COP 3502 Section 1 Exam 1B – Recursion, Sums/Recurrences, Alg. Anl. (Friday 10/13/2023) Solutions

1) (10 pts) Complete the code below so that it prints out all strings of length 10 that consist solely of the characters 'A' and 'B'. Please make sure strings are printed one per line, in alphabetical order.

```
#include <stdio.h>
#define SIZE 10

void go(char* str, int k, int n);

int main() {
    char str[SIZE+1];
    str[SIZE] = '\0';
    go (str, 0, SIZE);
    return 0;
}

void go(char* str, int k, int n) {


    // 1 pt
    if (k == n) {

        // 2 pts
        printf("%s\n", str);

        // 1 pt for this or else making sure this works…
        return;
    }

    // 1 pt
    str[k] = 'A';

    // 2 pts
    go(str, k+1, n);

    // 1 pt
    str[k] = 'B';

    // 2 pts
    go(str, k+1, n);




}
```

2) (10 pts) Solve the following recurrence relation, defined for all non-negative integers, using the iteration technique. Please give a closed-form formula for T(n) exactly (not a Big-Oh bound.)

$$T(n) = 2T(n-1) + 2^n, \text{ for all integers } n > 0.$$
$$T(0) = 3$$

$$T(n) = 2T(n-1) + 2^n$$

$$T(n) = 2(2T(n-2) + 2^{n-1}) + 2^n$$

$$T(n) = 4T(n-2) + (2)2^{n-1} + 2^n$$

$$T(n) = 4T(n-2) + 2^n + 2^n$$

$$T(n) = 4T(n-2) + 2(2^n)$$

$$T(n) = 4(2T(n-3) + 2^{n-2}) + 2(2^n)$$

$$T(n) = 8T(n-3) + 4(2^{n-2}) + 2(2^n)$$

$$T(n) = 8T(n-3) + (2^2)(2^{n-2}) + 2(2^n)$$

$$T(n) = 8T(n-3) + (2^n) + 2(2^n)$$

$$T(n) = 8T(n-3) + 3(2^n)$$

After k iterations, we have:

$$T(n) = 2^k T(n-k) + k(2^n)$$

Plug in k = n, and we have:

$$T(n) = 2^n T(0) + n(2^n) = 3(2^n) + n(2^n) = (n+3)2^n$$

Grading: 1 pt first iteration, 2 pts second iteration, 2 pts third iteration, 2 pts k iterations, 1 pt plug in k = n, 2 pts to get to final answer

3) (7 pts) Determine, as a function of n, the sum below. **Please provide your answer in standard polynomial form.**

$$\sum_{\boldsymbol{i=n+1}}^{\boldsymbol{3n}} (\boldsymbol{i}+\boldsymbol{5})$$

$$\sum_{i=n+1}^{3n} (i+5) = \sum_{i=1}^{3n} (i+5) - \sum_{i=1}^{n} (i+5)$$

$$= \frac{3n(3n+1)}{2} + 5(3n) - (\frac{n(n+1)}{2} + 5n)$$

$$= \frac{9n^2+3n}{2} + 15n - \frac{(n^2+n)}{2} - 5n$$

$$= \frac{9n^2+3n-n^2-n}{2} + 10n$$

$$= \frac{8n^2+2n}{2} + 10n$$

$$= 4n^2 + n + 10n$$

$$= 4n^2 + 11n$$

Grading: 1 pt split sum, 1 pt each for each of the four formulas or just adding all terms for the last two sums, 2 pts to simplify all the way to the end.

Alternate solution: Recognize the sum is an arithmetic sequence with 2n terms, with first term n+6 and last term 3n + 5. It follows that the desired sum is:

$$\left(\frac{n+6+3n+5}{2}\right) \times (2n) = \left(\frac{4n+11}{2}\right)(2n) = (4n+11)(n) = \mathbf{4n^2+11n}$$

Grading: 1 pt # of terms, 1 pt first term, 1 pt last term, 3 pts for simplification

4) (7 pts) An algorithm with a run-time of O(nlgn) takes 45 ms to run on an input of size n = 2^{15}. How long will the algorithm take when run on an input of size n = 2^{20}? **Please answer in seconds, rounded to the nearest hundredth second.**

Let the T(n) represent the run time of the algorithm with input size n. Then there exists some constant c such that $T(n) = cnlgn$. Plug in the given information:

$$T(2^{15}) = c(2^{15})\lg(2^{15}) = 45ms$$

$$c = \frac{45ms}{2^{15} \times 15 \times \lg(2)} = \frac{3ms}{2^{15}\lg(2)}$$

Now, let's plug in n = 2^{20}:

$$T(2^{20}) = \frac{3ms}{2^{15}\lg(2)} \times 2^{20}\lg(2^{20}) = \frac{(3ms) \times 2^{5} \times 20\lg(2)}{(lg2)} = 60 \times 32ms = 1920ms = 1.92\ seconds$$

Grading: 2 pts set up initial equation, 2 pts solve for c, 1 pt plug in new value, 2 pts arrive at final answer. (Only give full credit if answer is in the right form.)

5) (1 pt) In what Texas city does the Austin City Limits Music Festival take place? **Austin (Give to all)**