Poisson Image Editing

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How to write a paper for SIGGRAPH

- The applications are plentiful and the result is amazing.
- Idea could be simple but it really works and it is robust.
- The method is explained in good detail without too much fancy formulas.

Definitions

- Poisson's Equation: $\Delta F = 4\pi\rho$
- Laplacian's Equation: $\Delta F = 0$
- Dirichlet boundary condition: specify the value of the function on a surface.

Notations used in paper and this presentation

f : unknown scalar image.

f^* : known scalar image.

g : known scalar image of an object which will be cloned.

Ω : the selected area.

$\partial\Omega$: the boundary of the area.

Contribution of this paper

- Based on membrane interpolation

$$\min_f \iint_\Omega |\nabla f|^2 \text{ with } f|_{\partial\Omega} = f^*|_{\partial\Omega}$$

$$\Delta f = 0 \text{ over } \Omega, \text{ with } f|_{\partial\Omega} = f^*|_{\partial\Omega}$$

- Extend the above minimization with a vector field v

$$\min_f \iint_\Omega |\nabla f - v|^2 \text{ with } f|_{\partial\Omega} = f^*|_{\partial\Omega}$$

- Solution is a Poisson Equation with a boundary condition

$$\Delta f = div(v) \text{ over } \Omega, \text{ with } f|_{\partial\Omega} = f^*|_{\partial\Omega}$$

Implementation

- Np is the set of 4-connected neighbors which are in the image, |Np| is the Count Number. __4
- Discrete minimization function

$$\min_{f|\Omega} \sum_{<p,q>\cap\Omega\neq 0} (f_p - f_q - v_{pq})^2, \text{ with } f_p = f^*_p, \text{ for all } p \in \Omega$$

$$\text{for all } p \in \Omega, |N_p| f_p - \sum_{q \in N_p \cap \Omega} f_q = \sum_{q \in N_p \cap \Omega} f^*_q + \sum_{q \in N_p} v_{pq}$$

Implementation

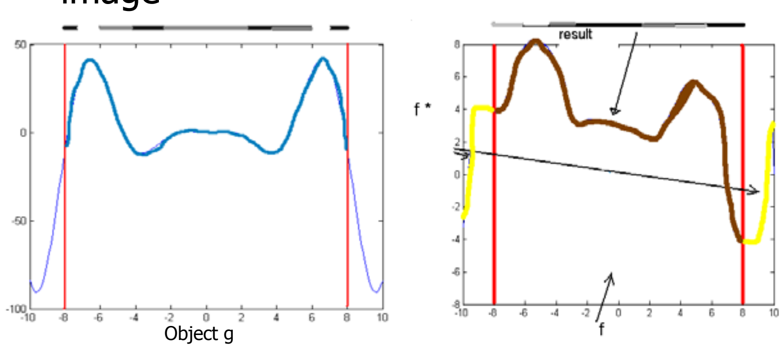
- Gauss-Seidel iteration

$$x_i^{(k)} = \frac{b_i - \sum_{j<i} a_{ij} x_j^{(k)} - \sum_{j>i} a_{ij} x_j^{(k-1)}}{a_{ii}}.$$

Pseudocode:
Choose an initial guess $x^{(0)}$ to the solution x.
for $k = 1, 2, \ldots$
  for $i = 1, 2, \ldots, n$
    $\sigma = 0$
    for $j = 1, 2, \ldots, i-1$
      $\sigma = \sigma + a_{ij} x_j^{(k)}$
    end
    for $j = i+1, \ldots, n$
      $\sigma = \sigma + a_{ij} x_j^{(k-1)}$
    end
    $x_i^{(k)} = (b_i - \sigma)/a_{ii}$
  end
  check convergence; continue if necessary.
end

Explanation of the fomulas

- Assume we work on one dimensional image



Applications

- Seamless cloning for opaque objects

$v = \nabla g$

$\Delta f = \Delta g \text{ over } \Omega, \text{with } f|_{\partial\Omega} = f^*|_{\partial\Omega}$

  - The texture is maintained, but not necessarily for the color.

Applications

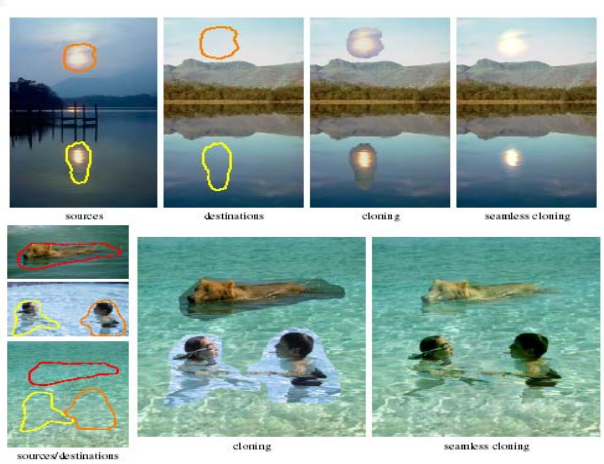
Applications

- Seamless cloning for objects with transparent parts or holes

$$\text{for all } \mathrm{x} \in \Omega,\ \mathrm{v}(\mathrm{x}) = \begin{cases} \nabla f^*(x) & \text{if } |\nabla f^*(x)| > |\nabla g(x)| \\ \nabla g(x) & \text{otherwise} \end{cases}$$

- Mixing gradient on different location can not really handle the transparency. Since a pixel with transparent foreground and opaque background should be a combinational value.

Applications



Applications

- Selection Editing
  - Texture flattening

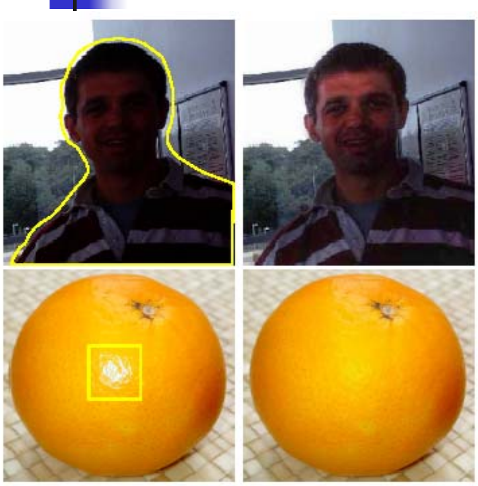
    The vector field has value only on edge pixel. Thus the detailed texture information is removed and better segmentation can be achieved.

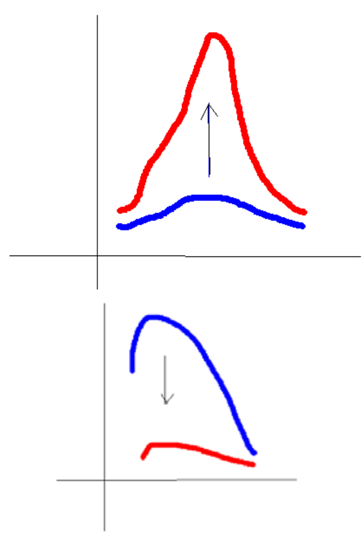
Applications

Applications

- Selection Editing
  - Local illumination changes
    - The gradient field is enlarged or reduced by logarithm. But the texture itself will not be changed.

Applications





Problems ? !

- Color intensity of the cloned object may be changed by the boundary color value.
- Check this poor 'pink' seagull out.

Problems

Problems

- Independency on color channels?
  - The author claims it is true, but we need a formal prove.
  - Recall something we discussed in last paper

Extended Usage

- Inpainting
  - Set the vector field to zeros. Then apply the method.
  - Can’t handle textured area.

Extended Usage

Extended Usage

- Shadow Removal
  - The problem is how to handle the edge of the shadow gradient field.