Survey: Bilateral Filter and Applications

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Related Papers

Gaussian Filter

- Normal distribution
  \[ \Phi = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\left(\frac{t}{\sigma}\right)^2} dt \]

- Gaussian kernal filter
  \[ Y(\mu) = \int_{\mu-\tau}^{\mu+\tau} X(t) \frac{1}{\sqrt{2\pi}\sigma} e^{-\left(\frac{t-\mu}{\sigma}\right)^2} dt \]

The filter’s width is 2\( \tau \), and center is at \( \mu \), bandwidth is \( \sigma \).
Bilateral Filter

- For image $I(u)$, at coordinate $u = (x,y)$:

$$
\hat{I}(u) = \frac{\sum_{p \in N(u)} W_c(\|p - u\|) W_s(|I(u) - I(p)|) I(p)}{\sum_{p \in N(u)} W_c(\|p - u\|) W_s(|I(u) - I(p)|)}
$$

- Two Gaussian filters:

$$
W_c(x) = e^{-x^2/2\sigma_c^2}
$$

$$
W_s(x) = e^{-x^2/2\sigma_s^2}
$$
Bilateral Filter: 2D spatial + 1D range

input
spatial kernel $f$
influence $g$ in the intensity domain for the central pixel
weight $f \times g$ for the central pixel
output
Example: 1D spatial + 1D range
Example: 2D spatial + 1D range
Example: 2D spatial + 1D range
Early Mean-Shift (ICCV1997)

- Kernel smooth on Probability Density Function (PDF).
- The feature space only includes color information (range data).
Range Data (Multiple Colors)
Recent Mean-Shift

- $x_i$ is a multiple dimension $(2+p)$ vector including 2D spatial and $p$D range feature.
- The mean of an initial vector $y_0$ shift from $y_0$ to $y_j$ if it is convergent.

$$y_{j+1} = \frac{\sum_{i=1}^{n} x_i g \left( \frac{\|y_j - x_i\|^2}{h} \right)}{\sum_{i=1}^{n} g \left( \frac{\|y_j - x_i\|^2}{h} \right)}$$

$$g_{h_s, h_r} (x) = \frac{C}{h_s^2 h_r^p} g \left( \frac{x_s^2}{h_s^2} \right) g \left( \frac{x_r^2}{h_r^2} \right)$$
Algorithm of Mean-Shift

Let \( x_i \) and \( z_i, i = 1, \ldots, n, \) be the \( d \)-dimensional input and filtered image pixels in the joint spatial-range domain. For each pixel,

1. Initialize \( j = 1 \) and \( y_{i,1} = x_i. \)
2. Compute \( y_{i,j+1} \) according to (20) until convergence, \( y = y_{i,c}. \)
3. Assign \( z_i = (x_i^s, y_{i,c}^r). \)

The superscripts \( s \) and \( r \) denote the spatial and range components of a vector, respectively. The assignment specifies that the filtered data at the spatial location \( x_i^s \) will have the range component of the point of convergence \( y_{i,c}^r. \)
2D Spatial + 1D Range (Gray)
My Results
Comparison
My Results
3D Application

- Using distance to tangent plane as range information.
- The normal of the tangent plane is mollified (smoothed).

\[ p' = \frac{1}{k(p)} \sum_{q \in S} \Pi_q(p) a_q \ f(||c_q - p||) \ g(||\Pi_q(p) - p||) \]

\[ k(p) = \sum_{q \in S} a_q \ f(||c_q - p||) \ g(||\Pi_q(p) - p||) \]
Result of Jones et al.
Algorithm by Fleishman et al.

DenoisePoint(Vertex $v$, Normal $n$)

$\{q_i\} = \text{neighborhood}(v)$

$K = |\{q_i\}|$

$sum = 0$

$\text{normalizer} = 0$

for $i := 1$ to $K$

$t = ||v - q_i||$

$h = \langle n, v - q_i \rangle$

$w_c = \exp(-t^2/(2\sigma_c^2))$

$w_s = \exp(-h^2/(2\sigma_s^2))$

$sum += (w_c \cdot w_s) \cdot h$

$\text{normalizer} += w_c \cdot w_s$

end

return Vertex $\hat{v} = v + n \cdot (sum/\text{normalizer})$
Results Comparison
Conclusion

- An very efficient approach for smoothing.
- May be similar to human eyes response procedure.
- Can be easily to extend to many areas.