1) (8 pts) What is the index of coincidence of the following set of letters: 10 As, 50 Bs, 50 Cs, and 40 Ds? For full credit, please express your answer as a fraction in lowest terms.

\[
IC = \frac{10 \times 9 + 50 \times 49 + 50 \times 49 + 40 \times 39}{150 \times 149} = \frac{10(9 \times 49 + 50 \times 49 + 40 \times 39)}{15 \times 15 \times 149} = \frac{9 + 490 + 156}{15 \times 149} = \frac{131}{3 \times 149} = \frac{131}{447}
\]

Grading: 4 pts numerator, 1 pt denominator, 3 pts simplification

2) (10 pts) The following ciphertext was encrypted using the Vigenere cipher with the keyword "SKY": KELSXBEYMF. What was the original plaintext?

K E L S X B E Y M F
S K Y S K Y S K Y S

10 - 18 = -8 = 18 (mod 26)  S
4 - 10 = -6 = 20 (mod 26)  U
11 - 24 = -13 = 13 (mod 26) N
18 - 18 = 0 A
23 - 10 = 13 N
1 - 24 = -23 = 3 (mod 26) D
4 - 18 = -14 = 12 (mod 26) M
24 - 10 = 14 O
12 - 24 = -12 = 14 (mod 26) O
5 - 18 = -13 = 13 (mod 26) N

SUNANDMOON (Grading: 1 pt per letter)

3) (5 pts) Encrypt "SEPTEMBER" using the shift cipher with an encryption key of 21.

Note that 21 \equiv -5 (mod 26), so we can equivalently add -5 to each numeric value for encryption.

S E P T E M B E R
18 4 15 19 4 12 1 4 17
-5 -5 -5 -5 -5 -5 -5 -5

13, -1, 10, 14, -1, 7, -4, -1, 12 (calculate mod 25)
13, 25, 10, 14, 25, 7, 22, 25, 12
N Z K O Z H W Z M

Ciphertext is NZKOZHWM. (Grading: 1/2 pt per letter, round score up to next integer if ends in 1/2)
4) (15 pts) Using the Extended Euclidean Algorithm determine $108^{-1} \mod 239$. Please answer with an integer in between 0 and 238, inclusive. Note: most of the credit will be for the steps of the algorithm and not the final answer.

\[
\begin{align*}
239 & = 2 \times 108 + 23 \\
108 & = 4 \times 23 + 16 \\
23 & = 1 \times 16 + 7 \\
16 & = 2 \times 7 + 2 \\
7 & = 3 \times 2 + 1 \\
\end{align*}
\]

(Grading: 5 pts)

\[
\begin{align*}
7 - 3 \times 2 &= 1 \\
7 - 3(16 - 2 \times 7) &= 1 \\
7 - 3 \times 16 + 6 \times 7 &= 1 \\
7 \times 7 - 3 \times 16 &= 1 \\
7(23 - 16) - 3 \times 16 &= 1 \\
7 \times 23 - 7 \times 16 - 3 \times 16 &= 1 \\
7 \times 23 - 10 \times 16 &= 1 \\
7 \times 23 - 10(108 - 4 \times 23) &= 1 \\
7 \times 23 - 10 \times 108 + 40 \times 23 &= 1 \\
47 \times 23 - 10 \times 108 &= 1 \\
47(239 - 2 \times 108) - 10 \times 108 &= 1 \\
47 \times 239 - 94 \times 108 - 10 \times 108 &= 1 \\
47 \times 239 - 104 \times 108 &= 1 \\
\end{align*}
\]

(Grading: 8 pts)

Take this equation mod 239 to yield

\[-104 \times 108 \equiv 1 \pmod{239} \]  
(Grading: 1 pt to get -104)

It follows that $108^{-1} \equiv -104 \equiv \boxed{135 \pmod{239}}$  
(Grading: 1 pt to convert to 135)
5) (10 pts) Consider an affine cipher with the encryption function \( f(x) = (19x + 12) \mod 35 \). What is the corresponding decryption function?

Switch \( x \) and \( y \) and solve for \( y \):

\[
x = (19y + 12) \mod 35
\]
\[
(x - 12) = 19y \mod 35
\]

We must find \( 19^{-1} \mod 35 \):

\[
35 = 1 \times 19 + 16
\]
\[
19 = 1 \times 16 + 3
\]
\[
16 = 5 \times 3 + 1
\]

We need to find \( 19^{-1} \mod 35 \):

\[
16 - 5 \times 3 = 1
\]
\[
16 - 5(19 - 16) = 1
\]
\[
16 - 5x19 + 5x16 = 1
\]
\[
6x16 - 5x19 = 1
\]
\[
6(35 - 19) - 5x19 = 1
\]
\[
6x35 - 6x19 - 5x19 = 1
\]
\[
6x35 - 11x19 = 1
\]

Take this equation mod 35 to yield

\[-11 \times 19 \equiv 1 \mod 35\]. It follows that \( 19^{-1} \equiv -11 \equiv 24 \mod 25 \)

\[
24(x - 12) = 24(19y) \mod 35
\]
\[
y \equiv 24(x - 12) \mod 35
\]
\[
y \equiv (24x - 288) \mod 35
\]
\[
y \equiv (24x + 27) \mod 35
\]

It follows that the decryption function is \( f^{-1}(x) = (24x + 27) \mod 35 \)

6) (2 pts) The world’s busiest airport, Hartsfield-Jackson Atlanta International Airport is named after which two former mayors of Atlanta?

Hartsfield and Jackson (Give to All)