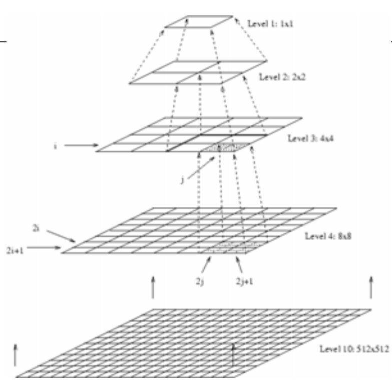
Pyrramids

Lecture-5

Pyramids

- Very useful for representing images.
- Pyramid is built by using multiple copies of image.
- Each level in the pyramid is 1/4 of the size of previous level.
- The lowest level is of the highest resolution.
- The highest level is of the lowest resolution.

Pyramid

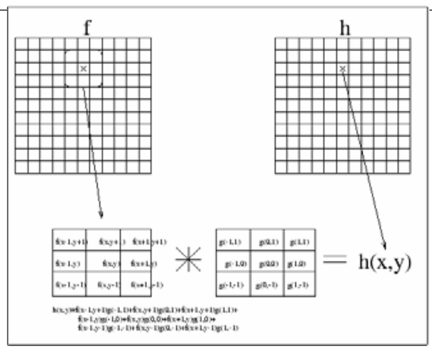


Gaussian Pyramids

$$g_l(i,j)=\sum_{m=-2}^{2}\sum_{n=-2}^{2} w(m,n)\,g_{l-1}(2i+m,2j+n)$$

$$g_l = REDUCE[g_{l-1}]$$

Convolution



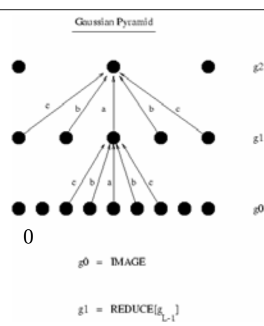
Reduce (1D)

$$g_l(i)=\sum_{m=-2}^{2}\hat{w}(m)g_{l-1}(2i+m)$$

$$g_l(2)=\hat{w}(-2)g_{l-1}(4-2)+\hat{w}(-1)g_{l-1}(4-1)+$$
$$\hat{w}(0)g_{l-1}(4)+\hat{w}(1)g_{l-1}(4+1)+\hat{w}(2)g_{l-1}(4+2)$$

$$g_l(2)=\hat{w}(-2)g_{l-1}(2)+\hat{w}(-1)g_{l-1}\hat{w}(3)+$$
$$\hat{w}(0)g_{l-1}(4)+\hat{w}(1)g_{l-1}(5)+\hat{w}(2)g_{l-1}(6)$$

Reduce



Gaussian Pyramids

$$g_{l,n}(i,j)=\sum_{p=-2}^{2}\sum_{q=-2}^{2}w(p,q)g_{l,n-1}\left(\frac{i-p}{2},\frac{j-q}{2}\right)$$

$$g_{l,n}=EXPAND[g_{l,n-1}]$$

Expand (1D)

$$g_{l,n}(i) = \sum_{p=-2}^{2} \hat{w}(p) g_{l,n-1}\left(\frac{i-p}{2}\right)$$

$$g_{l,n}(4) = \hat{w}(-2) g_{l,n-1}\left(\frac{4-2}{2}\right) + \hat{w}(-1) g_{l,n-1}\left(\frac{4-1}{2}\right) +$$

$$\hat{w}(0) g_{l,n-1}\left(\frac{4}{2}\right) + \hat{w}(1) g_{l,n-1}\left(\frac{4+1}{2}\right) + \hat{w}(2) g_{l,n-1}\left(\frac{4+2}{2}\right)$$

$$g_{l,n}(4) = \hat{w}(-2) g_{l,n-1}(1) + \hat{w}(0) g_{l,n-1}(2) + \hat{w}(2) g_{l,n-1}(3)$$

Expand (1D)

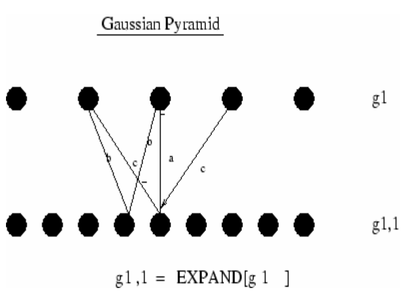
$$g_{l,n}(i) = \sum_{p=-2}^{2} \hat{w}(p) g_{l,n-1}\left(\frac{i-p}{2}\right)$$

$$g_{l,n}(3) = \hat{w}(-2) g_{l,n-1}\left(\frac{3-2}{2}\right) + \hat{w}(-1) g_{l,n-1}\left(\frac{3-1}{2}\right) +$$

$$\hat{w}(0) g_{l,n-1}\left(\frac{3}{2}\right) + \hat{w}(1) g_{l,n-1}\left(\frac{3+1}{1}\right) + \hat{w}(2) g_{l,n-1}\left(\frac{3+2}{2}\right)$$

$$g_{l,n}(3) = \hat{w}(-1) g_{l,n-1}(1) + \hat{w}(1) g_{l,n-1}(2)$$

Expand



Convolution Mask

$$[w(-2), w(-1), w(0), w(1), w(2)]$$

Convolution Mask

- Separable

$$w(m,n) = \hat{w}(m)\hat{w}(n)$$

•Symmetric

$$\hat{w}(i) = \hat{w}(-i)$$

$$[c, b, a, b, c]$$

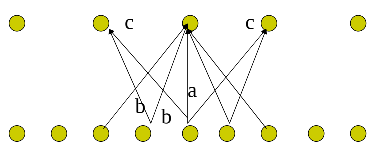
Convolution Mask

- The sum of mask should be 1.

$$a + 2b + 2c = 1$$

•All nodes at a given level must contribute the same total weight to the nodes at the next higher level.

$$a + 2c = 2b$$



Convolution Mask

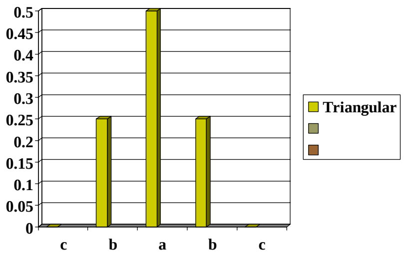
$$\hat{w}(0) = a$$

$$\hat{w}(-1) = \hat{w}(1) = \frac{1}{4}$$

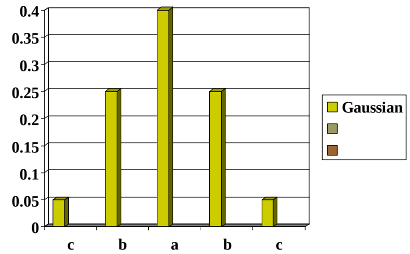
$$\hat{w}(-2) = \hat{w}(2) = \frac{1}{4} - \frac{a}{2}$$

a=.4 GAUSSIAN, a=.5 TRINGULAR

Triangular



Approximate Gaussian



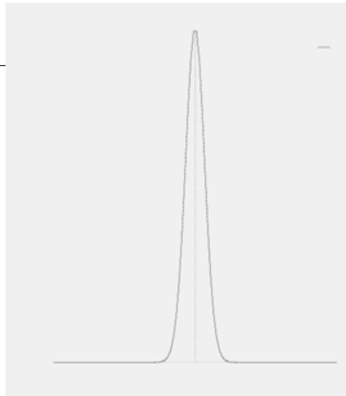
Gaussian

$$g(x) = e^{\frac{-x^2}{2\sigma^2}}$$

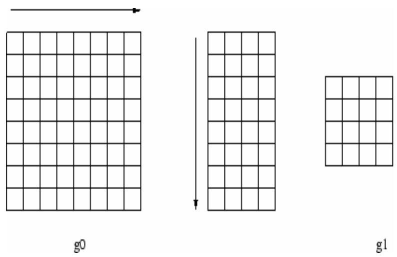
Gaussian

$$g(x) = e^{\frac{-x^2}{2\sigma^2}}$$

x	-3	-2	-1	0	1	2	3
g(x)	.011	.13	.6	1	.6	.13	.011



Separability



Algorithm

- Apply 1-D mask to alternate pixels along each row of image.
- Apply 1-D mask to each pixel along alternate columns of resultant image from previous step.

Gaussian Pyramid

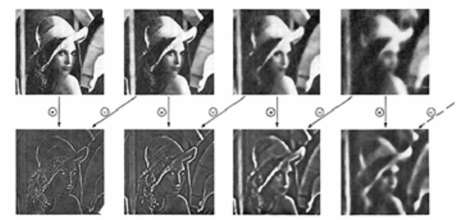
Laplacian Pyramids

- Similar to edge detected images.
- Most pixels are zero.
- Can be used for image compression.

$$L_1 = g_1 - EXPAND[g_2]$$

$$L_2 = g_2 - EXPAND[g_3]$$

$$L_3 = g_3 - EXPAND[g_4]$$



Coding using Laplacian Pyramid

•Compute Gaussian pyramid

$$g_1, g_2, g_3, g_4$$

•Compute Laplacian pyramid

$$L_1 = g_1 - EXPAND[g_2]$$
$$L_2 = g_2 - EXPAND[g_3]$$
$$L_3 = g_3 - EXPAND[g_4]$$
$$L_4 = g_4$$

•Code Laplacian pyramid

Decoding using Laplacian pyramid

- Decode Laplacian pyramid.
- Compute Gaussian pyramid from Laplacian pyramid.

$$g_4 = L_4$$
$$g_3 = EXPAND[g_4] + L_3$$
$$g_2 = EXPAND[g_3] + L_2$$
$$g_1 = EXPAND[g_2] + L_1$$

- is reconstructed image.

Gaussian

- Most natural phenomenon can be modeled by Gaussian.
- Take a bunch of random variables of any distribution, find the mean, the mean will approach to Gaussian distribution.
- Gaussian is very smooth function, it has infinite no of derivatives.

Gaussian

- Fourier Transform of Gaussian is Gaussian.
- If you convolve Gaussian with itself, it is again Gaussian.
- There are cells in human brain which perform Gaussian filtering.
  - Laplacian of Gaussian edge detector

Carl F. Gauss

- Born to a peasant family in a small town in Germany.
- Learned counting before he could talk.
- Contributed to Physics, Mathematics, Astronomy,…
- Discovered most methods in modern mathematics, when he was a teenager.

Carl F. Gauss

- Some contributions
  - Gaussian elimination for solving linear systems
  - Gauss-Seidel method for solving sparse systems
  - Gaussian curvature
  - Gaussian quadrature

Laplacian Pyramid

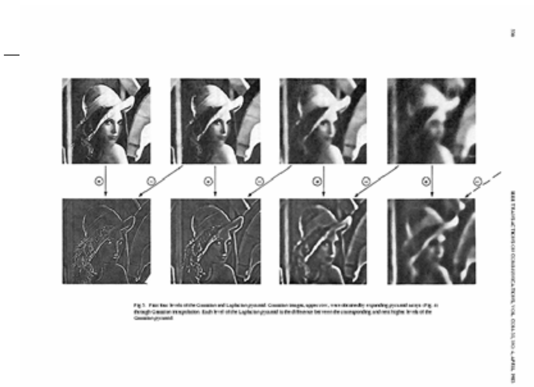
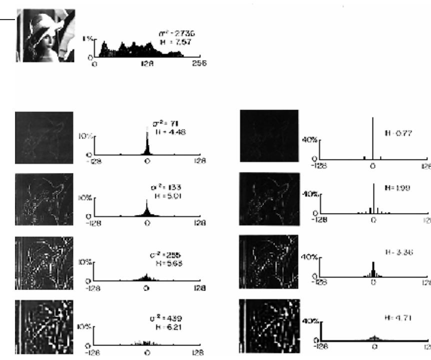


Image Compression (Entropy)



Huffman Coding (Example-1)

A_1 — P=.5 — 0

A_2 — P=.25 — 0 / 1

A_3 — P=.125 — 0 / 1

A_4 — P=.125 — 1

A_1 0
A_2 10
A_3 110
A_4 111

Huffman Coding

Entropy $H = -\sum_{i=0}^{255} p(i)\log_2 p(i)$

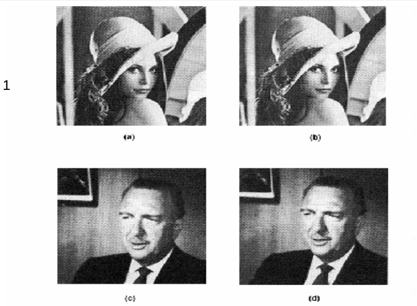
$$H = -.5\log .5 - .25\log .25 - .125\log .125 - .125\log .125 = 1.75$$

Image Compression

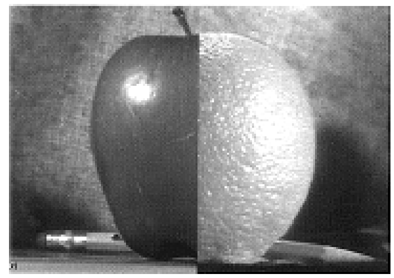
1.58

1

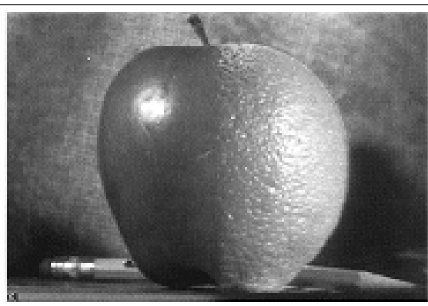
.73



Combining Apple & Orange



Combining Apple & Orange



Algorithm

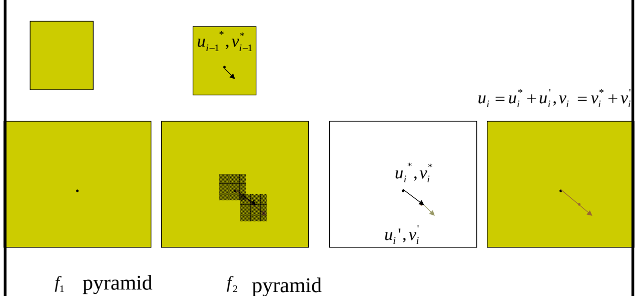
- Generate Laplacian pyramid Lo of orange image.
- Generate Laplacian pyramid La of apple image.
- Generate Laplacian pyramid Lc by copying left half of nodes at each level from apple and right half of nodes from orange pyramids.
- Reconstruct combined image from Lc.

- http://ww-bcs.mit.edu/people/adelson/papers.html
  - The Laplacian Pyramid as a compact code, Burt and Adelson, IEEE Trans on Communication, 1983.

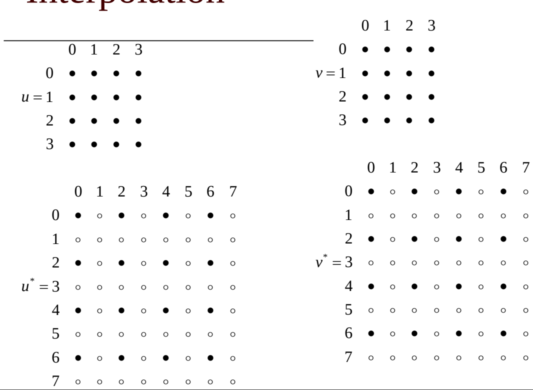
Lucas Kanade with Pyramids

- Compute 'simple' LK at highest level
- At level i
  - Take flow u_{i-1}, v_{i-1} from level i-1
  - bilinear interpolate it to create u_i^*, v_i^* matrices of twice resolution for level i
  - multiply u_i^*, v_i^* by 2
  - compute f_t from a block displaced by $u_i^*(x,y)$, $v_i^*(x,y)$
  - Apply LK to get $u_i'(x, y)$, $v_i'(x, y)$ (the correction in flow)
  - Add corrections u_i' v_i', *i.e.* $u_i = u_i^* + u_i'$, $v_i = v_i^* + v_i'$.

Pyramids



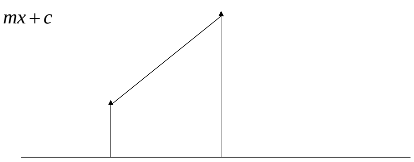
Interpolation



1-D Interpolation

$y = mx + c$

$f(x) = mx + c$



$f(1)$ $f(2)$

2-D Interpolation

$f(x,y) = a_1 + a_2x + a_3y + a_4xy$ Bilinear

X X
O X

Bi-linear Interpolation

Four nearest points of (x,y) are:

$$(\underline{x}, \underline{y}), (\overline{x}, \underline{y}), (\underline{x}, \overline{y}), (\overline{x}, \overline{y})$$

$$(3,5), (4,5), (3,6), (4,6)$$

$\underline{x} = \text{int}(x)$ 3 (3.2,5.6)

$\underline{y} = \text{int}(y)$ 5

$\overline{x} = \underline{x} + 1$ 4

$\overline{y} = \underline{y} + 1$ 6

X (3,6) X (4,6)
X(3,5) X (4,5)

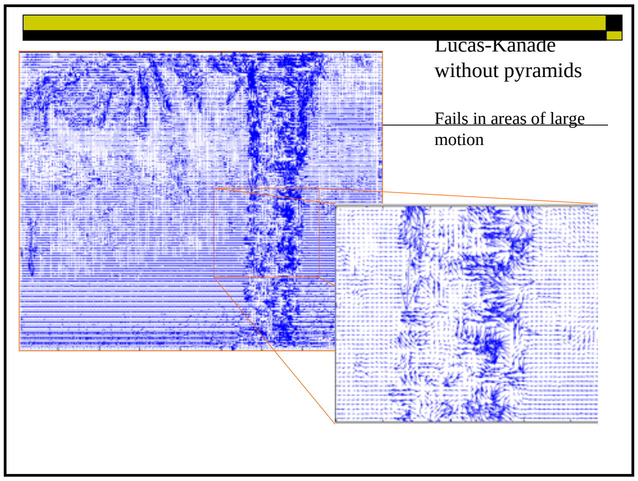
$$f'(x,y) = \overline{\varepsilon_x}\,\overline{\varepsilon_y} f(\underline{x}, \underline{y}) + \underline{\varepsilon_x}\,\overline{\varepsilon_y} f(\overline{x}, \underline{y}) + \overline{\varepsilon_x}\,\underline{\varepsilon_y} f(\underline{x}, \overline{y}) + \underline{\varepsilon_x}\,\underline{\varepsilon_y} f(\overline{x}, \overline{y})$$

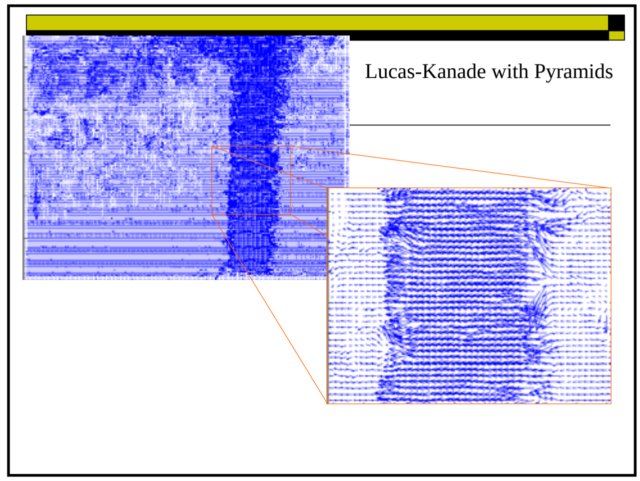
$\overline{\varepsilon_x} = \overline{x} - x$ $\overline{\varepsilon_x} = \overline{x} - x = 4 - 3.2 = .8$

$\overline{\varepsilon_y} = \overline{y} - y$ $\overline{\varepsilon_y} = \overline{y} - y = 6 - 5.6 = .4$

$\underline{\varepsilon_x} = x - \underline{x}$ $\underline{\varepsilon_x} = x - \underline{x} = 3.2 - 2 = .2$

$\underline{\varepsilon_y} = y - \underline{y}$ $\underline{\varepsilon_y} = y - \underline{y} = 5.6 - 5 = .6$


Lucas-Kanade without pyramids
Fails in areas of large motion


Lucas-Kanade with Pyramids