

Q CIS 3362 11/2/2022

Review Quiz 4 Review Tonight 8-9pm
(hope)

Aids

1 sheet notes (8.5" x 11") you bring

Calculator

Date: 11/9 12:30pm

Don't meet in person Monday 11/7 (watch recordings instead!)

Today: RSA encryption

Ron Rivest
Adi Shamir
Leonard Adleman] Interested in creating public
key crypto system

General set-up: Some public info posted by Alice
Alice's Public Keys. She also has corresponding
Private Keys, but knowledge of Public Keys don't
reveal info about private keys.

Allows others to send encrypted messages to
Alice knowing that only Alice should be able
to decrypt the message.

Private keys: 2 large primes p, q (Alice primes)

Public key #1: $n = pq$

Private info/key: $\phi(n) = (p-1)(q-1)$

~~Next~~

Public key #2: Choose e , such that $\gcd(e, \phi(n)) = 1$.

Next Private key: $d = \underline{e^{-1} \text{ mod } \phi(n)}$ ↑

Public keys: n, e

Private keys: p, q, d , [$\phi(n)$ should be kept secret]

Bob to send msg to Alice, M ,

$$0 \leq M < n$$

$$C = E(M) = M^e \text{ mod } n \text{ (encryption)}$$

$$\text{Alice } D(C) = C^d \equiv M \text{ mod } n$$

$$C^d = (M^e)^d$$

$$= M^{ed}$$

$$= M^{k\phi(n)+1}$$

$$= M^{\phi(n)k} \times M^1$$

$$= \left(M^{\phi(n)} \right)^k \times M$$

$$\equiv 1^k \times M \text{ mod } n, \text{ via Euler's Thm, assuming } \gcd(M, n) = 1$$

$$d = e^{-1} \text{ mod } \phi(n)$$

$$\Rightarrow ed \equiv 1 \text{ mod } \phi(n)$$

$$ed = k\phi(n) + 1 \text{ for some int } k.$$

How do I convert text to a number and vice versa?

① Base 26 use only letters

$$\text{COMPUTER} = 2 \times 26^7 + 14 \times 26^6 + 12 \times 26^5 + 15 \times 26^4 + 20 \times 26^3 + 19 \times 26^2 + 4 \times 26^1 + 17 \times 26^0$$

Find max of k such that $26^k < n$.
 Make the input into blocks of k characters.

* Ciphertext remains in numbers.

② Radix 64 (encodes upto 64 chars, letter lower, upper, digits, +, -)

③ Store data in bytes. Figure out max k s.t. $256^k < n$. Store k bytes in a single int.

0 1 2
 C A T
 —

←
 C

| | | |
|-----|--|---|
| i | $val = 0$ | C |
| 0 | $val = 26 + val + 2 = 2$ | A |
| 1 | $val = \underbrace{26 * val}_{\text{Shifts left all old values}} + \underbrace{0}_{\text{new val}} = 52$ | T |

←
 CA
 ↖ ↗

2 $val = 26 + val + 19 = 1371$

$$\begin{array}{r} 606 \\ \times 2 \\ \hline 1352 \\ 12 \\ \hline 1212 \end{array}$$