

$$1) \quad \begin{aligned} x &= \# \text{ zeros} \\ 100-x &= \# \text{ ones} \end{aligned}$$

$$\frac{x(x-1) + (100-x)(99-x)}{100 \times 99} = \frac{19}{33}$$

$$\frac{x^2 - x + 9900 - 199x + x^2}{2} = \frac{19 \times 100 \times 99}{33}$$

$$2x^2 - 200x + 9900 = 5700$$

$$2x^2 - 200x + 4200 = 0$$

$$x^2 - 100x + 2100 = 0$$

$$(x-30)(x-70)$$

$$x = 30, 70$$

$$\text{Answer} = 40$$

$$2) \quad f(x) = (28x + 33) \pmod{65}$$

$$x = (28y + 33) \pmod{65}$$

$$(x-33) \equiv 28y \pmod{65}$$

$$7(x-33) \equiv y \pmod{65}$$

$$y = (7x - 231) \pmod{65}$$

$$y = 7x + 29 \pmod{65}$$

$$65 = 2 \times 28 + 9$$

$$28 = 3 \times 9 + 1$$

$$28 - 3 \times 9 = 1$$

$$28 - 3(65 - 2 \times 28) = 1$$

$$28 - 3 \times 65 + 6 \times 28 = 1$$

$$7 \times 28 - 3 \times 65 = 1$$

3) $\begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 \\ \underline{0} & \underline{1} & \underline{0} & \underline{0} & \underline{1} & \underline{1} & \underline{1} & \underline{1} & \underline{0} & \underline{1} & \underline{1} & \underline{1} & \underline{1} & \underline{0} & \underline{0} & \underline{1} \end{matrix}$

Output: 0010 1101 1110 1011

2 D E B

4) $03 \times B6$

$$01 \times B6 = \boxed{1011 \ 0110}$$

$$02 \times B6 = \begin{array}{r} 1011 \ 0110 \\ \times 0110 \ 110 \\ \hline 11011 \end{array}$$

$$\boxed{11000001}$$

$$\begin{array}{r} 1011 \ 0110 \\ 1100 \ 0001 \\ \hline 0111 \ 0111 \end{array}$$

$$\boxed{77}$$

5) Want terms such that

$$g^1, g^2, \dots \not\equiv 1 \pmod{p}$$

$$\text{but } g^{\frac{p-1}{2}} \equiv 1 \pmod{p} \quad (\text{period to be half of } p-1)$$

$g^x \Rightarrow$ all terms can be written like this

$$(g^x)^{\frac{p-1}{2}} = g^{(p-1)y} \quad \text{for some int } y.$$

Once we cancel the 2 in x , it can't share any further factors with $p-1$. $\Rightarrow \underline{\underline{\gcd(x, p-1) = 2}}$

How many even number $\gcd(x, \frac{p-1}{2}) = 1$

$$\phi\left(\frac{p-1}{2}\right)$$

7) If both use the same n , they know the other person's p and q and knowing their other person's e can calculate the other person's d . Once they know this, they can sign as the other person!

8)

2 people Apr

2 people Dec

Sample Space = 12^4

A, A, D, D

A D A D

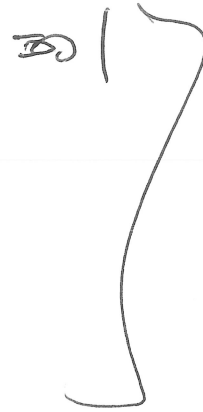
A D D A

D A A D

D A D A

D D A A

$6 = \binom{4}{2}$



6 ways fixed for 2 months

$\binom{12}{2}$ choose 2 month out of 12

$$\frac{12 \times 11}{2} = 66$$

$$P = \frac{6 \times 66}{12 \times 12 \times 12^2}$$

$$= \boxed{\frac{11}{576}}$$

$$\begin{array}{r} 144 \\ 4 \\ \hline 576 \end{array}$$