Summations – Practice Problems

For each of the summation problems below, either solve the summation or produce the closed-form solution.

Problems

1) $\sum_{k=1}^{8} k$

2) $\sum_{i=1}^{50} (i+1)$

3) $\sum_{i=1}^{n} (3i)$

4) $\sum_{i=0}^{50} 1$

5) $\sum_{i=0}^{10} 4$

6) $\sum_{k=1}^{2n^2} k$

7) $\sum_{i=1}^{22} (3i+3)$

8) $\sum_{i=1}^{5}(4i+21)$

9) $\sum_{k=1}^{20}(3k)$

10) $\sum_{k=1}^{15}(2k-3)$

11) $\sum_{i=10}^{37}(4i-3)$

12) $\sum_{i=5}^{15}(3i^2-4)$

Solutions

1) $\sum_{k=1}^{8}k=\frac{n(n+1)}{2}=\frac{8\times 9}{2}=\frac{72}{2}=36$

2) $\sum_{i=1}^{50}(i+1)=\sum_{i=1}^{50}i+\sum_{i=1}^{50}1=\frac{50\times 51}{2}+50=\frac{2550}{2}+50=1275+50=1325$

3) $\sum_{i=1}^{n}(3i)=3\sum_{i=1}^{n}i=\frac{3n(n+1)}{2}$

4) $\sum_{i=0}^{50} 1 = (n+1) = 51$

5) $\sum_{i=0}^{10} 4 = 4\sum_{i=0}^{10} 1 = 4(11) = 44$

6) $\sum_{k=1}^{2n^2} k = \frac{2n^2(2n^2+1)}{2}$

7) $\sum_{i=1}^{22} (3i+3) = 3\sum_{i=1}^{22} i + 3\sum_{i=1}^{22} 1 = 3\frac{22\times 23}{2} + 3(22) = 759 + 66 = 825$

8) $\sum_{i=1}^{5} (4i+21) = 4\sum_{i=1}^{5} i + 21\sum_{i=1}^{5} 1 = 4\frac{5\times 6}{2} + 21(5) = 60 + 105 = 165$

9) $\sum_{k=1}^{20} (3k) = 3\sum_{k=1}^{20} k = 3\frac{20\times 21}{2} = \frac{1260}{2} = 630$

10) $\sum_{k=1}^{15} (2k-3) = 2\sum_{k=1}^{15} k - 3\sum_{k=1}^{15} 1 = 2\frac{15\times 16}{2} - 3(15) = 240 - 45 = 195$

11) using range shifting method

$$\sum_{i=10}^{37} (4i-3) = \sum_{i=1}^{28} (4(i+9) - 3 = \sum_{i=1}^{28} (4i+33) = 4\sum_{i=1}^{28} i + 33\sum_{i=1}^{28} 1 = 4\frac{28\times 29}{2} + 33(28) = 2548$$

11) using subtraction method

$$\sum_{i=10}^{37} (4i-3) = \sum_{i=1}^{37} (4i-3) - \sum_{i=1}^{9} (4i-3) = \left(4\sum_{i=1}^{37} i - 3\sum_{i=1}^{37} 1\right) - \left(4\sum_{i=1}^{9} i - 3\sum_{i=1}^{9} 1\right)$$

$$= \left(4\frac{37\times 38}{2} - 3(37)\right) - \left(4\frac{9\times 10}{2} - 3(9)\right) = 2701 - 153 = 2548$$

12) using range shifting method

$$\sum_{i=5}^{15}(3i^2 - 4) = \sum_{i=1}^{11}(3(i+4)^2 - 4 = \sum_{i=1}^{11}(3i^2 + 24i + 16) - 4 = 3\sum_{i=1}^{11} i^2 + 24\sum_{i=1}^{11} i + 44\sum_{i=1}^{11} 1$$

$$= 3\left(\frac{2n^3 + 3n^2 + n}{6}\right) + 24\left(\frac{n(n+1)}{2}\right) + 44(n) = 3\left(\frac{2(11)^3 + 3(11)^2 + 11}{6}\right) + 24\left(\frac{11(12)}{2}\right) + 44(1$$

$$= 1518 + 1584 + 484 = 3586$$

12) using subtraction method

$$\sum_{i=5}^{15}(3i^2 - 4) = \sum_{i=1}^{15}\left(3i^2 - 4\right) - \sum_{i=1}^{4}\left(3i^2 - 4\right) = \left(3\sum_{i=1}^{15} i^2 - 4\sum_{i=1}^{15} 1\right) - \left(3\sum_{i=1}^{4} i^2 - 4\sum_{i=1}^{4} 1\right)$$

$$= \left(\left(3\frac{2(15)^3 + 3(15)^2 + 15}{6}\right) - 4(15)\right) - \left(\left(3\frac{2(4)^3 + 3(4)^2 + 4}{6}\right) - 4(4)\right)$$

$$= 3660 - 74 = 3586$$