Survey: Bilateral Filter and Applications

Presented by
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Related Papers

- C. Tomasi, R. Manduchi, "Bilateral filtering for gray and color images", ICCV 1998.
- D. Comaniciu, P. Meer, "Mean Shift: A Robust Approach toward Feature Space Analysis", PAMI, 2002.
- F. Durand, J. Dorsey, "Fast Bilateral Filtering for the Display of High-Dynamic-Range Images", SIGGRAPH 2002.
- S. Fleishman, I. Drori, D. Cohen-Or, "Bilateral Mesh Denoising", SIGGRAPH 2003.
- T. Jones, F. Durand, M. Desbrun. "Non-Iterative, Feature-Preserving Mesh Smoothing", SIGGRAPH 2003.

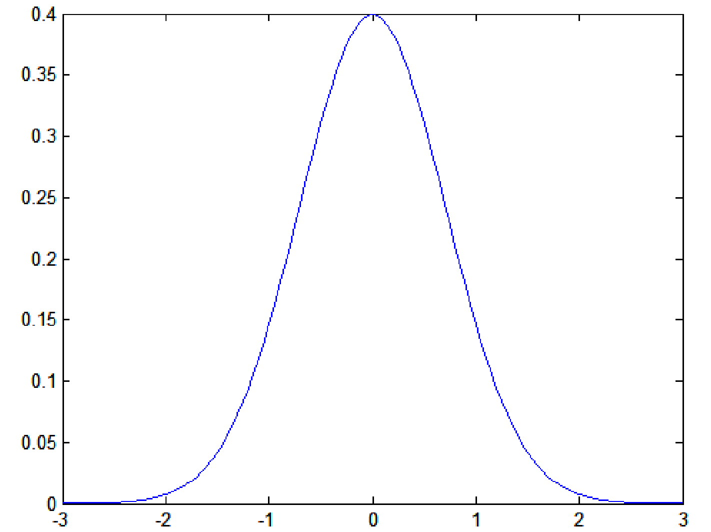
Gaussian Filter

- Normal distribution

$$\Phi = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma}} e^{-\left(\frac{t}{\sigma}\right)^2} dt$$

- Gaussian kernal filter

$$Y(\mu) = \int_{\mu-\tau}^{\mu+\tau} X(t) \frac{1}{\sqrt{2\pi\sigma}} e^{-\left(\frac{t-\mu}{\sigma}\right)^2} dt$$



The filter's width is 2τ, and center is at μ, bandwidth is σ.

Bilateral Filter

- For image I(u), at coordinate u=(x,y):

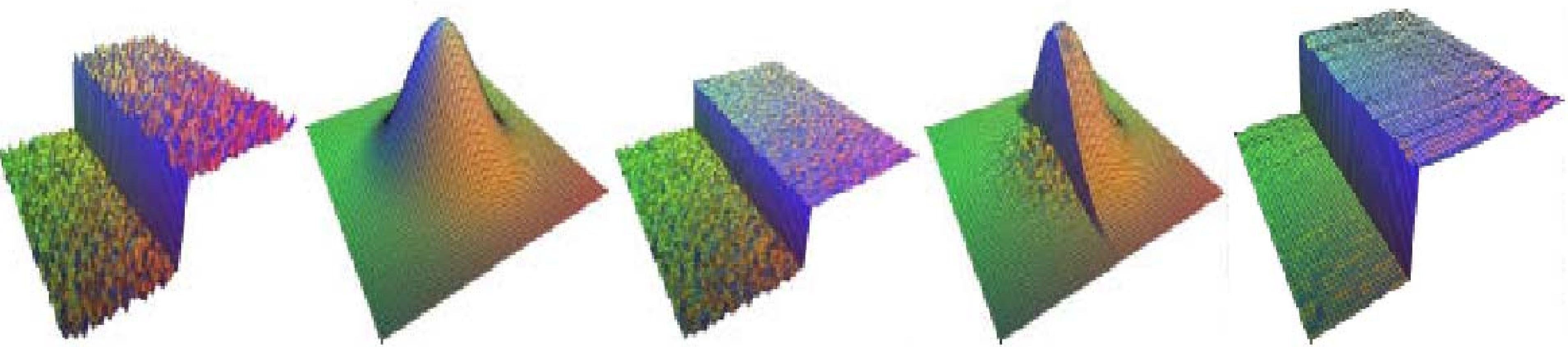
$$\hat{I}(\mathbf{u}) = \frac{\sum_{\mathbf{p}\in N(\mathbf{u})} W_c(\|\mathbf{p}-\mathbf{u}\|)W_s(|I(\mathbf{u})-I(\mathbf{p})|)I(\mathbf{p})}{\sum_{\mathbf{p}\in N(\mathbf{u})} W_c(\|\mathbf{p}-\mathbf{u}\|)W_s(|I(\mathbf{u})-I(\mathbf{p})|)},$$

- Two Gaussian filters:

$$W_c(x) = e^{-x^2/2\sigma_c^2}$$

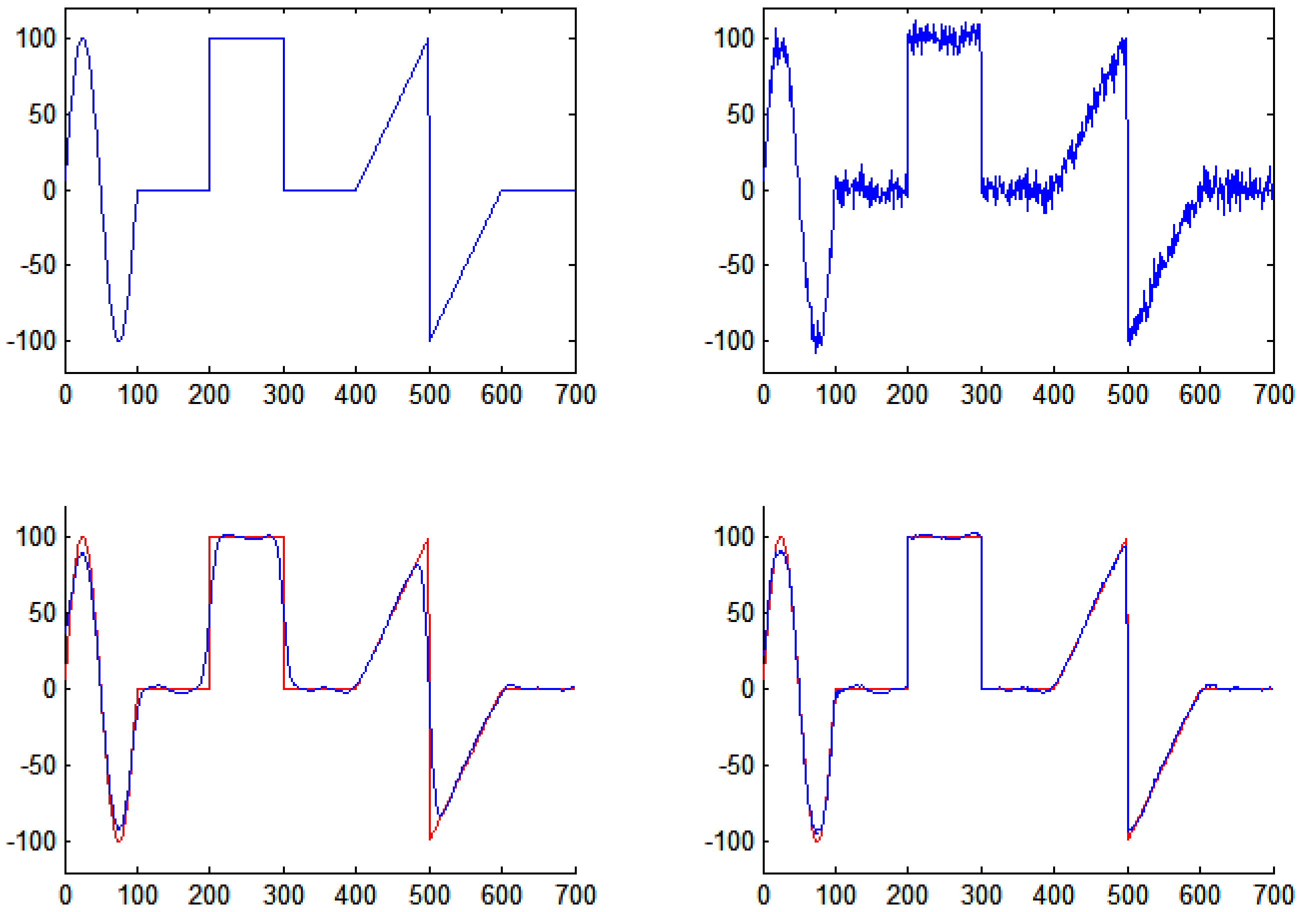
$$W_s(x) = e^{-x^2/2\sigma_s^2}$$

Bilateral Filter:
2D spatial +1D range



input | spatial kernel f | influence g in the intensity domain for the central pixel | weight $f \times g$ for the central pixel | output

Example: 1D spatial +1D range

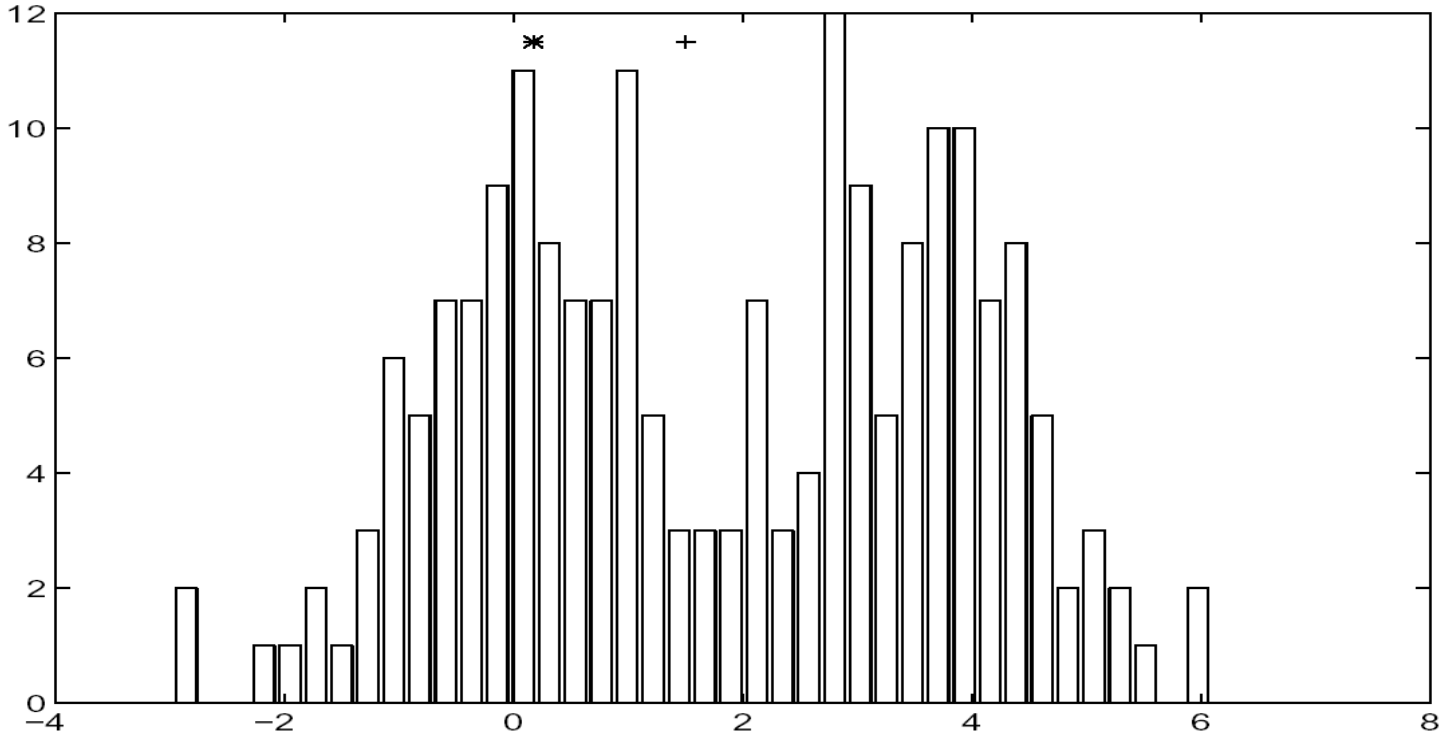


Example: 2D spatial +1D range

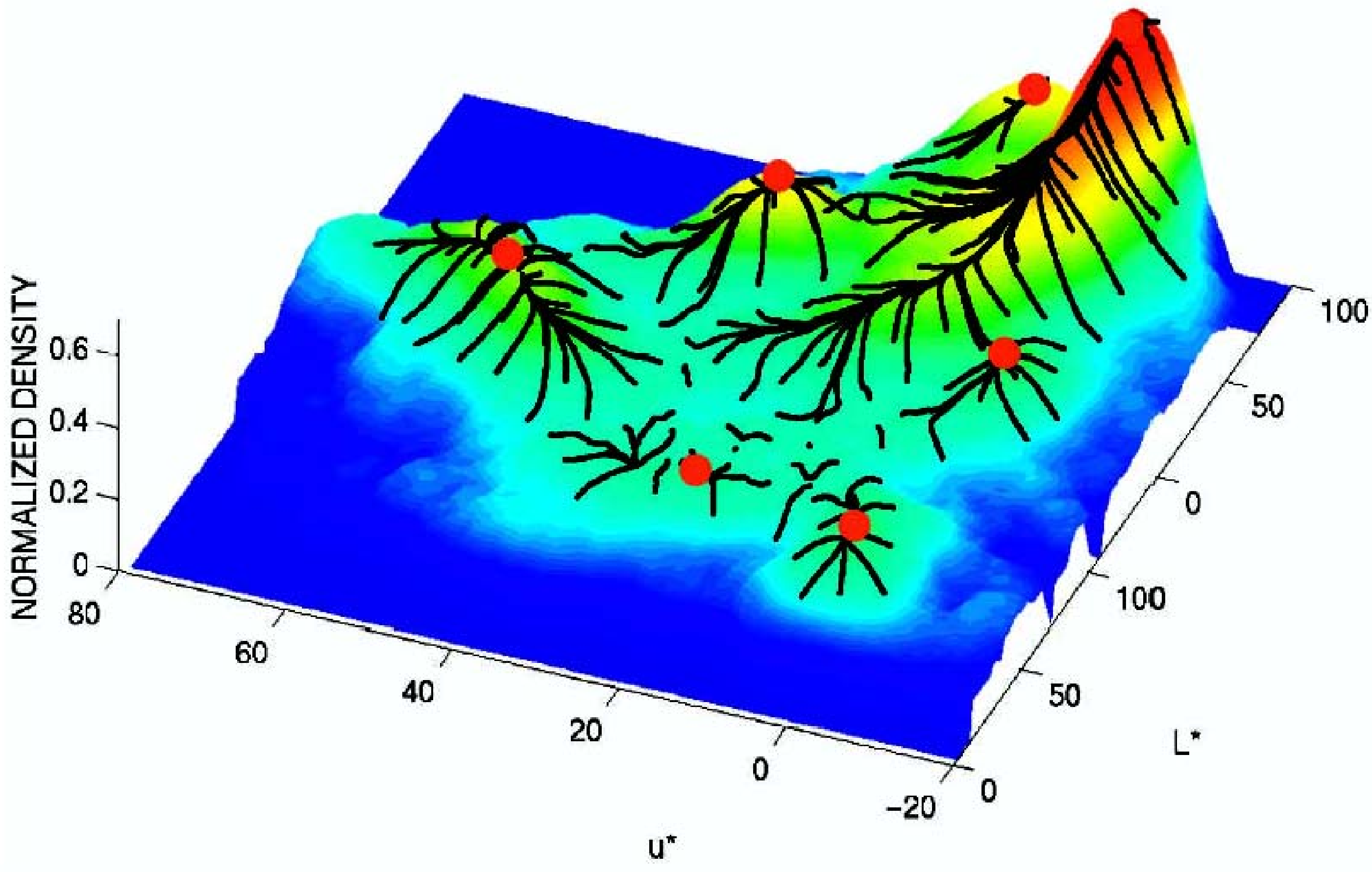
Example: 2D spatial +1D range

Early Mean-Shift (ICCV1997)

- Kernel smooth on Probability Density Function (PDF).
- The feature space only includes color information (range data).



Range Data (Multiple Colors)



Recent Mean-Shift

- x_i is a multiple dimension (2+p) vector including 2D spatial and pD range feature.
- The mean of an initial vector y_0 shift from y_0 to y_j if it is convergent.

$$y_{j+1} = \frac{\sum_{i=1}^{n} x_i g\left(\left\|\frac{y_j - x_i}{h}\right\|^2\right)}{\sum_{i=1}^{n} g\left(\left\|\frac{y_j - x_i}{h}\right\|^2\right)}$$

$$g_{h_s,h_r}(x) = \frac{C}{h_s^2 h_r^p} g\left(\left\|\frac{x^s}{h_s}\right\|^2\right) g\left(\left\|\frac{x^r}{h_r}\right\|^2\right)$$

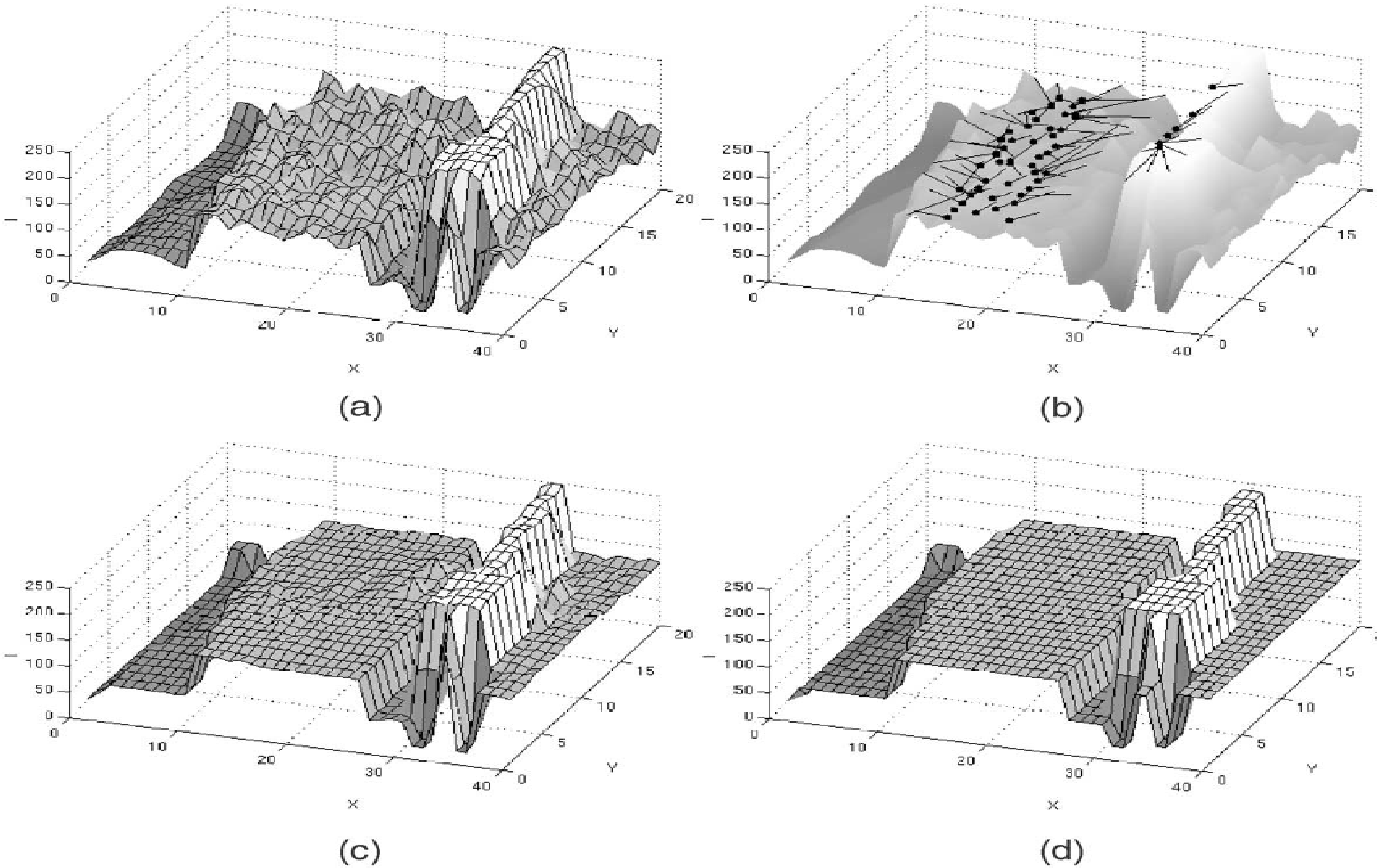
Algorithm of Mean-Shift

Let $\mathbf{x}_i$ and $\mathbf{z}_i, i = 1, \ldots, n$, be the d-dimensional input and filtered image pixels in the joint spatial-range domain. For each pixel,

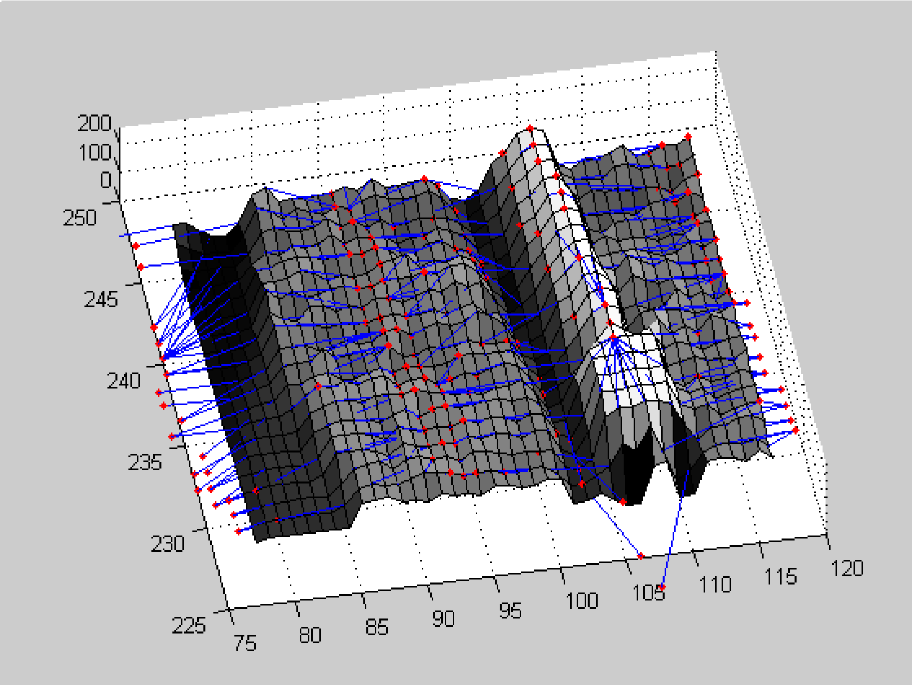
1. Initialize $j = 1$ and $\mathbf{y}_{i,1} = \mathbf{x}_i$.
2. Compute $\mathbf{y}_{i,j+1}$ according to (20) until convergence, $\mathbf{y} = \mathbf{y}_{i,c}$.
3. Assign $\mathbf{z}_i = (\mathbf{x}_i^s, \mathbf{y}_{i,c}^r)$.

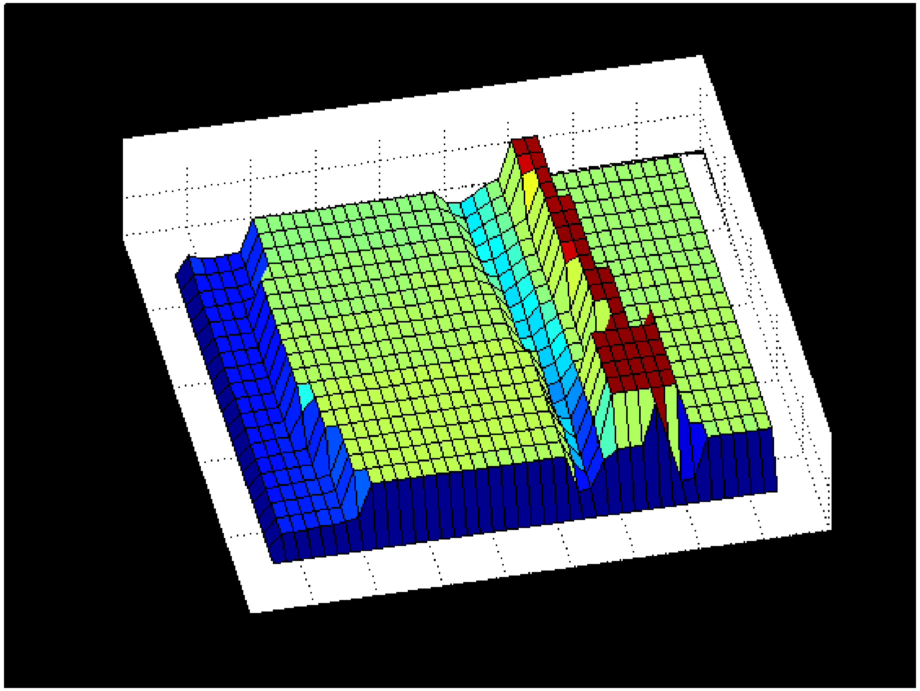
The superscripts s and r denote the spatial and range components of a vector, respectively. The assignment specifies that the filtered data at the spatial location $\mathbf{x}_i^s$ will have the range component of the point of convergence $\mathbf{y}_{i,c}^r$.

2D Spatial +1D Range (Gray)



My Results

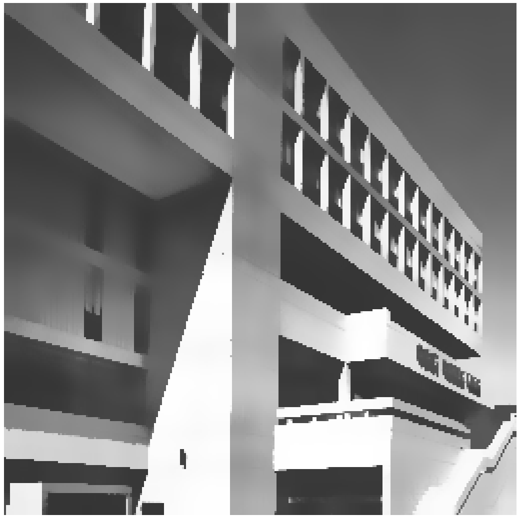

200
100
0
250
245
240
235
230
225
75
80
85
90
95
100
105
110
115
120



Comparison

My Results



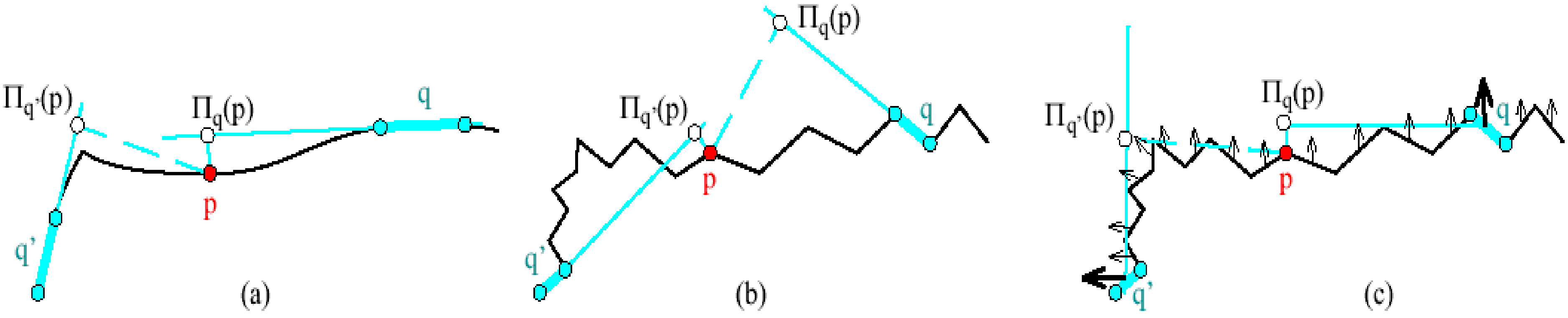



3D Application

- Using distance to tangent plane as range information.
- The normal of the tangent plane is mollified (smoothed).

$$p' = \frac{1}{k(p)} \sum_{q \in S} \Pi_q(p)\; a_q\; f(||c_q - p||)\; g(||\Pi_q(p) - p||)$$

$$k(p) = \sum_{q \in S} a_q\; f(||c_q - p||)\; g(||\Pi_q(p) - p||)$$

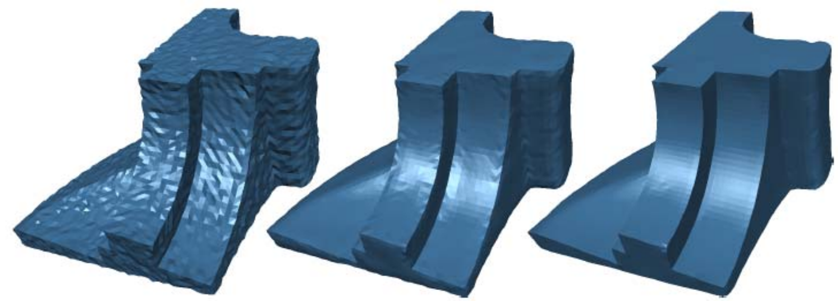


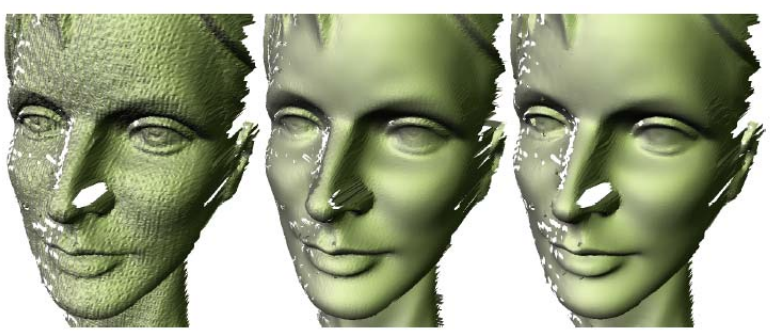
Result of Jones et al.

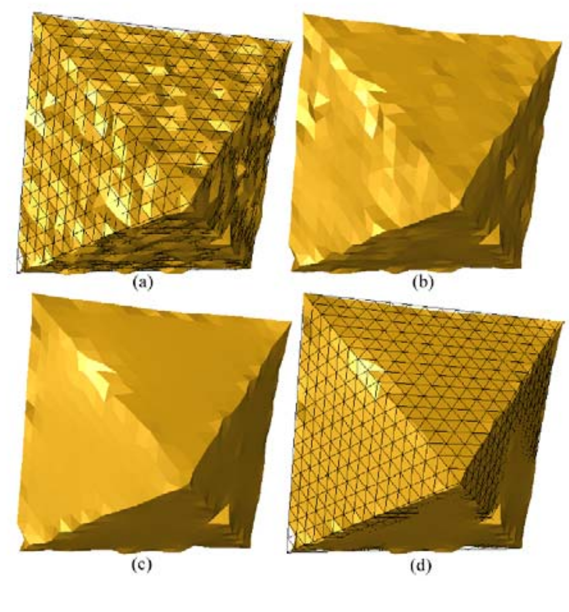
Algorithm by Fleishman et al.

DenoisePoint(Vertex $\mathbf{v}$, Normal $\mathbf{n}$)
$\{\mathbf{q_i}\} = \text{neighborhood}(\mathbf{v})$
$K = |\{q_i\}|$
$sum = 0$
$normalizer = 0$
for i := 1 **to** K
$t = ||\mathbf{v} - \mathbf{q_i}||$
$h = \langle \mathbf{n}, \mathbf{v} - \mathbf{q_i} \rangle$
$w_c = exp(-t^2/(2\sigma_c^2))$
$w_s = exp(-h^2/(2\sigma_s^2))$
$sum += (w_c \cdot w_s) \cdot h$
$normalizer += w_c \cdot w_s$
end
return Vertex $\hat{\mathbf{v}} = \mathbf{v} + \mathbf{n} \cdot (sum/normalizer)$

Results Comparison







Conclusion

- An very efficient approach for smoothing.
- May be similar to human eyes response procedure.
- Can be easily to extend to many areas.