

CIS 3362 - 10/28/22

Calculating  $a^{\text{exp}} \bmod n$

```
res = 1
for (int i = 0; i < exp; i++)
    res = (res * a) % n;
```

Run-time at least  $O(\text{exp})$

Problem  $\text{exp}$  will be very large!

Not viable!

Key Idea

$$a^{1000000} = \left( a^{500,000} \right)^2 \bmod n$$

500,000 steps

$$= X^2 \bmod n$$

1 step (huge savings!)

If exponent is even  
Calculate  $X = a^{\text{exp}/2} \bmod n$   
return  $(X * X) \bmod n$

else calculate =  $a \times \left[ a^{\text{exp}-1} \bmod n \right]$

base BAD CASE Subproblem  
(only happens once in a row, never twice!)

# mult+mods is  $O(\log \text{exp})$

Java - modPow

Python - pow (base, exp, mod)

Here  $Q3$  10101

$$3^{(43)} = 3^{32} \cdot 3^8 \cdot 3^2 \cdot 3^1$$

$$\text{pow}b = 3$$

$$3^1, 3^2, (3^2)^2 = 3^4, (3^4)^2 = 3^8$$