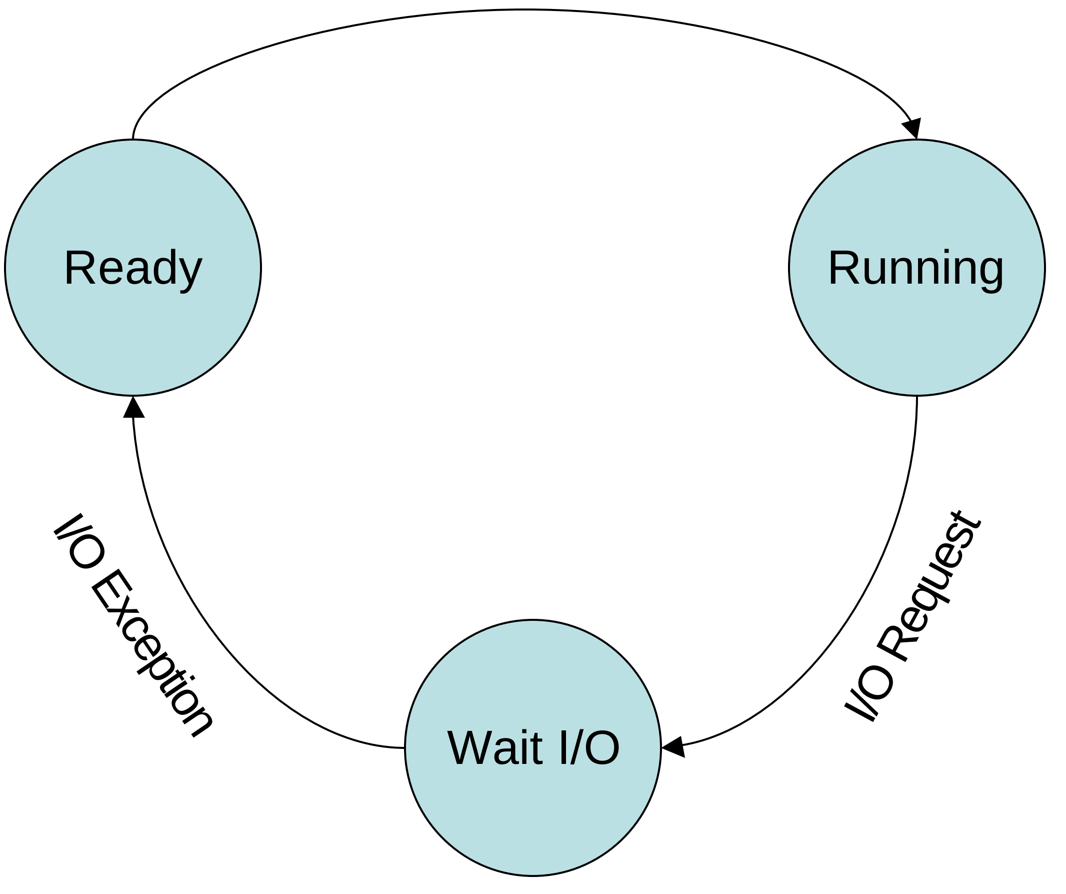
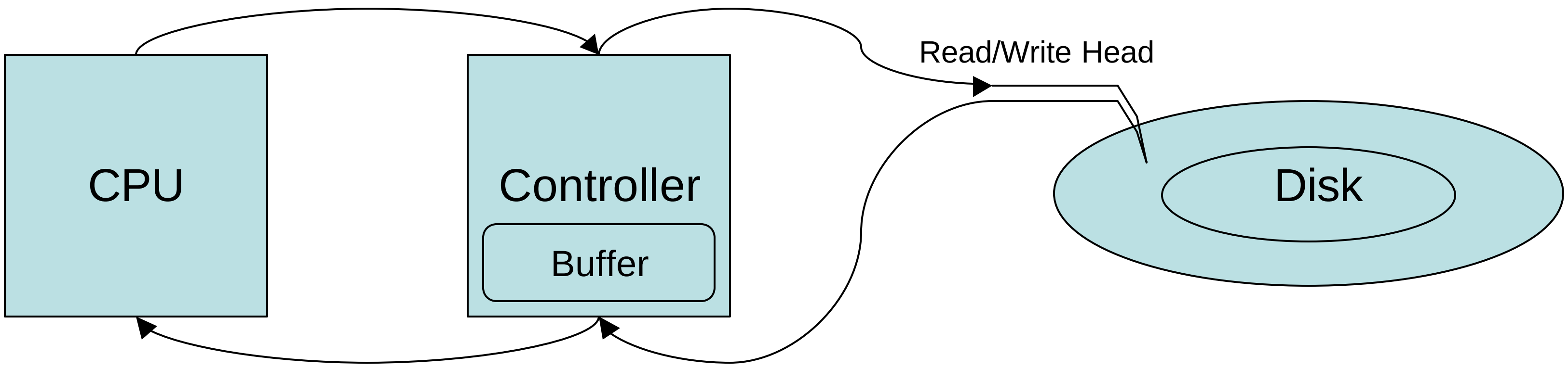
Input/Output



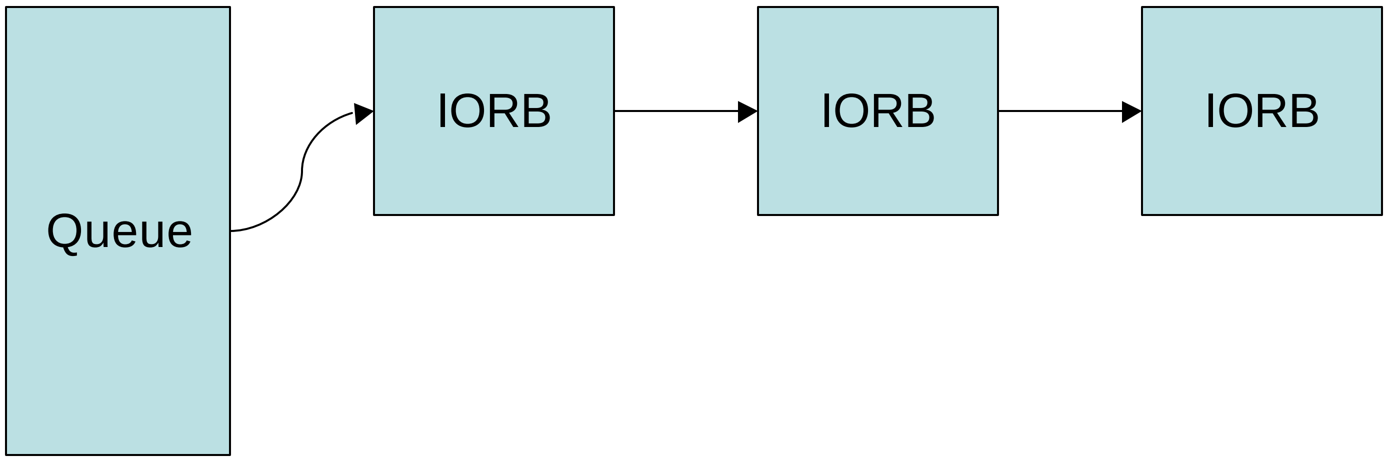
- Program is in running state unit it makes an I/O request.
- The I/O Request is a service request and is handled by the Operating System
- Once the request has been made, the program waits for the I/O request to be fulfilled
- Once the request is fulfilled, an I/O Exception signifies to the hardware that the I/O has completed and the program may continue

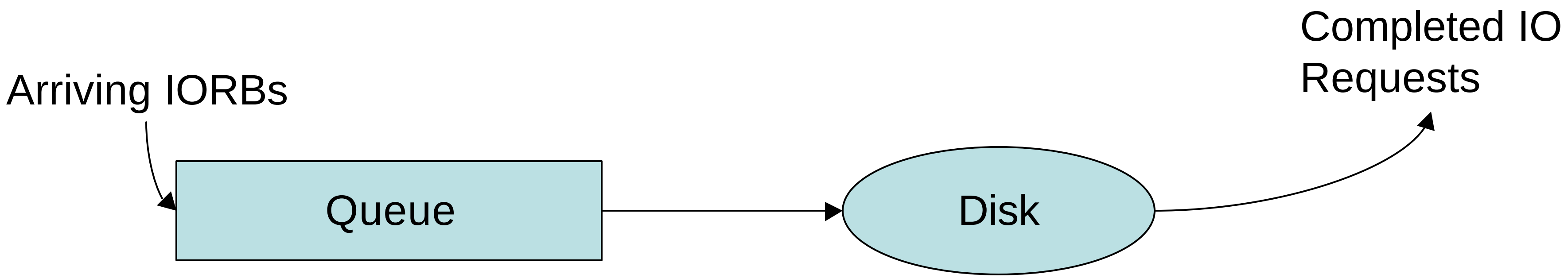
I/O Hardware



- CPU sends I/O request Blocks to controller, which in turn sends signals to the Read/Write Head to retrieve information from the disk
    - Disks are divided into tracks, (concentric disks on the surface of the media)
    - It is possible to arrange data in such a manner as to optimize the movement of the R/W Head across the tracks (i.e., rather than visiting tracks in the order 3-7-2, visit in the order 2-3-7 to allow one smooth motion)
- When the information has been retrieved, the controller sends an I/O Exception to the CPU

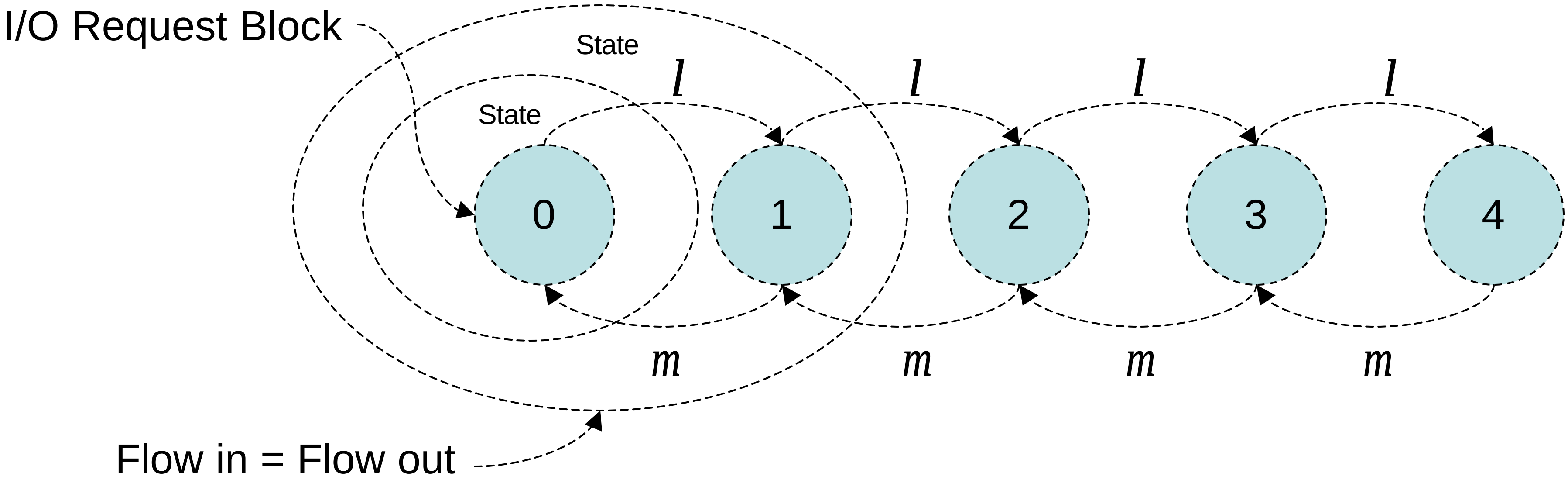
I/O Queue





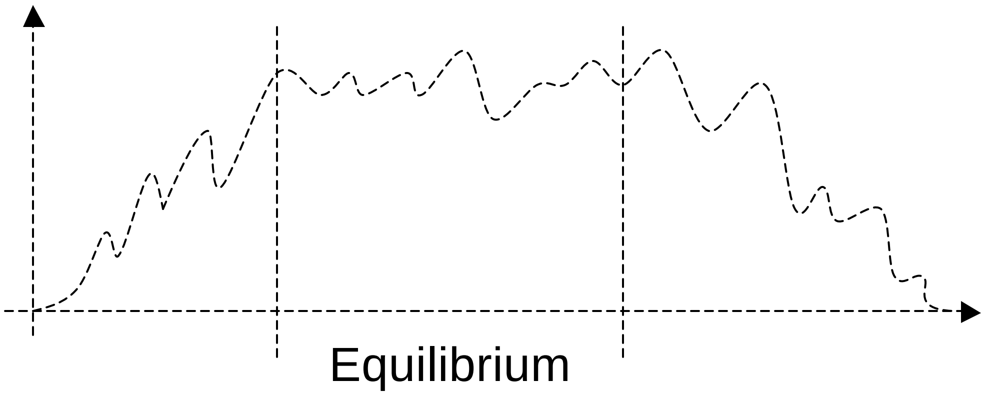
The queue is the final order of I/O’s that is to occur. It may be reordered when new requests come in to make best use of the R/W Head

I/O Equilibrium



After a certain period of build-up the system works into an equilibrium, shown by the IORB x Time graph to the right, and the diagram above.

In each state, the Flow of Incoming IORB equals the Flow of completed IORB's.



Some Definitions

A = Number of IO Request arrivals
C = Number of completed IO Requests
T = Length of time system is observed
B = Length of time the device is busy
S = Service time (per request)

Arrival Rate: $l = \frac{A}{T}$

Throughput: $x = \frac{C}{T}$

Utilization: $U = \frac{B}{T} = \frac{l}{m}$

Service Time: $s = \frac{1}{m}$

Probabilities of IO System

P_0 Probability of having 0 I/O requests in the system

P_1 Probability of having 1 I/O requests in the system

P_2 Probability of having 2 I/O requests in the system

P_i Probability of having i I/O requests in the system

Assuming an equal flow-in rate and flow-out rate we get the following…

$$mP_1 = l\,P_0 \Rightarrow P_1 = \frac{l}{m} P_0$$

$$mP_2 = l\,P_1 \Rightarrow P_2 = \frac{l}{m} P_1$$

$$\vdots$$

$$mP_i = l\,P_{i-1} \Rightarrow P_i = \frac{l}{m} P_{i-1}$$

Note: $$\frac{l}{m} = U$$

where *U* is the utilization of IO system

Finding P_i

Given that:

$$P_2 = \frac{l}{m} P_1$$

substituting the following $P_1 = \frac{l}{m} P_0$ and we get

$$P_2 = \left(\frac{l}{m}\right)\left(\frac{l}{m}\right) P_0 = \left(\frac{l}{m}\right)^2 P_0$$

Continue the pattern for P_i we get:

$$P_i = \left(\frac{l}{m}\right)^i P_0 = U^i P_0$$

where *i* is any given state

Sum of Probabilities

$$P_0 + P_1 + P_2 + \ldots + P_k = \sum_{i=0}^{k} P_i = 1$$

substituting $P_i = U^i P_0$ we get

$$\sum_{i=0}^{k} U^i P_o = 1 \Rightarrow P_o = \frac{1}{\sum_{i=0}^{k} U^i}$$

Geometric Series

Please recall the following about a geometric series

$$\sum_{i=0}^{\infty} r^i = \frac{1}{1-r}$$

where $|r| < 1$

Assume there exists an infinite number of states

$$P_o = \frac{1}{\sum_{i=0}^{k} U^i} = \frac{1}{\frac{1}{1-U}} = 1-U$$

where $|U| < 1$

In other words, the utilization, U, must always be less than 100% or else the system will not function

Average System State

$\overline{N}$ is the average system state at a finite small amount of time

N = The sum of the states times the probability of the state occurring, or…

$$\overline{N} = \lim_{t \to 0} \sum_{i=0}^{k} iP_i = \lim_{t \to 0} \sum_{i=0}^{k} i(1-U)U^i$$

$$\overline{N} = \lim_{t \to 0} U(1-U) \sum_{i=0}^{k} iU^{i-1}$$

Noting that the first derivative of U^i = *iU^{i-1} we can substitute this into the equation*

$$iU^{i-1} = \frac{\partial\left(U^i\right)}{\partial U}$$

$$\overline{N} = \lim_{t \to 0} U(1-U) \sum_{i=0}^{k} \left(\frac{\partial U^i}{\partial U} \right)$$

Average System State

Solving the for the geometric sum and the derivative of the result we get the following

$$\overline{N} = \lim_{t \to 0} U(1-U)\left(\frac{\partial\left(\frac{1}{1-U}\right)}{\partial U}\right) = \lim_{t \to 0} \frac{U(1-U)}{(1-U)^2}$$

Which, in turn, gives us the final equation:

$$\overline{N} = \lim_{t \to 0} \frac{U}{(1-U)}$$