

**Computer Science I – Spring 2012**  
**Summation Homework: Due Wednesday 2/15/12, IN CLASS (Solutions)**

**Evaluate the following sums:**

$$1) \sum_{i=10}^{100} (3i + 4) = 3 \sum_{i=10}^{100} i + \sum_{i=10}^{100} 4 = 3 \frac{(10+100)(100-10+1)}{2} + (100-10+1) * 4 = 15379$$

$$2) \sum_{i=5}^{2n} i = \frac{(5+2n)(2n-4)}{2} = (2n+5)(n-2)$$

$$3) \sum_{i=1}^{4n} n^3 i = n^3 \sum_{i=1}^{4n} i = n^3 \frac{(4n)(4n+1)}{2} = 2(4n+1)n^4$$

$$4) \sum_{i=1}^n \sum_{j=1}^i 4 = \sum_{i=1}^n 4 \sum_{j=1}^i 1 = \sum_{i=1}^n 4i = 4 \frac{n(n+1)}{2} = 2n(n+1)$$

$$5) \sum_{i=10}^{100} (1 + \sum_{j=1}^i 12j) = \sum_{i=10}^{100} 1 + \sum_{i=10}^{100} 12 \sum_{j=1}^i 12j = \sum_{i=10}^{100} 1 + \sum_{i=10}^{100} 6i(i+1)$$

$$= 91 + 6 \frac{(10+100)(100-10+1)}{2} + 6 \frac{100(100+1)(200+1)}{6} - 6 \frac{9(9+1)(18+1)}{6}$$

$$= 91 + 30030 + 2030100 - 1710 = 2058511$$

$$6) \sum_{i=n-1}^{2n} i = \sum_{i=1}^{2n} i - \sum_{i=1}^{n-2} i = \frac{2n(2n+1)}{2} - \frac{(n-2)(n-1)}{2} = \frac{3n^2 + 5n - 2}{2}$$

$$7) \sum_{i=n+1}^{2n} (2i + 3n^2) = \sum_{i=n+1}^{2n} 2i + \sum_{i=n+1}^{2n} 3n^2 = \sum_{i=1}^{2n} 2i - \sum_{i=1}^n 2i + \sum_{i=1}^{2n} 3n^2 - \sum_{i=1}^n 3n^2$$

$$= 2 \frac{2n(2n+1)}{2} - 2 \frac{n(n+1)}{2} + 6n^3 - 3n^3 = 2n(2n+1) - n(n+1) + 3n^3$$

$$= n(2(2n+1) - (n+1) + 3n^2) = n(4n+2 - n - 1 + 3n^2) = n(3n^2 + 3n + 1)$$

$$8) \sum_{i=1}^n \left( 3 \sum_{j=1}^n 2ij \right) = \sum_{i=1}^n \left( 6i \sum_{j=1}^n j \right) = \sum_{i=1}^n \left( 6i \frac{n(n+1)}{2} \right) = 6 \frac{n(n+1)}{2} \sum_{i=1}^n i = 6 \frac{n(n+1)}{2} \frac{n(n+1)}{2} = \frac{3n^2(n+1)^2}{2}$$