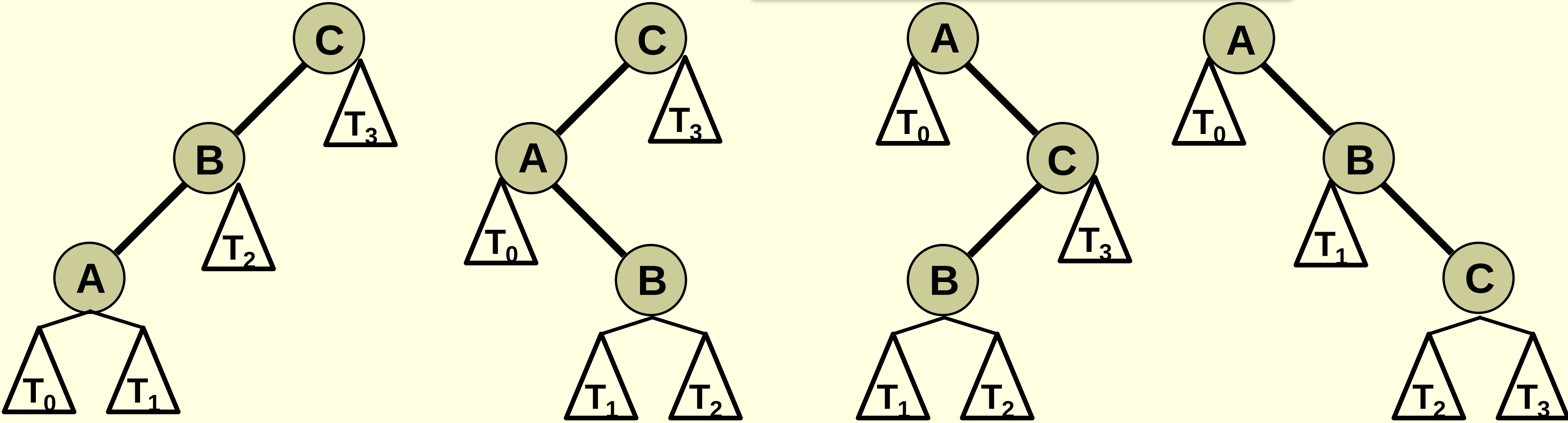

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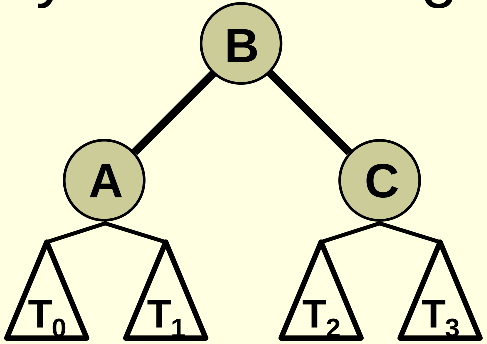

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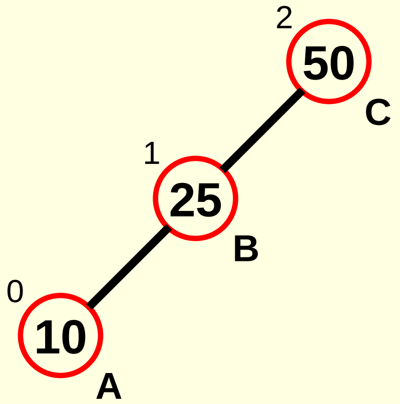

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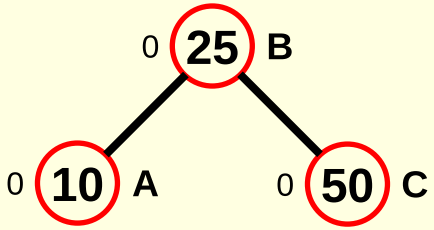

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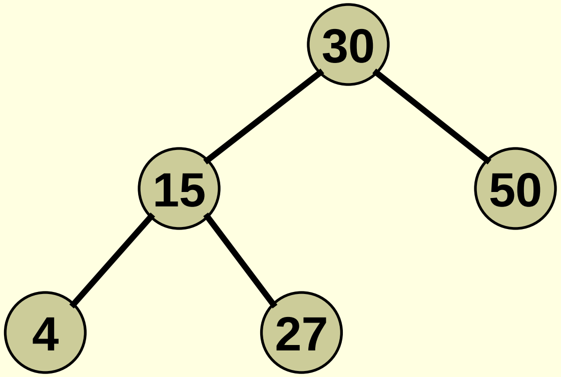

      - We insert a node with the value 20
      - This node clearly goes to the left of 27

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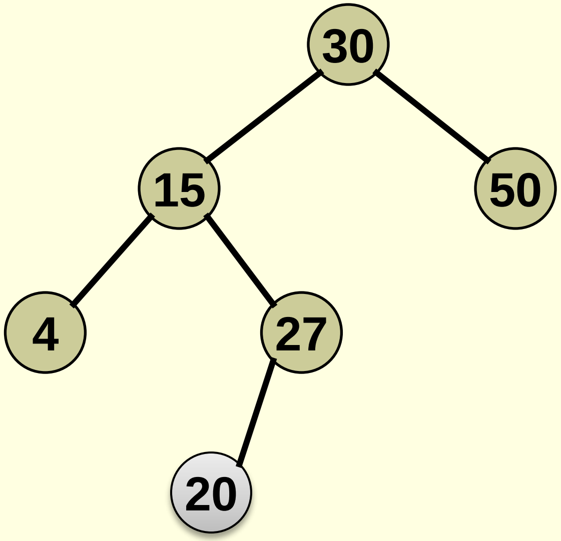
    - Given this tree:



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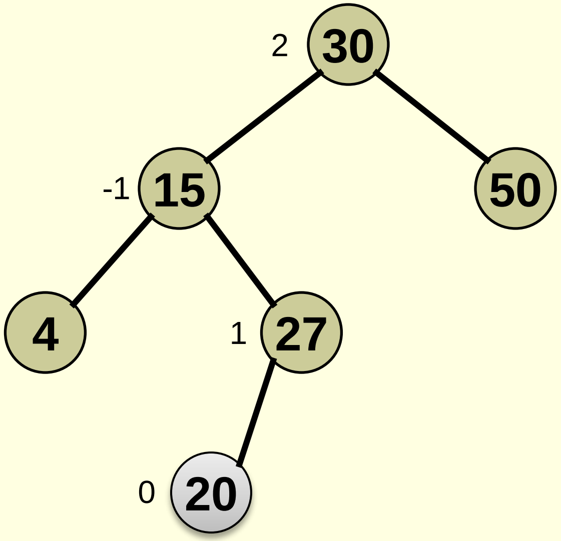

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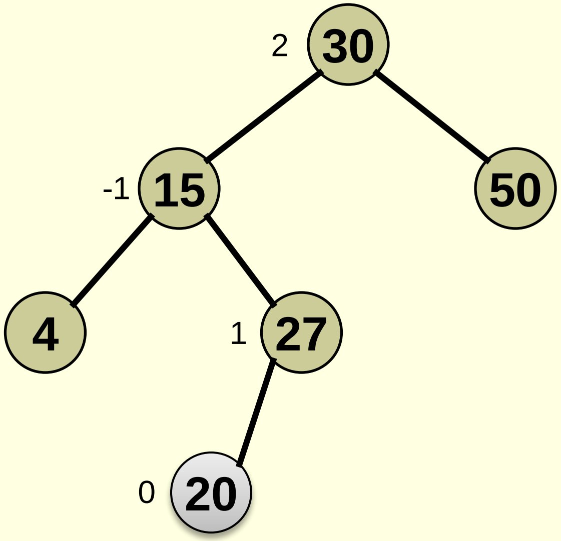

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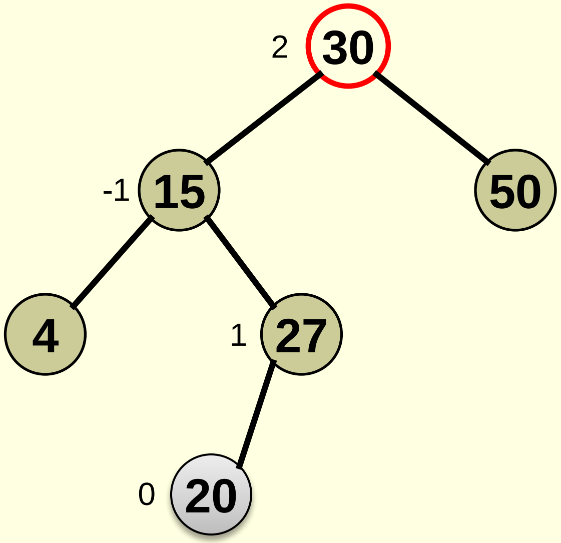

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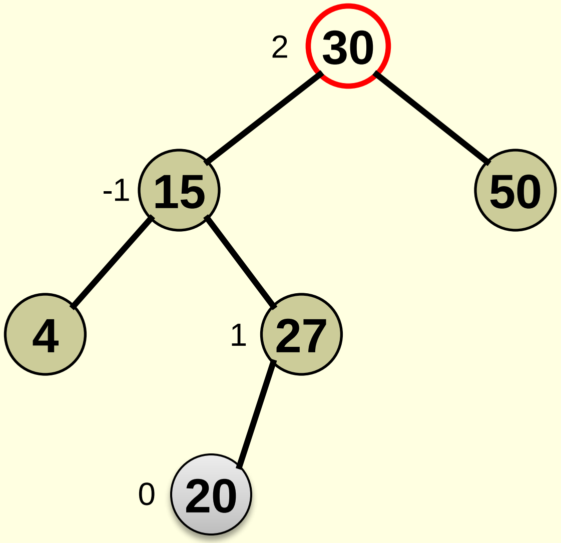

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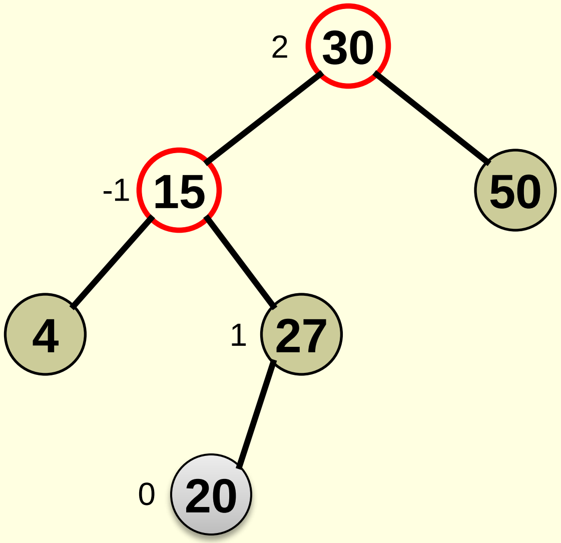

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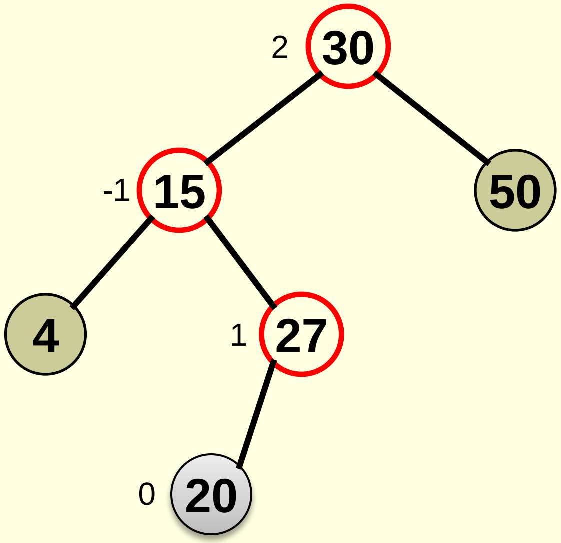

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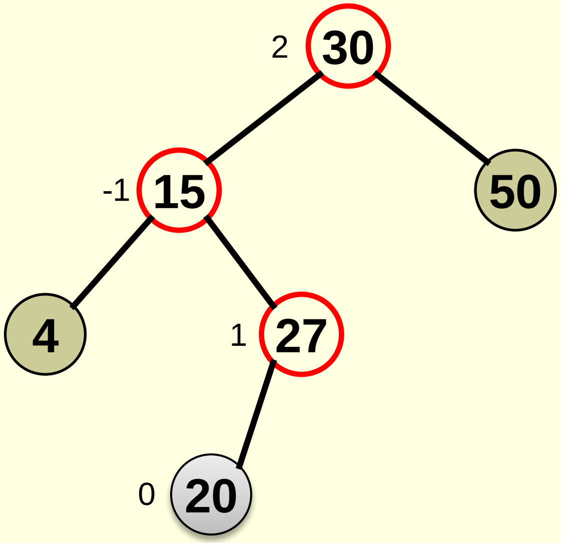

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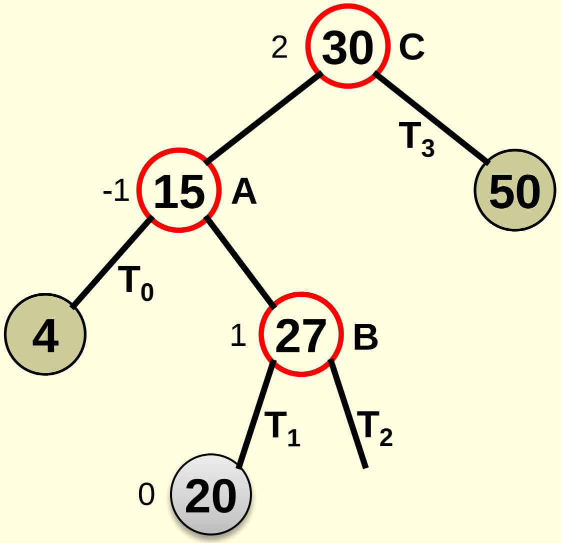

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4. Label those ‘A, B, C’ nodes appropriately (and subtrees)
   - Don’t forget to label the subtrees from smallest to largest (from T_0 to T_3)

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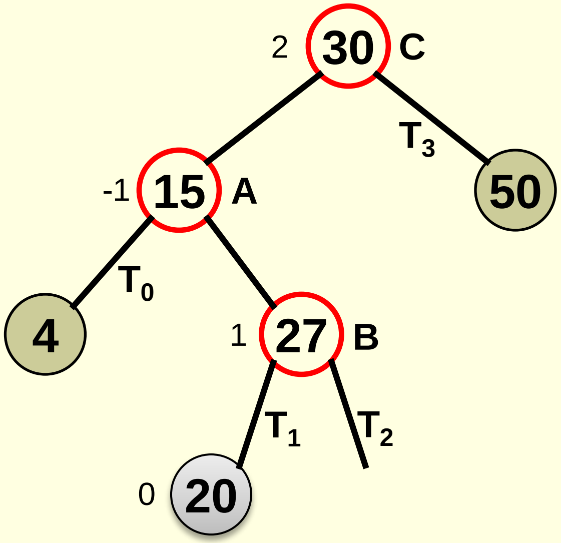

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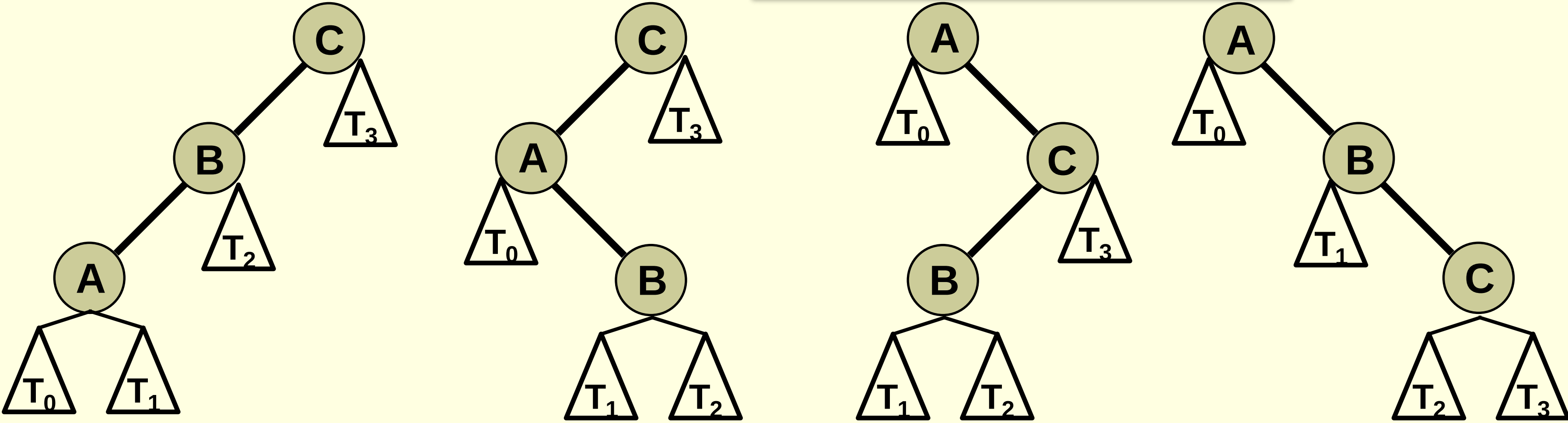

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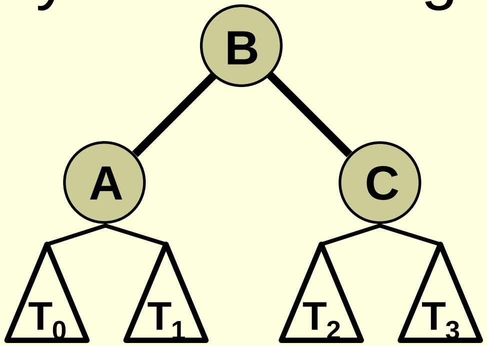

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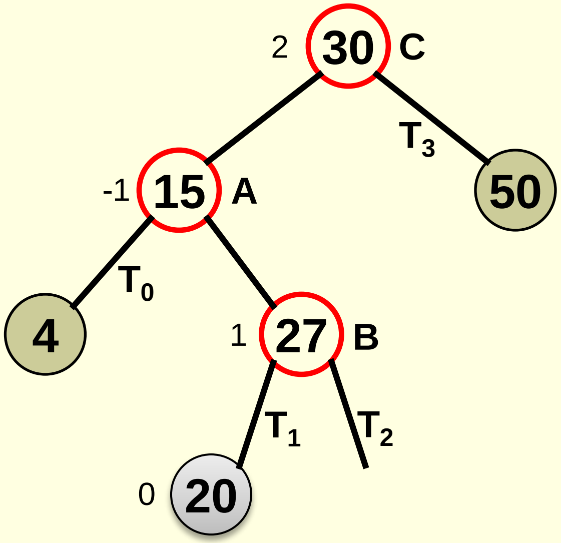

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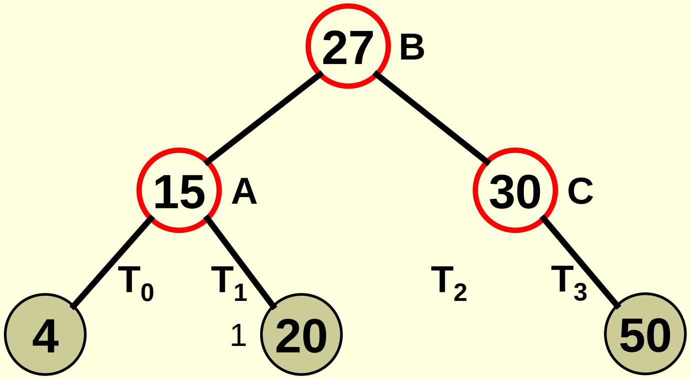

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Brief Interlude: FAIL Picture

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- More examples given in class!
- See PDF of Arup’s Insertion notes
  - maybe not the most exciting notes
  - but it has the same examples

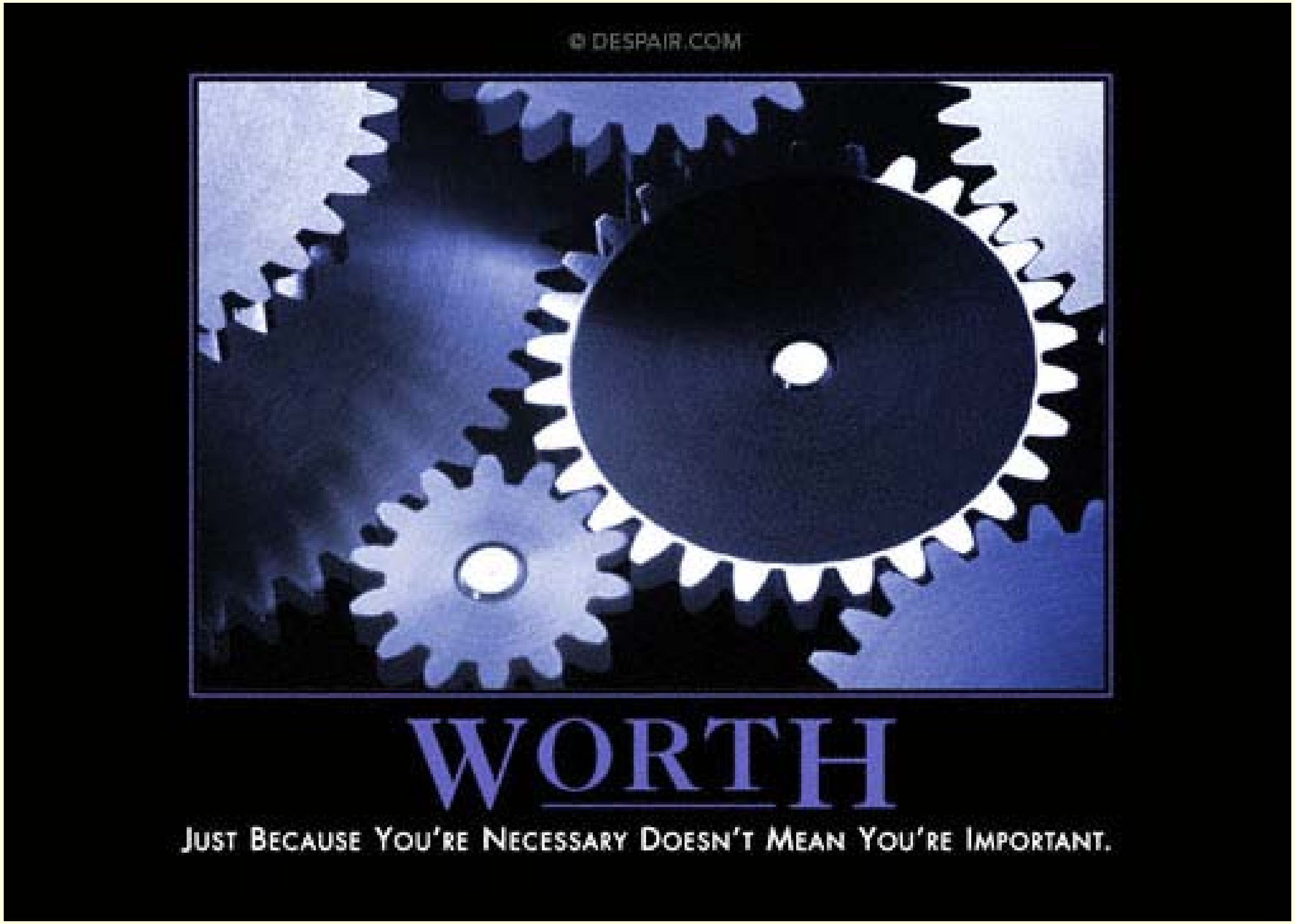
AVL Trees: Insertion Revisited

WASN’T

THAT

MOMENTOUS!

Daily Demotivator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WORTH
Just Because You're Necessary Doesn't Mean You're Important.

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Computer Science Department
University of Central Florida

COP 3502 – Computer Science I