And More Recursion

Computer Science Department
University of Central Florida

COP 3502 – Computer Science I

Binary Search – **A reminder**

- Array Search
  - We are given the following **sorted** array:

index	0	1	2	3	4	5	6	7	8
value	2	6	19	27	33	37	38	41	118

  - We are searching for the value, 19 (for example)
  - Remember, we said that you search the middle element
    - If found, you are done
    - If the element in the middle is greater than 19
      - Search to the LEFT (cuz 19 MUST be to the left)
    - If the element in the middle is less than 19
      - Search to the RIGHT (cuz 19 MUST then be to the right)

Binary Search – **A reminder**

- Array Search
  - We are given the following **sorted** array:

index	0	1	2	3	4	5	6	7	8
value	2	6	19	27	33	37	38	41	118

  - We are searching for the value, 19
  - So, we MUST start the search in the middle INDEX of the array.
  - In this case:
    - The lowest index is 0
    - The highest index is 8
    - So the middle index is 4

Binary Search

- Array Search
  - Correct Strategy
    - We would ask, “is the number I am searching for, 19, greater or less than the number stored in index 4?
      - Index 4 stores 33
    - The answer would be “less than”
    - So we would modify our search range to in between index 0 and index 3
      - Note that index 4 is no longer in the search space
    - We then continue this process
      - The second index we’d look at is index 1, since (0+3)/2=1
      - Then we’d finally get to index 2, since (2+3)/2 = 2
      - And at index 2, we would find the value, 19, in the array

Binary Search

- Binary Search code:

```
int binsearch(int a[], int len, int value) {

        int low = 0, high = len-1;
        while (low <= high) {
                int mid = (low+high)/2;
                if (value < a[mid])
                        high = mid-1;
                else if (value > a[mid])
                        low = mid+1;
                else
                        return 1;
        }

        return 0;
}
```

Binary Search

- Binary Search code:
  - At the end of each array iteration, all we do is update either low or high
  - This modifies our search region
    - Essentially halving it
  - As we saw previously, this runs in **log n** time
  - But this iterative code isn't the easiest to read
  - We now look at the recursive code
    - MUCH easier to read and understand

Binary Search – Recursive

- Binary Search using recursion:
  - We need a stopping case:
    - We need to STOP the recursion at some point

  - So when do we stop:

1) When the number is found!
2) Or when the search range is nothing
    - huh?
    - The <u>search range is empty</u> when `(low > high)`

  - So how let us look at the code…

Binary Search – Recursive

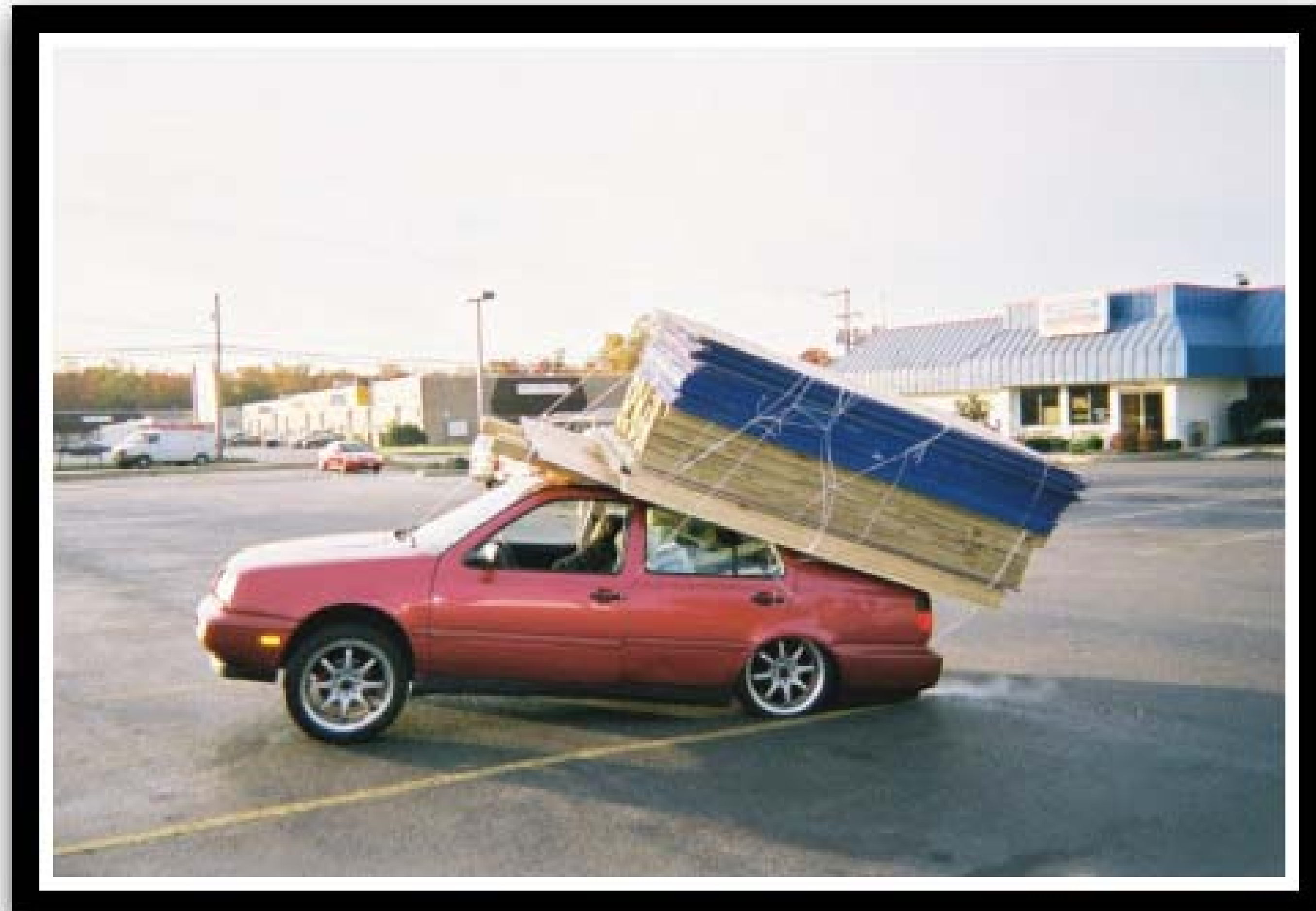
- Binary Search Code (using recursion):
  - We see how this code follows from the explanation of binary search quite easily

```
int binSearch(int *values, int low, int high, int searchval)
        int mid;
        if (low <= high) {
                mid = (low+high)/2;
                if (searchval < values[mid])
                        return binSearch(values, low, mid-1, searchval);
                else if (searchval > values[mid])
                        return binSearch(values, mid+1, high, searchval);
                else
                        return 1;
        }
        return 0;
}
```

Binary Search – Recursive

- Binary Search Code (using recursion):
  - So if the value is found
    - We `return 1`
  - Otherwise,
    - `if (searchval < values[mid])`
      - Then recursively call binSearch to the LEFT
    - `else if (searchval > values[mid])`
      - Then recursively call binSearch to the RIGHT
  - If `low` ever becomes greater than `high`
    - This means that `searchval` is NOT in the array

Brief Interlude: Human Stupidity



Recursive Exponentiation

- Example from Previous lecture
  - Our function:
    - Calculates b^e
      - Some base raised to a power, e
      - The input is the base, b, and the exponent, e
      - So if the input was 2 for the base and 4 for the exponent
        - The answer would be $2^4 = 16$
  - How do we do this recursively?
    - We need to solve this in such a way that part of the solution is a sub-problem of the EXACT same nature of the original problem.

Recursive Exponentiation

- Example from Previous lecture
  - Our function:
    - Using b and e as input, here is our function
      - $f(b,e) = b^e$
    - So to make this recursive, can we say:
      - $f(b,e) = b^e = b*b^{(e-1)}$
    - Does that "look" recursive?
    - YES it does!
    - Why?
    - Cuz the right side is indeed a sub-problem of the original
    - We want to evaluate b^e
    - And our right side evaluates $b^{(e-1)}$

Recursive Exponentiation

- Example from Previous lecture
  - Our function:
    - $f(b,e) = b*b^{(e-1)}$
    - So we need to determine the terminating condition!
    - We know that $f(b,0) = b^0 = 1$
      - So our terminating condition can be when e = 1
    - Additionally, our recursive calls need to return an expression for f(b,e) in terms of f(b,k)
      - for some k < e
    - We just found that $f(b,e) = b*b^{(e-1)}$
    - So now we can write our actual function…

Recursive Exponentiation

- Example from Previous lecture
  - Code:

```
// Pre-conditions: e is greater than or equal to 0.
// Post-conditions: returns b^e.
int Power(int base, int exponent) {

      if ( exponent == 0 )
            return 1;
      else
            return (base*Power(base, exponent-1));
}
```

Recursive Exponentiation

- Example from Previous lecture
  - Say we initially call the function with 2 as our base and 8 as the exponent
  - The final return will be
    - return 2*2*2*2*2*2*2*2
    - Which equals 256
  - You notice **we have 7 multiplications** (exp was 8)
  - The number of multiplications needed is one less than the exponent value
  - So if **n** was the exponent
    - The # of multiplications needed would be **`n-1`**

Fast Exponentiation

- Example from Previous lecture
  - This works just fine
  - BUT, it becomes <u>**VERY slow**</u> for large exponents
    - If the exponent was 10,000, that would be 9,999 mults!
  - How can we do better?
  - One key idea:
    - Remembering the <u>laws of exponents</u>
      - Yeah, algebra…the thing you forgot about two years ago
    - So using the laws of exponents
      - We remember that $\mathbf{2^8 = 2^4 * 2^4}$

Fast Exponentiation

- Example from Previous lecture
  - One key idea:
    - Remembering the laws of exponents
    - $2^8 = 2^4 * 2^4$
    - Now, <u>if we know 2^4</u>
      - **<u>we can calculate 2^8 with one multiplication</u>**
    - What is 2^4?
      - $2^4 = 2^2 * 2^2$
      - and $2^2 = 2*(2)$
    - So… $2*(2) = 4$, $4*(4) = 16$, $16*(16) = 256 = 2^8$
    - So we've calculated 2^8 using <u>only three multiplications</u>
      - <u>MUCH better than 7 multiplications</u>

Fast Exponentiation

- Example of Fast Exponentiation
  - So, in general, we can say:
  - **$b^n = b^{n/2} * b^{n/2}$**
  - <u>So to find b^n, we find $b^{n/2}$</u>
    - HALF of the original amount
  - <u>And to find $b^{n/2}$, we find $b^{n/4}$</u>
    - Again, HALF of $b^{n/2}$
  - This smells like a **`log n`** running time
    - log n number of multiplications
    - Much better than n multiplications
  - But as of now, this only works if n is even

Fast Exponentiation

- Example of Fast Exponentiation
  - So, in general, we can say:
  - $b^n = b^{n/2} * b^{n/2}$
  - This works when n is even
  - But what if n is odd?
  - Notice that $2^9 = 2^4 * 2^4 * 2$
  - So, in general, we can say:

$$a^n = \begin{cases} a^{n/2}(a^{n/2}) & \text{if n is even} \\ a^{n/2}(a^{n/2})(a) & \text{if n is odd} \end{cases}$$

Fast Exponentiation

- Example of Fast Exponentiation
  - Also, this method relies on "**integer division**"
    - We've briefly discussed this
    - Basically **if n is 9, then n/2 = 4**
      - Integer division
      - Think of it as dividing
      - AND then rounding down, if needed, to the nearest integer
    - So if n is 121, then n/2 = 60
    - Finally, if n is 57, then n/2 = 28
  - Using the same base case as the previous power function, here is the code…

Fast Exponentiation

- Example of Fast Exponentiation
  - Code:

```
int powerB(int base, int exp) {
        if (exp == 0)
                return 1;
        else if (exp == 1)
                return base;
        else if (exp%2 == 0)
                return powerB(base*base, exp/2);
        else
                return base*powerB(base, exp-1);
}
```

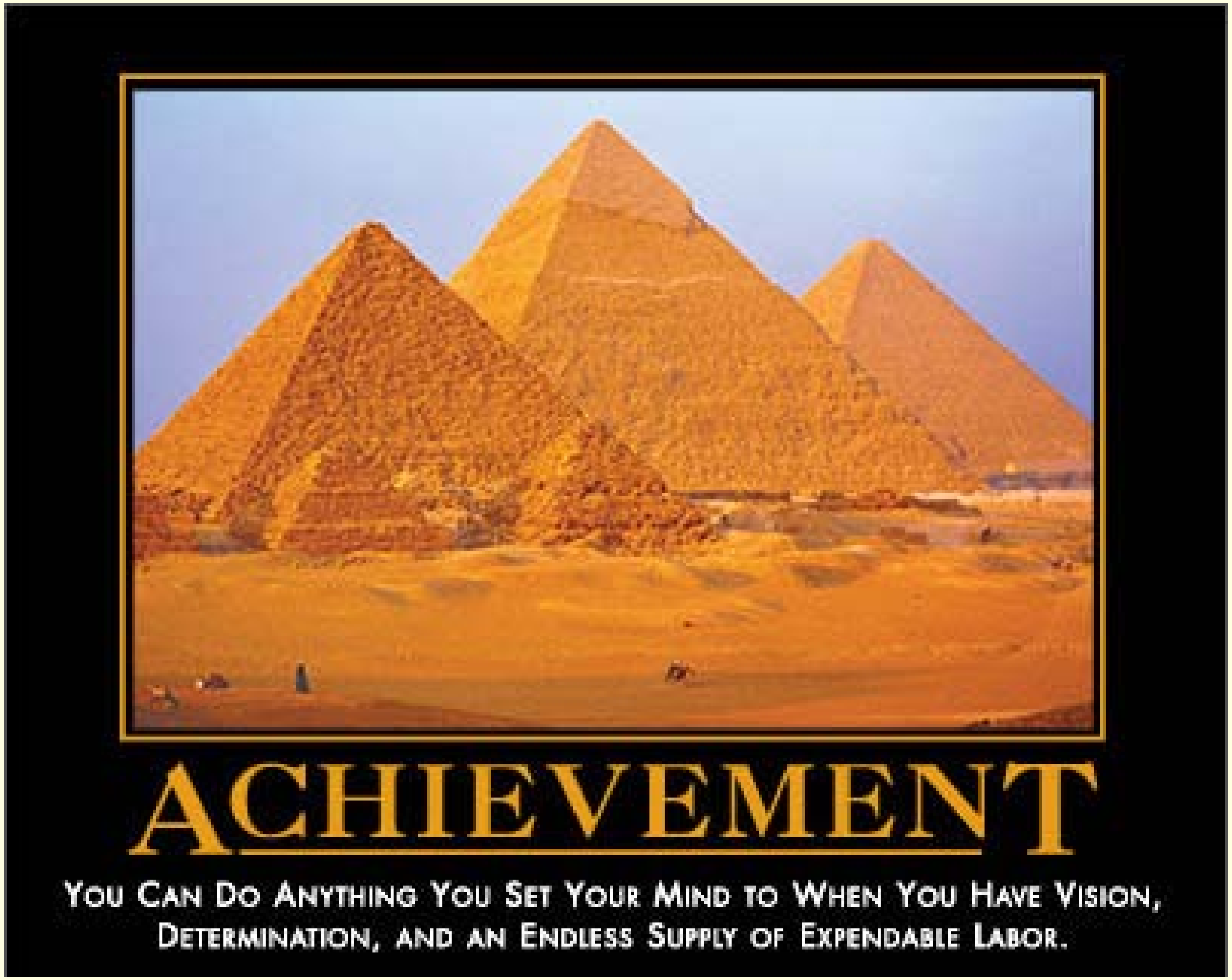
Recursion

WASN'T

THAT

BODACIOUS!

Daily Demotivator



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