Sorting: Quick Sort

Computer Science Department
University of Central Florida

COP 3502 – Computer Science I

Announcement

- Quiz 4 is TODAY
- Exam 2 is this Friday
  - March 30th
- Want to avoid Program 6?
  - And want a FREE 100 in its place?
  - Do the Community Service
  - Absolute DEADLINE is this Wednesday
    - March 28th by 12:30 PM sharp in MY OFFICE

Sorting: Quick Sort

- Quick Sort
  - Most common sort used in practice
  - Why?
    - cuz it is usually the quickest in practice!
  - Quick Sort uses two main ideas to achieve this efficiency:
  1) The idea of making partitions
  2) Recursion

  - Let's look at the partition concept…

Sorting: Quick Sort

- Quick Sort – Partition
  - **A partition works as follows:**
  - Given an array of n elements
    - You must manually select an element in the array to partition by
    - You must then compare ALL the remaining elements against this element
    - If they are greater,
      - Put them to the “right” of the partition element
    - If they are less,
      - Put them to the “left” of the partition element

Sorting: Quick Sort

- Quick Sort – Partition
  - **A partition works as follows:**
    - Once the partition is complete, what can we say about the position of the partition element?
    - We can say (we KNOW) that **the partition element is in its CORRECTLY sorted location**
    - In fact, after you partition the array, you are left with:
      - all the elements to the left of the partition element, in the array, that still need to be sorted
      - all the elements to the right of the partition element, in the array, that still need to be sorted
    - And if you sort those two sides, the entire array will be sorted!

Sorting: Quick Sort

- Quick Sort
  - Partition:
    - Essentially breaks down the sorting problem into two smaller sorting problems
      - …what does that sound like?
  - Code for Quick Sort (at a real general level):
    1) Partition the array with respect to a random element
    2) Sort the left part of the array using Quick Sort
    3) Sort the right part of the array using Quick Sort
  - Notice there is no “merge” step like in Merge Sort
    - at the end, all elements are already in their proper order

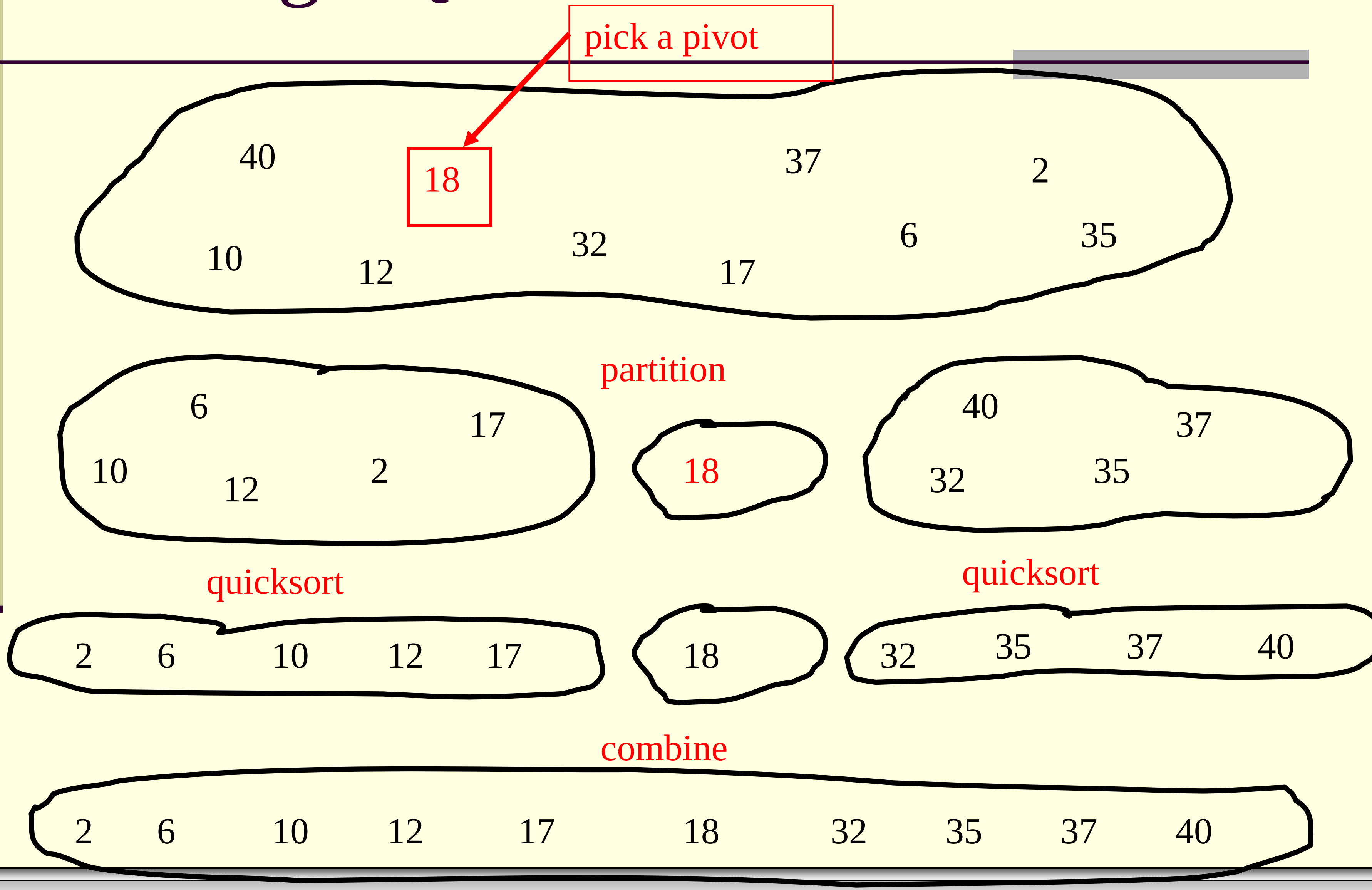
Sorting: Quick Sort

- Quick Sort
  - Code for Quick Sort (at a real general level):
    1) Partition the array with respect to a random element
    2) Sort the left part of the array using Quick Sort
    3) Sort the right part of the array using Quick Sort
  - Quick Sort is a recursive algorithm:
    - We need a base case
      - A case that does NOT make recursive calls
    - Our base case, or terminating condition, will be when we sort an array with only one element
      - We know the array is already sorted!

Sorting: Quick Sort

- Quick Sort
  - Let S be the input set.
  1. If $|S| = 0$ or $|S| = 1$, then return.
  2. Pick an element v in S. Call v the partition element.
  3. Partition $S - \{v\}$ into two disjoint groups:
     - $S_1 = \{x \in S - \{v\} \mid x \leq v\}$
     - $S_2 = \{x \in S - \{v\} \mid x \geq v\}$
  4. Return { quicksort(S_1), v, quicksort(S_2) }

Sorting: Quick Sort



Sorting: Quick Sort

- The idea of "in place"
  - In Computer Science, an "in-place" algorithm is one where the output usually overwrites the input
    - There is more detail, but for our purposes, we stop with that
  - Example:
    - Say we wanted to reverse an array of n items
      - Here is a simple way to do that:

```
function reverse(a[0..n]) {
        allocate b[0..n]
        for i from 0 to n
                b[n - i] = a[i]
        return b
}
```

Sorting: Quick Sort

- The idea of "in place"
  - Example:
    - Say we wanted to reverse an array of n items
      - Here is a simple way to do that:

```
function reverse(a[0..n]) {
        allocate b[0..n]
        for i from 0 to n
                b[n - i] = a[i]
        return b
}
```

    - Unfortunately, this method requires O(n) extra space to create the array b
      - And allocation can be a slow operation

Sorting: Quick Sort

- The idea of "in place"
  - Example:
    - Say we wanted to reverse an array of n items
    - If we no longer need the original array a
    - We can overwrite it using the following in-place algorithm

```
function reverse-in-place(a[0..n])
       for i from 0 to floor(n/2)
              swap(a[i], a[n-i])
```

    - Many Sorting algorithms are in-place algorithms
    - Quick sort is NOT an in-place algorithm
    - BUT, **the Partition algorithm can be in-place**

Sorting: Quick Sort

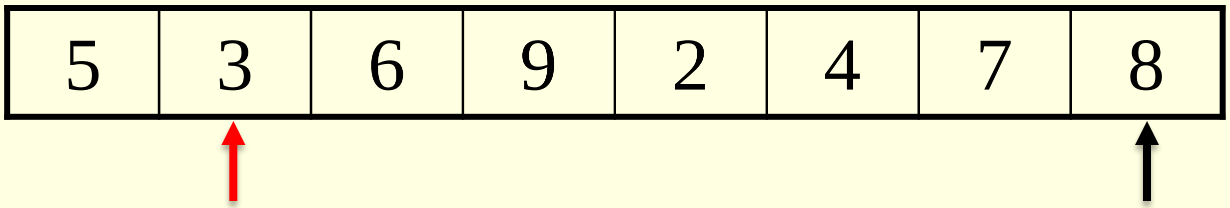
- How to Partition "in-place"
  - Consider the following list of values that we want to partition

5	3	6	9	2	4	7	8

  - Let us assume for the time being that we will partition based on the first element in the array
  - The algorithm will partition these elements "in-place"

Sorting: Quick Sort

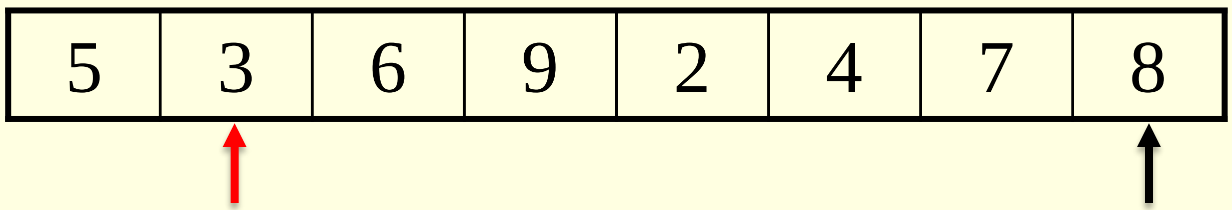
- How to Partition "in-place"



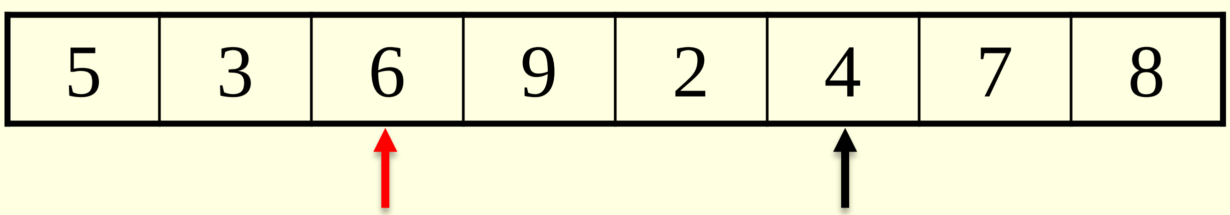
- Here's how the partition will work:
  - Start two counters, one at index one and one at index 7
    - The last element in the array
  - Advance the left counter forward until an element greater than the partition element is encountered
  - Advance the right counter backwards until a value less than the pivot is encountered

Sorting: Quick Sort

- How to Partition "in-place"

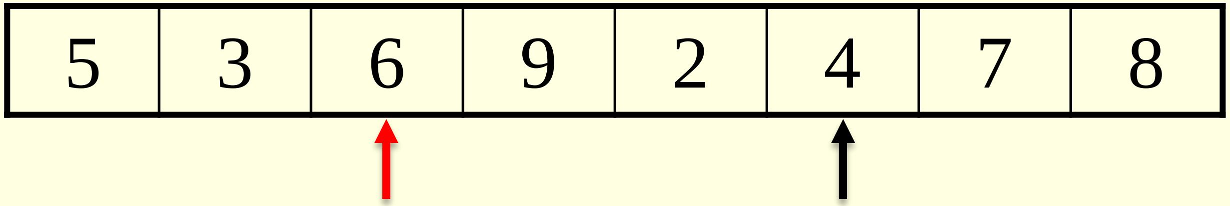


  - After these two steps are performed, we have:

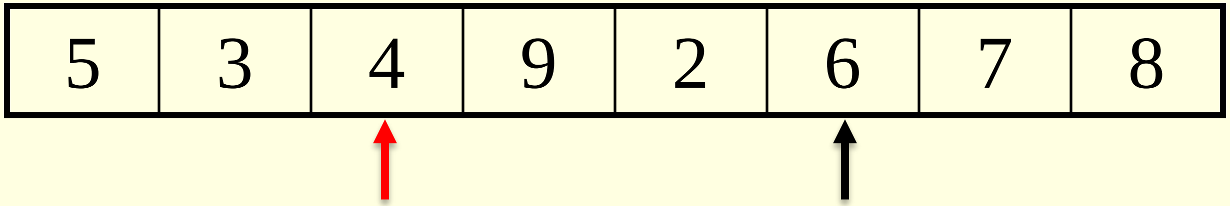


Sorting: Quick Sort

- How to Partition "in-place"

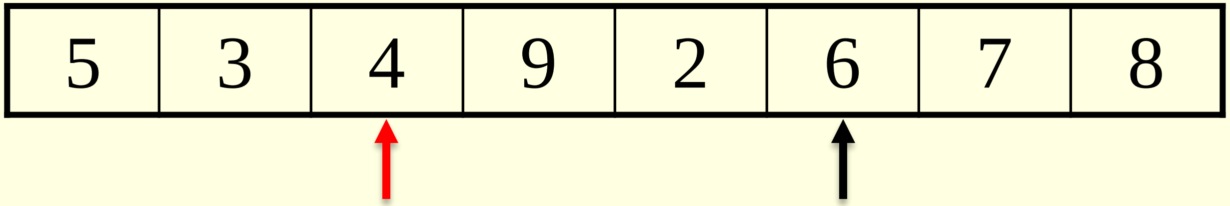


  - We know that these two elements are on the "wrong" side of the array ...so SWAP them!

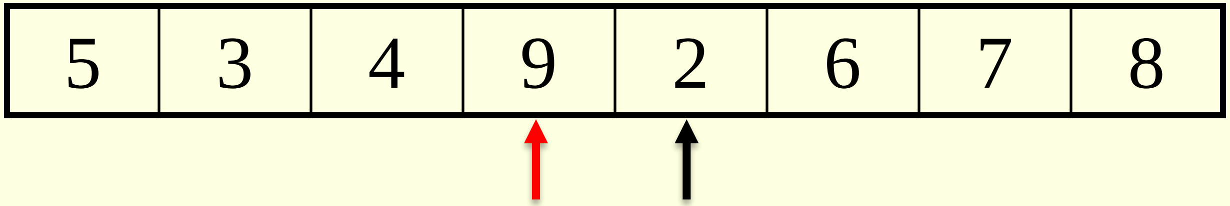


Sorting: Quick Sort

- How to Partition "in-place"



  - Now continue to advance the pointers as before



Sorting: Quick Sort

- How to Partition "in-place"

5	3	4	9	2	6	7	8

  - Then SWAP as before:

5	3	4	2	9	6	7	8

    - At some point, the counters will cross over each other

Sorting: Quick Sort

- How to Partition "in-place"

5	3	4	2	9	6	7	8

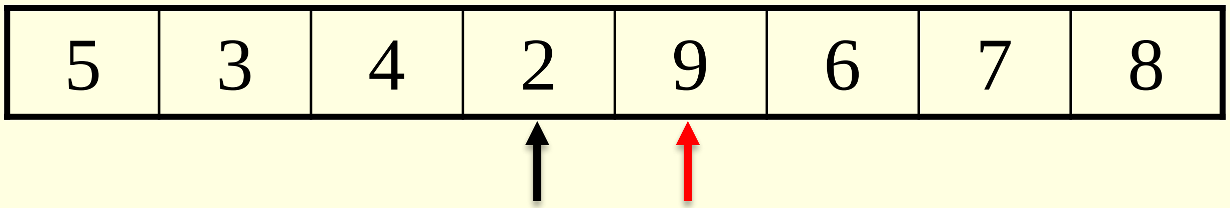
  - Again, advance the pointers as before

5	3	4	2	9	6	7	8

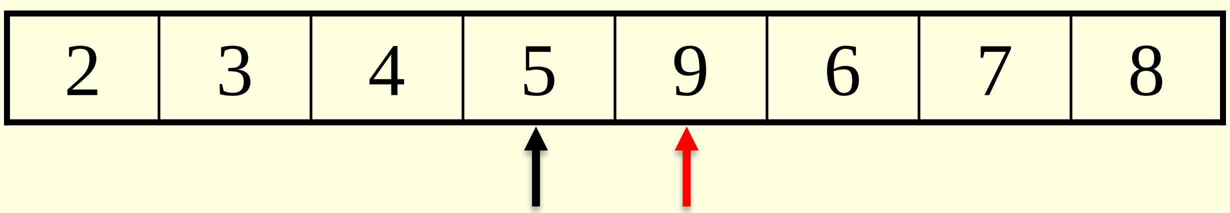
    - So we see that the counters crossed over each other

Sorting: Quick Sort

- How to Partition "in-place"



  - Now, SWAP the value stored in the original right counter (black arrow) with the partition element



    - Finally, RETURN the index the five is stored in (the right pointer) to indicate where the partition element ended up

Sorting: Quick Sort

- Partition Code

```
int partition(int* vals, int low, int high) {
        int temp;
        int i, lowpos;

        // A base case that should never really occur.
        if (low == high) return low;

        // Pick a random partition element and swap it into index low.
        i = low + rand()%(high-low+1);
        temp = vals[i];
        vals[i] = vals[low];
        vals[low] = temp;

        // Store the index of the partition element.
        lowpos = low;

        // Update our low pointer.
        low++;
```

Sorting: Quick Sort

- Partition Code

```
        // Run Partition so long as low and high counters don't cross.
        while (low <= high) {
                // Move the low pointer forwards.
                while (low <= high && vals[low] <= vals[lowpos]) low++;

                // Move the high pointer backwards.
                while (high >= low && vals[high] > vals[lowpos]) high--;

                // Now swap the values at those two pointers.
                if (low < high)
                        swap(&vals[low], &vals[high]);
        }

        // Swap the partition element into it's correct location.
        swap(&vals[lowpos], &vals[high]);

        return high; // Return the index of the partition element.
}
```

Sorting: Quick Sort

- Quick Sort Code

```
void quicksort(int* numbers, int low, int high) {

        // Only have to sort if we are sorting more than one number
        if (low < high) {

                // Partition the elements
                // Parition function returns the index of the
                // partition element.  Saved into “split”.
                int split = partition(numbers,low,high);

                // Recursively Quick Sort the left side
                quicksort(numbers,low,split-1);

                // Recursively Quick Sort the right side
                quicksort(numbers,split+1,high);
        }
}
```

Sorting: Quick Sort

- Choosing a Partition Element
  - For correctness, we can choose any pivot.
  - For efficiency, one of following is best case, the other worst case:
    - pivot partitions the list roughly in half
    - pivot is greatest or least element in list
  - Which case above is best?
    - Clearly, a partition element in the middle is ideal
    - As it splits the list roughly in half
  - But we don't know where that element is
  - So we have a few ways of choosing pivots

Sorting: Quick Sort

- Choosing a Partition Element
  - first element
    - bad if input is sorted or in reverse sorted order
    - bad if input is nearly sorted
    - variation: particular element (e.g. middle element)
  - random element
    - You could get lucky and choose the middle element
    - You could be unlucky and choose the smallest or greatest element
      - Resulting in a partition with ZERO elements on one side
  - median of three elements
    - choose the median of the left, right, and center elements

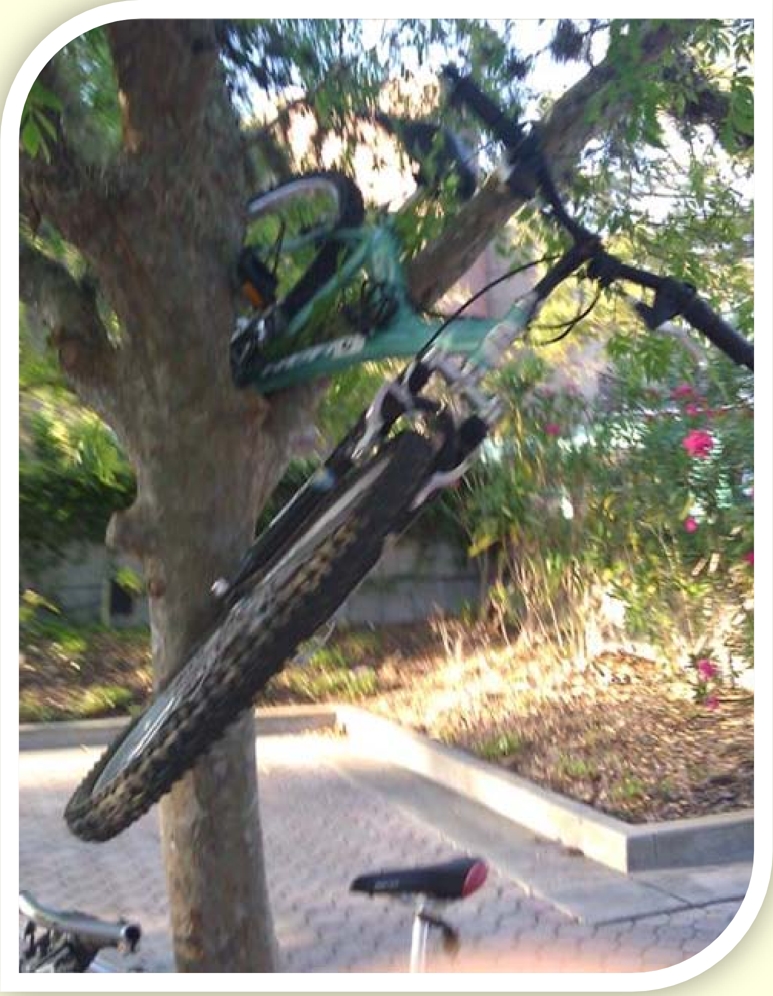
Sorting: Quick Sort

- Choosing a Partition Element
  - median of three elements
    - choose the median of the left, right, and center elements
    - There is extra expense with this method
      - Picking three values
      - Doing three comparisons
    - But if the array is large, doing this little extra work will be small compared to the gains of a better partition
  - You could also pick the median of 5 or 7 elements
    - The more you pick the better partition you get

Brief Interlude: FAIL Picture

Daily UCF Bike Fail

Finding new and innovative ways to get your bike stolen!



Courtesy of
Benjamin Stanchina

Sorting: Quick Sort

- Quick Sort Analysis
  - This is more difficult to do than Merge Sort
    - Why?
    - With Merge Sort, we knew that our recursive calls always had equal sized inputs
      - Remember: we would split the array of size n into two arrays of size n/2 (so the smaller arrays were always the same size)
  - How is Quick Sort different? (more difficult?)
    - Each recursive call of Quick Sort could have a different sized set of numbers to sort
      - Because the size of the sets is based on our partition element
      - If partition element is in the middle, each set has about half
      - Otherwise, one set is large and one is small

Sorting: Quick Sort

- Quick Sort Analysis
  - Location of partition element determines difficulty

1) If we get lucky
   - and the partition element is ALWAYS in the middle:
   - Then this is the BEST case
     - As we will always be **halving** the amount of work left
2) If we are unlucky:
   - and we ALWAYS choose the first or the last element in the list as our partition
   - Then this is the WORST case
     - As we will have not really sorted anything
     - We simply reduced the 2-be-sorted amount by 1

Sorting: Quick Sort

- Quick Sort Analysis
  - Location of partition element determines difficulty
  3) If we are neither lucky or unlucky:
     - Most likely, we will have some great partitions
     - Some bad partitions
     - And some okay partitions

  - So we need to analyze each case:
     - Best case
     - Average case
     - Worst case

And we **omit** the Average Case due to its difficulty.
*You'll get to see it in CS2.

Sorting: Quick Sort

- Quick Sort Analysis
  - Analysis of Best Case:
    - As mentioned, in the best case, we get a perfect partition every single time
    - Meaning, if we have n elements before the partition,
      - we "luckily" pick the middle element as the partition element
      - Then we end up with n/2 - 1 elements on each side of the partition
    - So if we had 101 unsorted elements
      - we "luckily" pick the 51st element as the partition element
      - Then we end up with 50 elements smaller than this element, on the left
      - And 50 elements, greater than this element, on the right

Sorting: Quick Sort

- Quick Sort Analysis
  - Analysis of Best Case:
    - Again, here are the steps of Quick Sort:
      1) Partition the elements
      2) Quick Sort the smaller half (recursive)
      3) Quick Sort the larger half (recursive)
    - So at each recursive step, the input size is **halved**
    - Let T(n) be the running time of Quick Sort on n elements
      - And remember that Partition runs on O(n) time
    - So we get our recurrence relation for the best case:
      - $T(n) = 2*T(n/2) + O(n)$
        - This is the same recurrence relation as Merge Sort
      - So in the best case, Quick Sort runs in $O(n\log n)$ time

Sorting: Quick Sort

- Quick Sort Analysis
  - Analysis of Worst Case:
    - Assume that we are horribly unlucky
    - And when choosing the partition element, we somehow end up always choosing the greatest value remaining
    - **Now for this worst case:**
      - How many times will the Partition function run?
        - Think: when we choose the greatest element (for example)
        - We have the partition element, then ALL other elements are to the left in one partition
        - The "partition" to the right will have ZERO elements
      - So Partition will run n-1 times
        - The first time results in comparing n-1 values, then comparing n-2 values the second time, followed by n-3, etc.

Sorting: Quick Sort

- Quick Sort Analysis
  - Analysis of Worst Case:
    - How many times will the Partition function run?
      - Partition will run n-1 times
        - The first time results in comparing n-1 values, then comparing n-2 values the second time, followed by n-3, etc.
    - When we sum the number of compares, we get:
      - 1 + 2 + 3 + … + (n - 1)
      - You should know what this equals:

$$\frac{(n-1)n}{2}$$

    - Thus, the worst case running time is $O(n^2)$

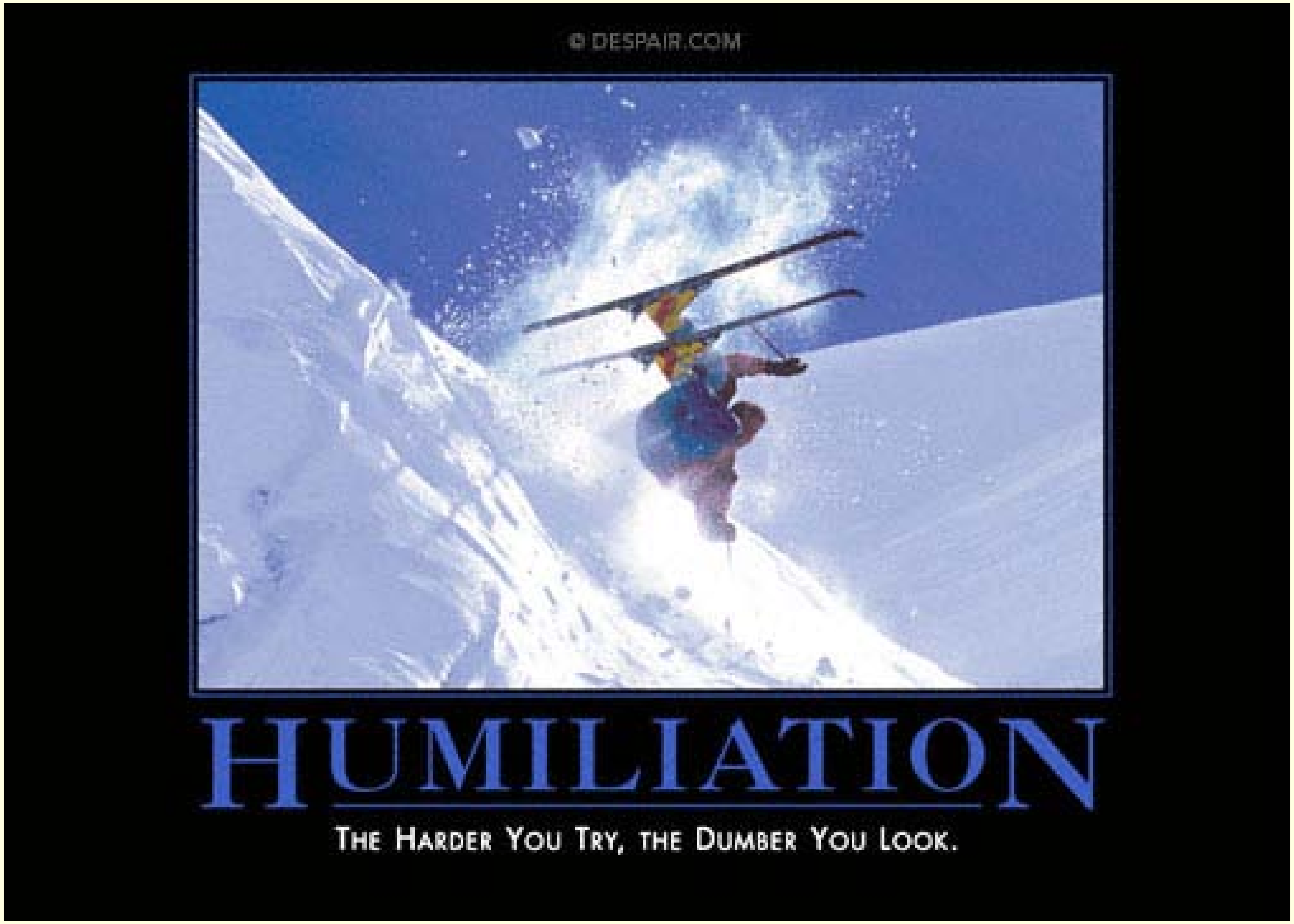
Sorting: Quick Sort

- Quick Sort Analysis
  - Summary:
    - Best Case: O(nlogn)
    - **Average Case: O(nlogn)**
    - Worst Case: $O(n^2)$
  - Compare Merge Sort and Quick Sort:
    - Merge Sort: guaranteed O(nlogn)
    - Quick Sort: best and average case is O(nlogn) but worst case is $O(n^2)$

Sorting: Quick Sort

WASN'T
THAT
THE GREATEST!

Daily Demotivator



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