Outline

- M-bin Histograms
- Exhaustive Search.
- Gradient-based Optimization.
- Gradient-based Optimization modified.
- Extra notes.

M-bin Histograms

- M-bin histogram is a chart with m number of bins where each bin shows the number of same color pixels.
- Target model: $\hat{q} = \{\hat{q}_u\}_{u=1...m}$
  - m : bin number in the histogram.
  - $\hat{q}_u$: number of same color pixels divided by total number of pixels in the ellipse.
  - Sum of the probabilities should equal to one.

M-bin Histograms

- Target candidate: $p^{\wedge}(y) = \{p^{\wedge}_u(y)\}_{u = 1...m}$

➢ y : x & y coordinates of the target in the next frame.

➢ $p^{\wedge}(y)$: probability of same color pixels (# same color pixels / total ellipse pixels).

➢ Sum of probabilities should equal to one.

M-bin histogram Modification

- Assigning smaller weights to pixels farther from the center of ellipse.

$$q = C \sum_{i=1}^{n} k(\|x_i\|) \delta [b(x_i) - u]$$

➢ x_i :normalized pixel locations in the region defined as the target model.

➢ $k(x)$:differentiable decreasing function to assign less weight for farther pixels.

➢ $b(x_i)$: which bin this pixel belong to.

➢ u :bin number in the histogram.

M-bin histogram Modification

- δ :Kronecker delta function.

$$\delta = \begin{cases} 1\,when\,u = b(x_i) \\ 0\,otherwise \end{cases}$$

- C: normalization constant since the sum of all pixel probabilities is equal to one.
- For target candidates $p_u(y)$, the target center location (y) and number of pixels considered in the localization process (h) are introduced to the k(x) profile.

$$p_u(y) = C_h \sum_{i=1}^{nh} k\left(\left\|\frac{y - x_i}{h}\right\|\right) \delta\left[b(x_i) - u\right]$$

Similarity function

- The similarity function defines a distance among target model and candidates.
- The maximum multiplication between probabilities of target model $\hat{q}$ and candidates $\hat{p}(y)$ will give the least error.
- How:

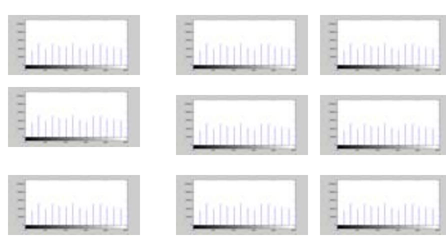
$$(\hat{p}(y) - \hat{q})^2 = \hat{p}(y)^2 + \hat{q}^2 - 2\,\hat{p}(y)\,\hat{q}$$

Exhaustive Search

- Computed the m-bin histogram probabilities of the target model $\hat{q}$.
- Computed probabilities of the target $\hat{p}(y)$ candidates for different locations of (y) where y is an (m*m) window and m-bin histogram will be computed for each position in the window.
- Apply similarity function(i.e: distance) between each $\hat{q}$ and $\hat{p}(y)$ find the minimum.

$$d(y) = \sqrt{1 - \rho\left[\hat{p}(y), \hat{q}\right]}$$

(y1)	(y2)	(y3)
(y4)	(yo)	(y6)
(y7)	(y8)	(y9)

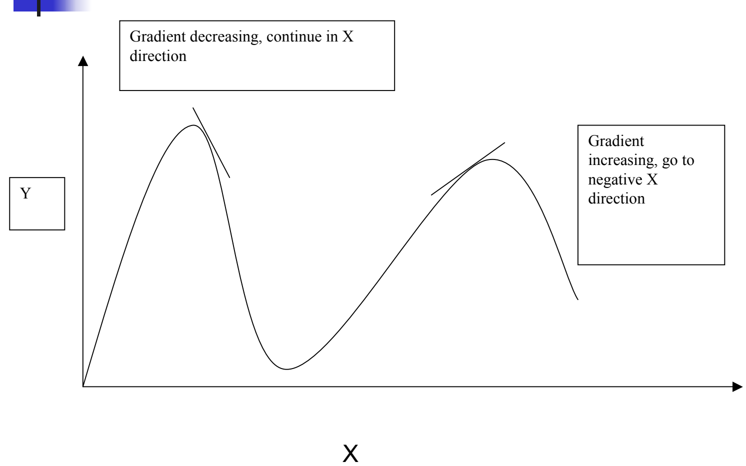


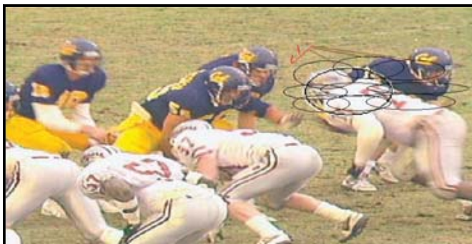
d(y1)	d(y2)	d(y3)
d(y4)	d(y5)	d(y6)
d(y7)	d(y8)	d(y9)

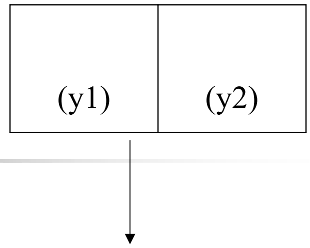
gradient-based optimization procedures

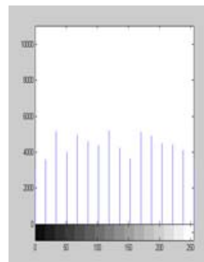
- Compute the gradient between two points.
- If the gradient is decreasing, continue in the same direction, otherwise reverse.
- In the paper, gradient is computed from two similarity distances.
- If gradient decreasing, continue computing till finding local maxima or minima.

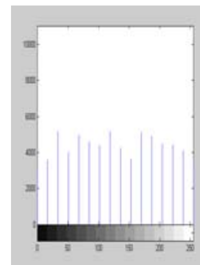
d(y1)	d(y2)	d(y3)
d(y4)	d(y5)	d(y6)
d(y7)	d(y8)	d(y9)











• (d(y2) – d(y1))/(y2 –y1)

•If gradient is decreasing, continue in increased y direction.

Modification To Gradient optimization

- Note: paper authors mention in p.566 that gradient-based optimization procedures are difficult to apply and only an expensive exhaustive search can be used and on p.567 a modification to gradient-based is proposed.

Modification To Gradient optimization

- Instead of using the gradient-based method to find the location of the target candidate and computing the similarity distance, a formula derived from Using Taylor expansion around the values is used: $\{\hat{p}_u(\hat{y}_0)\}_{u=1...m}$

1. Compute $\{\hat{p}_u(\hat{y}_0)\}_{u=1...m}$ where $\hat{y}_0$ is the same center location of the previous frame in the new frame and evaluate similarity function $\sum_{u=1}^{m}\sqrt{\hat{p}_u(\hat{y}_0)\hat{q}_u}$

Modification To Gradient optimization

2. Derive weights according to this relation. $w_i = \sum_{u=-1}^{m}\sqrt{\frac{\hat{q}_u}{\hat{p}_u(\hat{y}_o)}}\delta[b(x_i)-u]$
3. Find new location of target candidate according to $\hat{y}_1 = \frac{\sum_{i=1}^{nh} x_i w_i}{\sum_{i=1}^{nh} w_i}$
4. If $\|\hat{y}_1 = \hat{y}_0\| < \epsilon$ stop, otherwise $\hat{y}_0 = \hat{y}_1$

Adaptive Scale

- The ellipse should be adjusted incase it contains unwanted information.
- Run the target localization algorithm three times with bandwidths $h = h_{prev}$, $h = h_{prev} - \Delta h$,and $h = hprev + \Delta h$.
- h_{opt} is chosen where the similarity function is largest.

$$hnew = \gamma hopt + (1 - \gamma)hprev$$

Background Modification

- There are some cases when some of the target features are also present in the background, their relevance for the localization of the target is diminished.

- Compute the histogram of the background ellipse and probabilities of ellipse pixels.

$$\{\hat{O}_u\}_{u=1..m}$$

- find the minimum probability $\hat{O}^*_u$ and divide it by $\{\hat{O}_u\}_{u=1..m}$ to get v_u.
- Note: the paper did not mention if the histogram computed is the difference between background ellipse and object ellipse or is it the whole thing.

- New target model representation is then defined by: $\hat{q}_u = C v_u \sum_{i=1}^{n} k(\|x_i^*\|^2)\delta[b(x_i) - u]$

Where $$C = \frac{1}{\sum_{i=1}^{n} k(\|x_i\|^2)\sum_{u=1}^{m} v_u \delta[b(x_i^*-u)]}$$

- And new target candidate is represented by

$$\hat{p}_u = C_h v_u \sum_{i=1}^{n} k(\|(y - x_i^*)/h\|^2)\delta[b(x_i) - u]$$

Where $$C_h = \frac{1}{\sum_{i=1}^{nh} k(\|(y - x_i)/h\|^2)\sum_{u=1}^{m} v_u \delta[b(x_i^*-u)]}$$

Extra Notes

- Note one: in the previous formula there could be an error since v_u is cancelled with v_u in eqation(18).
- Other features can be combined with color (like gradient) to calculate probability and it was used in face tracking in this paper.