Symbol Recognition in Sketch-Based Interfaces

Lecture #9: Symbol Recognition
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Recall Pen-Based Interface Dataflow

- Raw Stroke Data
- Preprocessing
- Segmentation
- Sketch Understanding
- Ink Parsing
- Classification
- Feature Extraction And Analysis
  - Make Inferences
Symbol Recognition

- Want to recognize handwritten symbols
  - characters
  - shapes
  - gestures
- Use machine learning approach
- Which algorithm?
  - depends on number of symbols in alphabet
  - complexity (i.e., similarity of symbols)
  - distribution assumptions

Recognition Algorithms

- Many different approaches
- Machine learning techniques (classification)
  - linear classifiers
  - k-means classifiers
  - neural networks
  - Hidden Markov Models
  - template matching
  - support vector machines
  - AdaBoost
- Curve matching
  - elastic matching
- Primitive decomposition
Rubine’s Gesture Recognition Algorithm (Rubine 1991)

- Simple linear classifier
- Utilizes rejection metrics
- Assumes normality for features
- Simple to implement
- Does not need a lot of training samples

Recall Rubine’s Feature Set

- Cosine and sine of initial angle
- Length and angle of bounding box diagonal
- Distance between first and last point
- Cosine and sine of angle between first and last point
- Total gesture length
- Total angle traversed
- Sum of absolute value of the angle at each point
- Sum of squared values of the angle at each point
- Maximum speed
- Stroke duration
Rubine Classifier

\[ v_{\hat{c}} = w_{\hat{c}0} + \sum_{i=1}^{F} w_{\hat{c}i} f_i \quad 0 \leq c < C \]

where \( F \) is the number of features, \( w_{\hat{c}} \) is the weights, and the classification of symbol \( g \) is the \( c \) that maximizes \( v_{\hat{c}} \).

- Evaluate each gesture \( 0 \leq c < C \).
- \( v_{\hat{c}} \) = value = goodness of fit for that gesture \( c \).

Rubine Classifier Training

- Collect \( E \) samples for each symbol class
- Calculate feature vector for each sample for each class
  - \( f_{\hat{c}ei} \) = the feature value of the \( i^{th} \) feature for the \( e^{th} \) sample of the \( c^{th} \) symbol
- For each symbol calculate the mean value for each feature
  \[
  \bar{f}_{\hat{c}i} = \frac{1}{E_{\hat{c}}} \sum_{e=0}^{E_{\hat{c}}-1} f_{\hat{c}ei} \quad \text{where } 0 \leq e < E_{\hat{c}}
  \]
  - and \( E_{\hat{c}} \) is the number of training samples per class
Rubine Classifier – Computing Weights

- We first need the covariance matrix of each class $c$

$$
\Sigma_{\hat{c}ij} = \frac{1}{E_{\hat{c}} - 1} \sum_{c=0}^{E_{\hat{c}}-1} (f_{\hat{c}ei} - \bar{f}_{\hat{c}i})(f_{\hat{c}ej} - \bar{f}_{\hat{c}j})
$$

Rubine Classifier – Computing Weights (2)

- Using the covariance matrices from each class, find the common covariance matrix
  - numerator = non-normalize total covariance
  - denominator = normalization factor = total number of examples – total number of shapes

$$
\Sigma_{ij} = \frac{\sum_{c=0}^{C-1} \sum_{\hat{c}=0}^{E_{\hat{c}}-1} \hat{c}ij}{-C + \sum_{c=0}^{C-1} E_{\hat{c}}}
$$
Rubine Classifier – Computing Weights (3)

- Using the common covariance matrix and the mean feature vectors from each class, we can compute the weights

\[
 w_{\hat{c}_j} = \sum_{i=1}^{F} \left( \Sigma^{-1} \right)_{ij} \bar{f}_{\hat{c}_i}, \quad 1 \leq j \leq F
\]

\[
 w_{\hat{c}_0} = -\frac{1}{2} \sum_{i=1}^{F} w_{\hat{c}_i} \bar{f}_{\hat{c}_i}
\]

Rubine Classifier – Rejection Measures

- Linear classifier always will classify a symbol as one of the \( C \) classes
  - want to try to reject outliers and ambiguous symbols
  - two approaches
    - probabilistic
    - distance measure
Rubine Classifier – Probabilistic Rejection Measure

- Given a symbol $g$ with feature vector $f$ classified as class $i$ ($v_i > v_j, \forall j \neq i$)

$$
\tilde{P}(i \mid g) = \frac{1}{\sum_{j=0}^{C-1} e^{(v_j-v_i)}}
$$

Reject symbols with $\tilde{P}(i \mid g) < 0.95$

Rubine Classifier – Rejection based on Distance

- Mahalanobis distance – the number of standard deviations a symbol $g$ is away from the mean of its chosen class $i$

$$
\delta^2 = \sum_{j=1}^{F} \sum_{k=1}^{F} (\Sigma^{-1})_{jk} (f_j - \bar{f}_j)(f_k - \bar{f}_k)
$$

Rejecting symbols for which $\delta^2 > \frac{1}{2} F^2$

- May need to be careful not to reject too many good symbols (a simple alternate list to correct mistakes will be helpful)
AdaBoost (Schapire 1997)

- Not really a classification algorithm – more like a framework
- Can use many different classification algorithms within AdaBoost framework
- Works with series of weak (base) classifiers
  - Want to increase the importance of incorrectly classified examples
    - series of weak hypotheses and weights form a strong hypothesis
    - need to ensure weak learners output either 1 or -1
- Many different variants (M1, M2, etc…)

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**AdaBoost Algorithm**

Given \((x_1, y_1), \ldots, (x_m, y_m)\) where \(x_i \in X, y_i \in \{−1, +1\}\)

Initialize \(D_1(i) = 1/m\)

For \(t = 1 \ldots T\)

- Train weak learner using distribution \(D_t\)
- Get weak hypothesis \(h_t : X \rightarrow \{−1, +1\}\) with error
  \[\varepsilon_t = \Pr_{y \sim D_t} [h_t(x) \neq y] = \sum_{i \mid h_t(x_i) \neq y_i} D_t(i)\]

- Compute \(\alpha_t = \frac{1}{2} \ln \left( \frac{1-\varepsilon_t}{\varepsilon_t} \right)\)

- Update \(D_{t+1}(i) = \frac{D_t(i) e^{-\alpha_t h_t(x_i)}}{Z_t}\)

Final hypothesis is \(H(x) = \text{sign} \left( \sum_{t=1}^{T} \alpha_t h_t(x) \right)\)
More Information on Machine Learning


Readings