



SORTING

COP 3502

Sorting a List

- Let's say we have a list of the names of people in the class and we want to sort alphabetically
 - We are going to describe an algorithm (or systematic methods) for putting these names in order
 - The algorithms we will cover today:
 - Selection Sort Insertion Sort Bubble Sort



BOB



JOE



ABE



SAM



ANN



Sorting a List

■ Selection Sort

- Finds the smallest element (alphabetically the closest to a)
 - Swaps it with the element in the first position
- Then finds the second smallest element
 - Swaps it with the element in the second position
- Etc. until we get to the last position, and then we're done!



BOB



JOE



ABE



SAM



ANN

Sorting a List

■ Selection Sort

Min = "Abe"

C
U
R
R

C
H
E
C
K
E
R

C
H
E
C
K
E
R

C
H
E
C
K
E
R

C
H
E
C
K
E
R

"Joe" < "Bob"?

"Abe" < "Bob"?

"Sam" < "Abe"?

"Ann" < "Abe"?



BOB



JOE



ABBE



SAM



ANN

Sorting a List

■ Selection Sort

- Finds the smallest element (alphabetically the closest to a)
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ABE



JOE



BOB



SAM

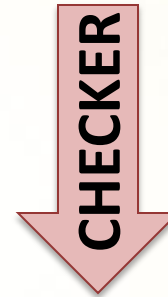
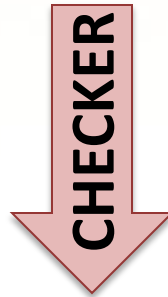
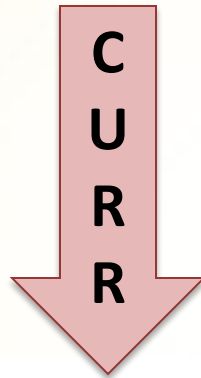


ANN

Sorting a List

■ Selection Sort

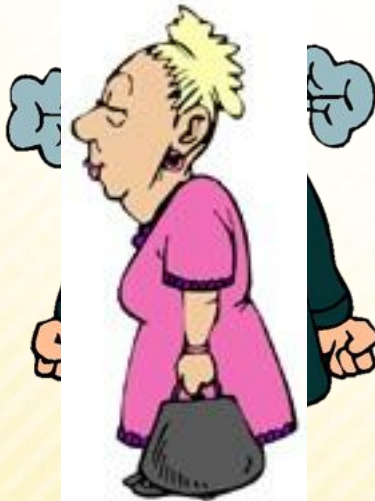
Min = "Ann"



"Bob" < "Joe"? "Sam" < "Bob"? "Ann" < "Bob"?



ABE



ANNE



BOB



SAM

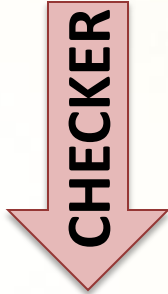
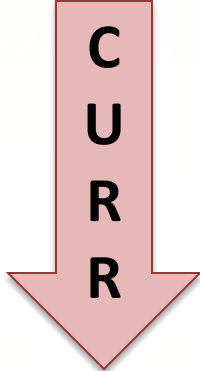


JOE

Sorting a List

■ Selection Sort

Min = "Bob"



"Sam" < "Bob"? "Joe" < "Bob"?"



ABE



ANN



BOB



SAM



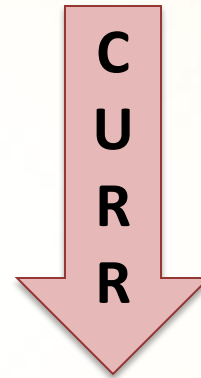
JOE

Sorting a List

■ Selection Sort

Min = "Joe"

Notice that now the list is sorted!
So we can stop when **Curr** is on the
2nd to last element.



"Joe" < "Sam"?



ABE



ANN



BOB



SAM



JOE

Sorting a List

■ Insertion Sort

- Take each element one by one, starting with the second and “insert” it into the already sorted list to its left in the correct order.



BOB



JOE



ABE



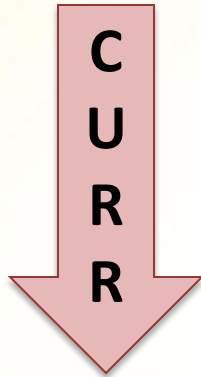
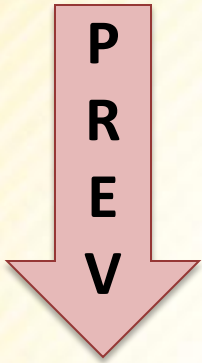
SAM



ANN

Sorting a List

■ Insertion Sort



“Joe” < “Bob”?

Pos = 1



BOB



JOE



ABE



SAM



ANN

Sorting a List

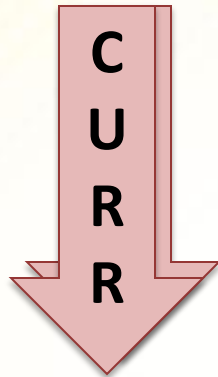
■ Insertion Sort



“Abe” < “Bob”?



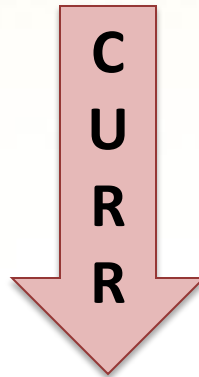
BBB



“Abe” < “Joe”?



BBB



Pos = 2



ABBE



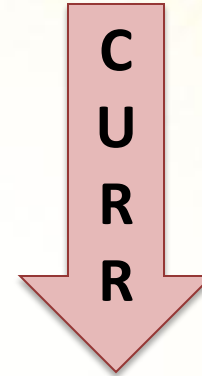
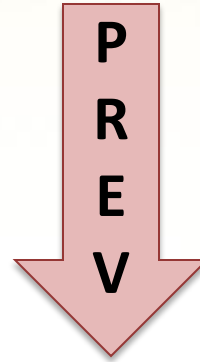
SAM



ANN

Sorting a List

■ Insertion Sort



"Sam" < "Joe"?

Pos = 3



ABE



BOB



JOE



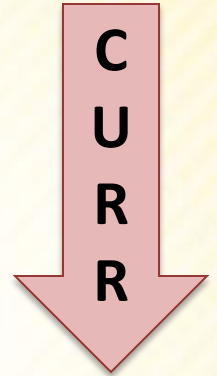
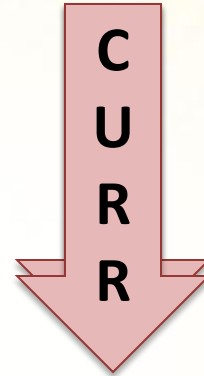
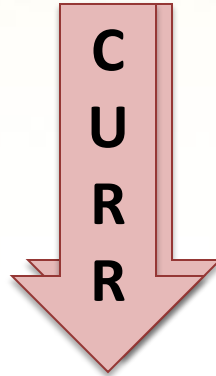
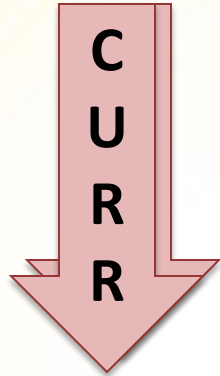
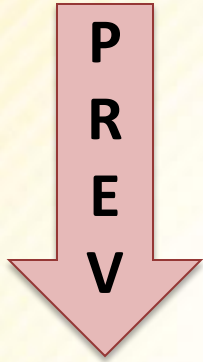
SAM



ANN

Sorting a List

■ Insertion Sort



“Ann” < “Abe”? “Ann” < “Bob”? “Ann” < “Joe”? “Ann” < “Sam”?

Pos = 4



ABE



ANN



BOB



JOE



SAM

Sorting a List

Bubble Sort

- The basic idea behind bubble sort is that you always compare consecutive elements, going left to right.
 - Whenever two elements are out of place, swap them.
 - At the end of a single iteration, the max element will be in the last spot.
- Now, just repeat this n times
- On each pass, one more maximal element will be put in place.
- As if the maximum elements are slowly “*bubbling*” up to the top.



BOB



JOE



ABE



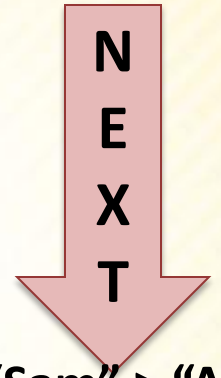
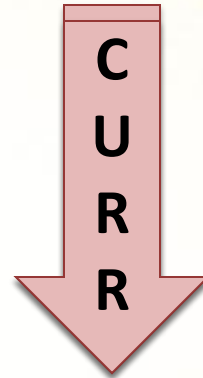
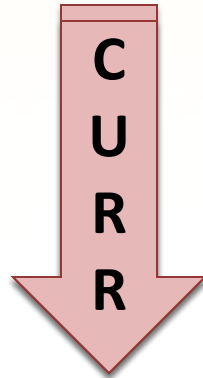
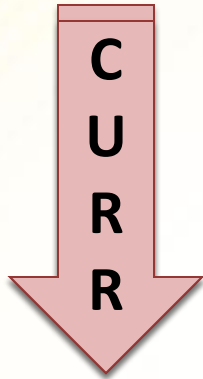
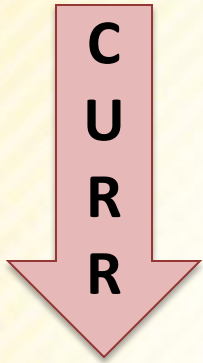
SAM



ANN

Sorting a List

■ Bubble Sort



"Bob" > "Joe"?

"Joe" > "Abe"?

"Joe" > "Sam"? "Sam" > "Ann"?



BOB



ABE



ABE



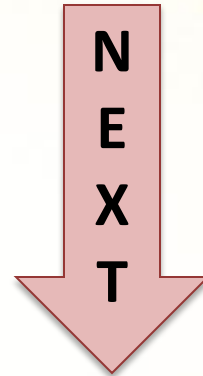
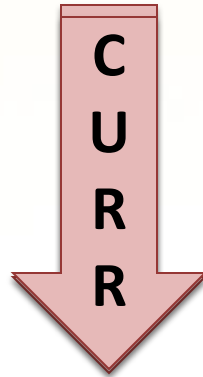
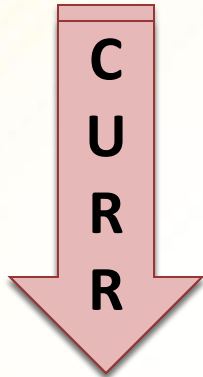
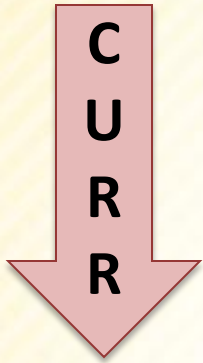
SAM



SAM

Sorting a List

■ Bubble Sort



"Bob" > "Abe"?

"Rob" > "Joe"?

"Joe" > "Ann"?



BBB



BBB



ADEN



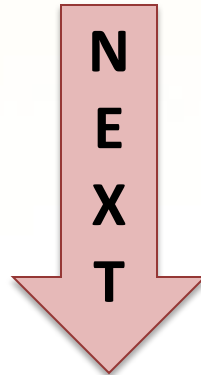
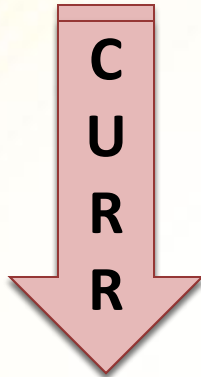
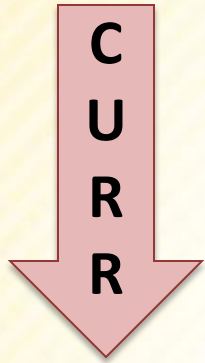
ADEN



SAM

Sorting a List

■ Bubble Sort



"Abe" > "Bob"?

"Bob" > "Ann"?



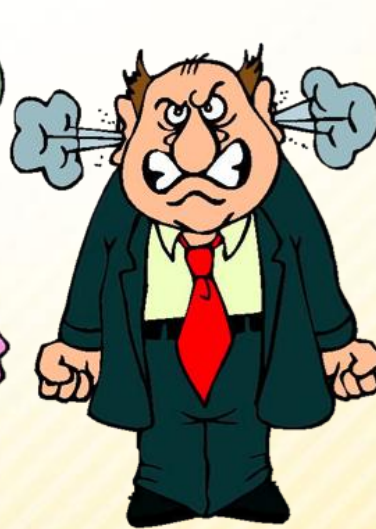
ABE



ANN



BOB



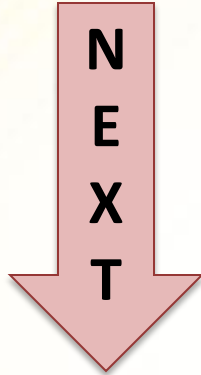
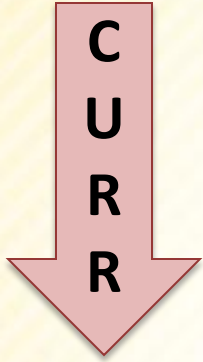
JOE



SAM

Sorting a List

■ Bubble Sort



"Abe" > "Anne"?



ABE



ANN



BOB



JOE



SAM

Limitation of Sorts that only swap adjacent elements

- A sorting algorithm that only swaps adjacent elements can only run so fast.
 - In order to see this, we must first define an inversion:
 - An inversion is a pair of numbers in a list that is out of order.
 - In the following list: 3, 1, 8, 4, 5
 - the inversions are the following pairs of numbers: (3, 1), (8, 4), and (8, 5).
 - When we swap adjacent elements in an array, we can remove at most one inversion from that array.



Limitation of Sorts that only swap adjacent elements

- Note that if we swap non-adjacent elements in an array, we can remove multiple inversions. Consider the following:
 - 8 2 3 4 5 6 7 1
 - Swapping 1 and 8 in this situation removes every inversion in this array (there are 13 of them total).
- Thus, the run-time of an algorithm that swaps adjacent elements only is constrained by the total number of inversions in an array.



Limitation of Sorts that only swap adjacent elements

- Let's consider the average case.
 - There are $\binom{n}{2} = \frac{(n-1)n}{2}$ pairs of numbers in a list of n numbers.
 - Of these pairs, on average, half of them will be inverted.
 - Thus, on average, an unsorted array will have $\frac{(n-1)n}{4} = \Omega(n^2)$ number of inversions,
 - and any sorting algorithm that swaps adjacent elements only will have a $\Omega(n^2)$ run-time.

