

SUMMATIONS

COP 3502

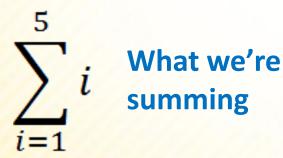
- Why do we need to go over summations?
 - This isn't a math class!
- Many times, analyzing an algorithm to determine its efficiency requires adding up many numbers.
 - This can be represented by a summation



For example,

- If we had the sequence 1+2+3+4+5
- This can be represented by the following summation:

Stopping condition



Starting condition

Does this remind you of anything we've seen in code?



If we're given a summation,

$$\sum_{k=2}^{14} 2k + 1$$

- **Total = θ +7 +9 +... 29**
- k = 2 📫 2k+1 = 5
- k = 3 📫 2k+1 = 7
- $k = 4 \implies 2k+1 = 9$
- k = 14) 2k+1 = 29

We can evaluate it in this way:

- 1) Create a running total set to 0.
- Set the variable in the bottom (k) of the sum equal to the initial value given, (2)
- 3) Plug this value into the expression, (2k+1)
- 4) Add this to your running "total".
- 5) If your variable equals the last value listed, (14) stop and your answer is what is stored in total.
 - -- Otherwise plug in the next integer value for the variable and go to step 3.

In code we would have this:

int total = 0;
for (k=2; k<=14; k++)
 total += (2*k+1);</pre>

In general we would say the following:

$$\sum_{k=a}^{b} f(k) = f(a) + f(a+1) + f(a+2) + \dots + f(b)$$

Let's use our example from before,
$$\sum_{k=2}^{14} 2k + 1$$

Where f(k) = 2k + 1

$$\sum_{k=2}^{14} f(k) = f(2) + f(3) + f(4) + \dots + f(14)$$

 $-5 + 7 + 9 + \dots + 29$ But what if we den

But what if we don't want to add up all these #'s? We can apply our formulas for solving summations...

Formula 1 – can take out constants

- The first formula we have is for a summation with just a constant.
 - Notice that c does not change with k,
 - ➢so it's constant
 - With constants we can pull them outside the summation: $\sum_{c}^{b} c = \sum_{c}^{b} c \sum_{c}^{b} 1$

Summations Formula 2 – Summing a constant

$$\sum_{k=a}^{b} c = c \sum_{k=a}^{b} 1 = (b-a+1)c$$

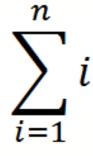
• Let's look at a specific example $\sum_{i=3}^{7} 5 = 5 \sum_{i=3}^{7} 1 = 5 * (7 - 3 + 1)$



Summations Formula 3 – Sum of i

If we look at a more difficult summation

 (that we saw last time) we can derive the formula for it using a clever trick.



S = 1+2+3+4+...+(n-1)+n

+ S = n+(n-1)+(n-2)+...+2+1

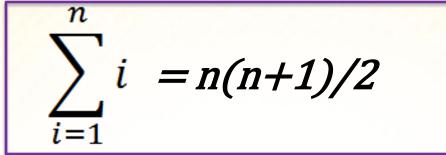
<mark>2S = (n+1)+ (n+1)+ (n+1)+...+ (n+1)</mark>

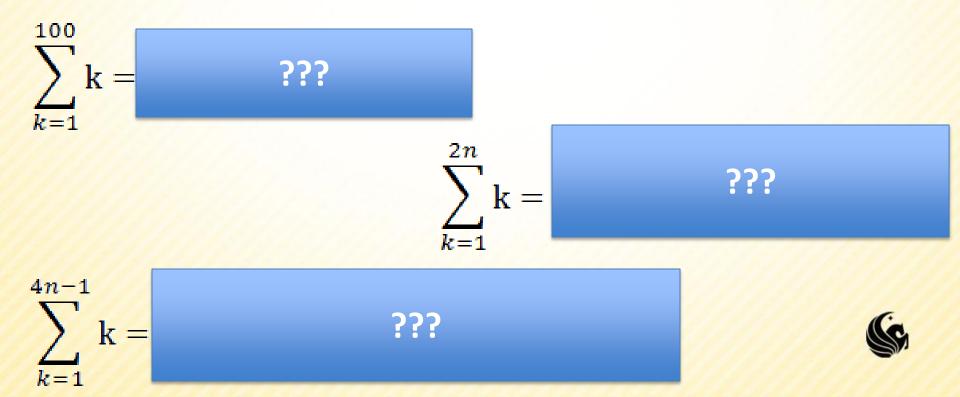
2S = n(n+1)

S = n(n+1)/2



Now let's look at a few quick uses of this formula:





Summations Formula 4 – Splitting up expressions

You can split up the terms in a summation into separate summations

$$\sum_{k=a}^{b} (f(k) + g(k)) = \sum_{k=a}^{b} f(k) + \sum_{k=a}^{b} g(k)$$

$$\sum_{k=1}^{n} (k+3) = \sum_{k=1}^{n} k + \sum_{k=1}^{n} 3 = \frac{n(n+1)}{2} + 3n = \frac{n^2 + 7n}{2}$$



Formula 5 – Change start to 1

Sometime summations don't start from 1 and we need them to to apply our formula

So this is what we can do:



In general our formula looks like this:

$$\sum_{k=a}^{b} f(k) = \sum_{k=1}^{b} f(k) - \sum_{k=1}^{a-1} f(k)$$



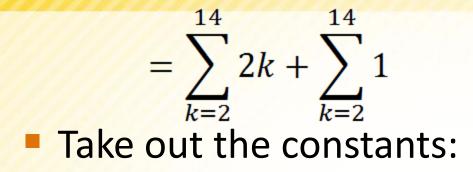
So we now we have all the pieces to solve our original example: $\sum_{k=1}^{14} 2k + 1$

Formula 4 – split up the terms:

$$\sum_{k=a}^{b} (f(k) + g(k)) = \sum_{k=a}^{b} f(k) + \sum_{k=a}^{b} g(k)$$

$$=\sum_{k=2}^{14} 2k + \sum_{k=2}^{14} 1$$





$$= 2\sum_{k=2}^{14} k + \sum_{k=2}^{14} 1$$



$$= 2\sum_{k=2}^{14} k + \sum_{k=2}^{14} 1$$

Formula 1 for the right side: *c*

t side:
$$c \sum_{k=a}^{b} 1 = (b - a + 1)c$$

$$\sum_{k=2}^{14} 1 = 14-2+1 = 13$$

And we get:

$$2\sum_{k=1}^{14} k + 13$$



$$2\sum_{k=2}^{14} k + 13$$

Formula 4 to change start of left side to 1:

$$\sum_{k=20}^{40} f(k) = \sum_{k=1}^{40} f(k) - \sum_{k=1}^{19} f(k)$$

$$2\left(\sum_{k=1}^{14} k - \sum_{k=1}^{2} k\right) + 13$$



$$2\left(\sum_{k=1}^{14} k - \sum_{k=1}^{2} k\right) + 13$$

Apply Formula 3 to each sum of k:

$$\sum_{i=1}^{n} i = n(n+1)/2$$

2(14*15/2 – 2*3/2)

= = 14*15-2*3 = 210

Don't forget about +13!!

Final answer = 210 + 13 = 223



Closed form solutions

- Not all summations result in a number for an answer.
 - Often the answer has one or more variables in it (usually n for our examples).
 - This is called the "closed form" of the summation



Examples on the board of finding the closed form of summations.

