And More Algorithm Analysis

Computer Science Department
University of Central Florida

COP 3502 – Computer Science I

And More Algorithm Analysis

- Examples of Analyzing Code:
  - Last time we went over examples of analyzing code
    - We did this in a somewhat naïve manner
      - Just analyzed the code and tried to “trace” what was going on
  - This Lecture:
    - We will do this in a more structured fashion
    - We mentioned that summations are a tool for you to help coming up with a running time of iterative algorithms
    - Today we will look at some of those same code fragments, as well as others, and show you how to use summations to find the Big-O running time

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- Example 1:
  - Determine the Big O running time of the following code fragment:
    - We have two for loops
    - They are NOT nested
      - The first runs from k = 1 up to (and including) n/2
      - The second runs from j = 1 up to (and including) n^2

```
for (k = 1; k <= n/2; k++) {
      sum = sum + 5;
}
for (j = 1; j <= n*n; j++) {
      delta = delta + 1;
}
```

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- Example 1:
  - Determine the Big O running time of the following code fragment:
    - Here's how we can express the number of operations in the form of a summation:

$$\sum_{k=1}^{n/2} 1 + \sum_{j=1}^{n^2} 1$$

The constant value, 1, inside each summation refers to the one, and only, operation in each for loop.

```
for (k = 1; k <= n/2; k++) {
       sum = sum + 5;
}
for (j = 1; j <= n*n; j++) {
       delta = delta + 1;
}
```

Now you simply solve the summation!

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- Example 1:
  - Determine the Big O running time of the following code fragment:
    - Here's how we can express the number of operations in the form of a summation:

$$\sum_{k=1}^{n/2} 1 + \sum_{j=1}^{n^2} 1$$

You use the formula: $\sum_{i=1}^{n} k = k * n$

$$\sum_{k=1}^{n/2} 1 + \sum_{j=1}^{n^2} 1 = \frac{n}{2} + n^2$$

    - This is a **<u>CLOSED FORM</u>** solution of the summation
    - So we approximate the running time as O(n^2)

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- Example 2:
  - Determine the Big O running time of the following code fragment:
    - Here we again have two for loops
    - But this time they are nested

```
int func2(int n) {
        int i, j, x = 0;
        for (i = 1; i <= n; i++) {
                for (j = 1; j <= n; j++) {
                        x++;
                }
        }
        return x;
}
```

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- Example 2:
  - Determine the Big O running time of the following code fragment:
    - Here we again have two for loops
    - But this time they are nested
      - The outer loop runs from i = 1 up to (and including) n
      - The inner loop runs from j = 1 up to (and including) n
    - The sole (only) operation is a “x++” within the inner loop

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- Example 2:
  - Determine the Big O running time of the following code fragment:
    - We express the number of operations in the form of a summation and then we solve that summation:

$$\sum_{i=1}^{n} \sum_{j=1}^{n} 1$$

You use the formula: $\sum_{i=1}^{n} k = k * n$

$$\sum_{i=1}^{n} \sum_{j=1}^{n} 1 = \sum_{i=1}^{n} n = n^2$$

All we did is apply the above formula twice.

- This is a **CLOSED FORM** solution of the summation
- So we approximate the running time as $O(n^2)$

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- Example 3:
  - Determine the Big O running time of the following code fragment:
    - Here we again have two for loops
    - And they are nested. So is this $O(n^2)$?

```
int func3(int n) {
        sum = 0;
        for (i = 0; i < n; i++) {
                for (j = 0; j < n * n; j++) {
                        sum++;
                }
        }
}
```

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- Example 3:
  - Determine the Big O running time of the following code fragment:
    - Here we again have two for loops
    - And they are nested. So is this $O(n^2)$?
      - The outer loop runs from i = 0 up to (and not including) n
      - The inner loop runs from j = 0 up to (and not including) n^2
    - The sole (only) operation is a “sum++” within the inner loop

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- Example 3:
  - Determine the Big O running time of the following code fragment:
    - We express the number of operations in the form of a summation and then we solve that summation:

$$\sum_{i=0}^{n-1} \sum_{j=0}^{n^2-1} 1$$

You use the formula: $\sum_{i=1}^{n} k = k * n$

$$\sum_{i=0}^{n-1} \sum_{j=0}^{n^2-1} 1 = \sum_{i=0}^{n-1} n^2 = n^2 \sum_{i=0}^{n-1} 1 = n^3$$

All we did is apply the above formula twice.

- This is a **<u>CLOSED FORM</u>** solution of the summation
- So we approximate the running time as $O(n^3)$

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- Example 4:
  - Write a summation that describes the <u>number of multiplication operations</u> in this code fragment:
    - Here we again have two for loops
    - Pay attention to the limits (bounds) of the for loop

```
int func3(int n) {
        bigNumber = 0;
        for (i = 100; i <= 2n; i++) {
                for (j = 1; j < n * n; j++) {
                        bigNumber += i*n + j*n;
                }
        }
}
```

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- Example 4:
  - Write a summation that describes the <u>number of multiplication operations</u> in this code fragment:
    - Here we again have two for loops
    - Pay attention to the limits (bounds) of the for loop
      - The outer loop runs from i = 100 up to (and including) 2n
      - The inner loop runs from j = 1 up to (and not including) n^2
    - Now examine the number of **<u>multiplications</u>**
      - Because this problem specifically said to “describe the number of multiplication operations, we do not care about ANY of the other operations
      - bigNumber += i*n + j*n;
      - There are TWO multiplication operations in this statement

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- Example 4:
  - Write a summation that describes the <u>number of multiplication operations</u> in this code fragment:
    - We express the number of multiplications in the form of a summation and then we solve that summation:

$$\sum_{i=100}^{2n} \sum_{j=1}^{n^2-1} 2$$

$$\sum_{i=100}^{2n} \sum_{j=1}^{n^2-1} 2 = \sum_{i=100}^{2n} 2(n^2-1) = 2(n^2-1)\sum_{i=100}^{2n} 1 = 2(n^2-1)(2n-99)$$

- This is a <u>**CLOSED FORM**</u> solution of the summation
- Shows the number of multiplications

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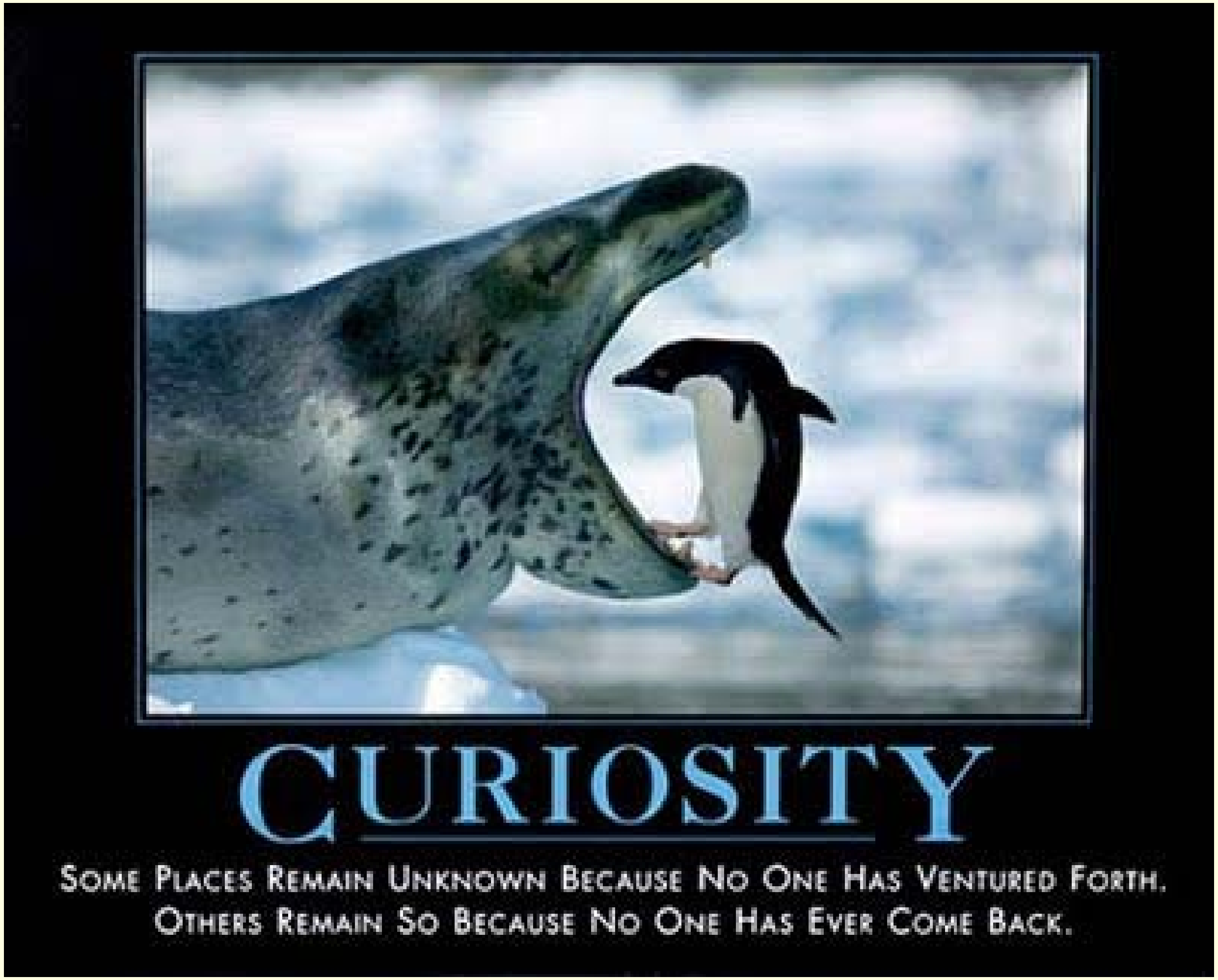

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