From Arup's Algorithm Analysis Lecture:

## Loop Tracing Example

Here is an example from a previous foundation exam:

Question: Find the value of x in terms of n after the following code segment below has executed. You may assume that n is a positive even integer.

```
x = 0;
for (i = 1; i <= n*(8*n+8); i++) {
     for (j = n/2; j <=n; j++) {
           x = x + (n - j);
    }
}
```

Easier Solution:

First notice that all we are doing is repeatedly adding numbers into x. Furthermore, since the inner loop is NOT dependant on the value of i, we are adding the same value into x for each iteration of the outer loop. Thus, we must first figure out how much is being added into to x each time the entire inner loop runs.

| Iteration | value of j | value of n-j |
|---|---|---|
| 1 | n/2 | n/2 |
| 2 | n/2+1 | n/2 - 1 |
| 3 | n/2 +2 | n/2 - 2 |
| ... | | |
| n/2+1 | n | 0 |

Thus, we must add all the values in the right-hand column to figure out the value that gets added to x for a complete run of the inner loop.

$0+1+2+...+n/2 = n/2*(n/2+1)/2 = (n^2 + 2n)/8$

Now, we see that we add this value into x exactly n(8n+8) number of times. Repeated addition is multiplication, so the value of x after the loops are done will be

$(n^2 + 2n)/8*n(8n+8) = n(n+2)/8*n*8*(n+1)$

$= n^2(n+1)(n+2)$

Harder Solution, Find the closed form solution of:

$$\sum_{i=1}^{n(8n+8)} \underbrace{\sum_{j=n/2}^{n} (n-j)}_{\text{Solve this 1st}}$$

$$= \sum_{j=n/2}^{n} n - \sum_{j=n/2}^{n} j$$

$$= n(n - n/2 + 1) - \left[ \sum_{j=1}^{n} j - \sum_{j=1}^{n/2-1} j \right]$$

$$= n(n/2 + 1) - \left[ \frac{n(n+1)}{2} - \frac{(n/2 - 1)\, n/2)}{2} \right]$$

$$= \cancel{\frac{n^2}{2}} + n - \cancel{\frac{n^2}{2}} - \frac{n}{2} + \frac{n^2}{8} - \frac{n}{4}$$

$$= \frac{n}{2} + \frac{n^2}{8} - n/4 = \underline{\frac{n^2}{8} + \frac{n}{4}}$$

$$\sum_{i=1}^{n(8n+8)} \underbrace{(n^2/8 + n/4)}_{\text{A constant}} = n(8n+8)(n^2/8 + n/4)$$

$$= \frac{8\cancel{n}(n+1)n(n+2)}{\cancel{8}}$$

$$= n^2(n+1)(n+2)$$