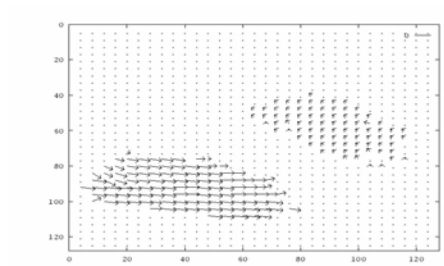
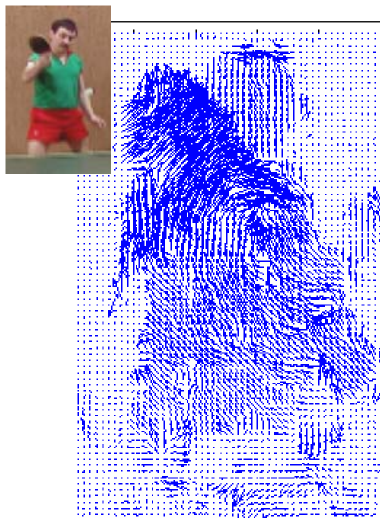
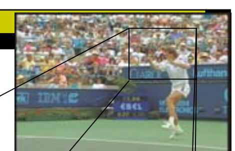
Lecture-4

Computing Optical Flow

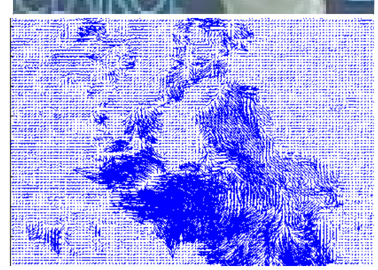
Hamburg Taxi seq

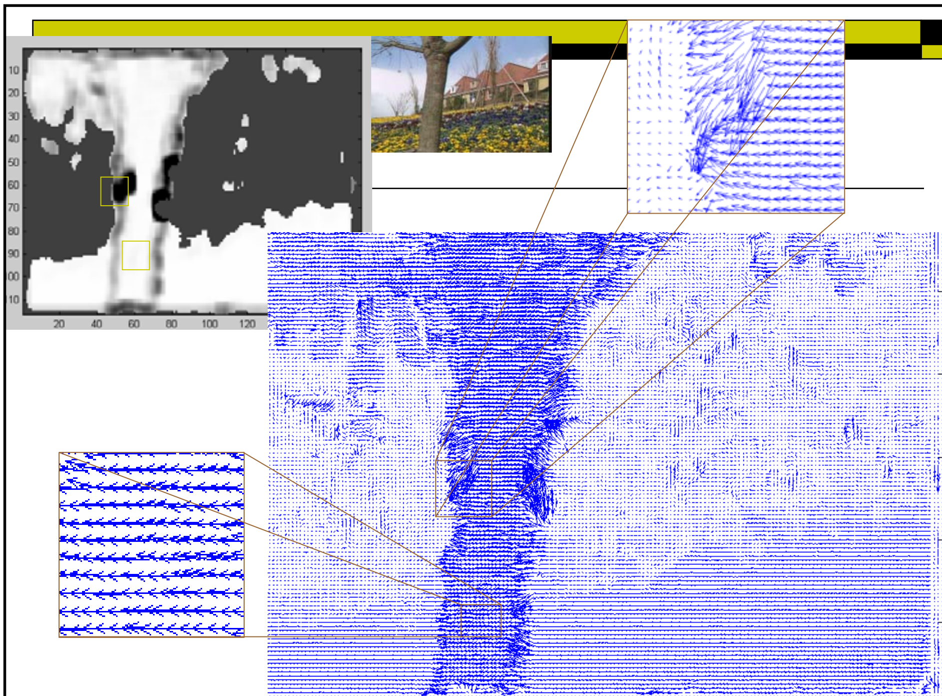







CLAIROL





Horn&Schunck Optical Flow

$f(x, y, t)$ Image Sequence

$$\frac{df(x,y,t)}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial t} = 0$$

$f_x u + f_y v + f_t = 0$ brightness constancy eq

Horn&Schunck Optical Flow

$$f(x,y,t)=f(x+dx,y+dy,t+dt)$$

Taylor Series

$$f(x,y,t)=f(x,y,t)+\frac{\partial f}{\partial x}dx+\frac{\partial f}{\partial y}dy+\frac{\partial f}{\partial t}dt$$

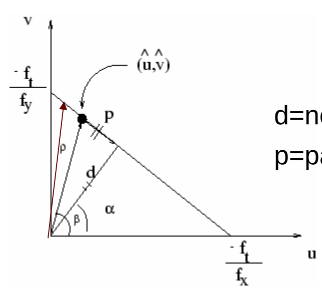
$$f_x dx+f_y dy+f_t dt=0$$

$$\boldsymbol{f_x u+f_y v+f_t=0}$$ **brightness constancy eq**

Interpretation of optical flow eq

$$f_x u+f_y v+f_t=0$$

$$v=-\frac{f_x}{f_y}u-\frac{f_t}{f_y}$$



d=normal flow

p=parallel flow

$$d=\frac{f_t}{\sqrt{f_x^2+f_y^2}}$$

Equation of st.line

Horn&Schunck (contd)

$$\iint \{(f_x u + f_y v + f_t)^2 + \lambda(u_x^2 + u_y^2 + v_x^2 + v_y^2)\}dxdy$$

min

variational calculus

$$(f_x u + f_y v + f_t) f_x + \lambda(\Delta^2 u) = 0$$

$$(f_x u + f_y v + f_t) f_y + \lambda((\Delta^2 v) = 0$$

$$u = u_{av} - f_x \frac{P}{D}$$

$$v = v_{av} - f_y \frac{P}{D}$$

discrete version

$$P = f_x u_{av} + f_y v_{av} + f_t$$

$$D = \lambda + f_x^2 + f_y^2$$

$$(f_x u + f_y v + f_t) f_x + \lambda(u - u_{av}) = 0$$

$$(f_x u + f_y v + f_t) f_y + \lambda((v - v_{av}) = 0$$

$$\Delta^2 u = u_{xx} + u_{yy}$$

Algorithm-1

- k=0
- Initialize $u^K \quad v^K$

- Repeat until some error measure is satisfied (converges)

$$u = u_{av} - f_x \frac{P}{D}$$

$$v = v_{av} - f_y \frac{P}{D}$$

$$P = f_x u_{av} + f_y v_{av} + f_t$$

$$D = \lambda + f_x^2 + f_y^2$$

Derivatives

- Derivative: Rate of change of some quantity
  - Speed is a rate of change of a distance
  - Acceleration is a rate of change of speed

Derivative

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x) - f(x - \Delta x)}{\Delta x} = f'(x) = f_x$$

$$v = \frac{ds}{dt}$$ speed

$$a = \frac{dv}{dt}$$ acceleration

Examples

$$y = x^2 + x^4$$

$$\frac{dy}{dx} = 2x + 4x^3$$

$$y = \sin x + e^{-x}$$

$$\frac{dy}{dx} = \cos x + (-1)e^{-x}$$

Second Derivative

$$\frac{df_x}{dx} = f''(x) = f_{xx}$$

$$y = x^2 + x^4$$

$$\frac{dy}{dx} = 2x + 4x^3$$

$$\frac{d^2y}{dx^2} = 2 + 12x^2$$

Discrete Derivative

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x) - f(x - \Delta x)}{\Delta x} = f'(x)$$

$$\frac{df}{dx} = \frac{f(x) - f(x-1)}{1} = f'(x)$$

$$\frac{df}{dx} = f(x) - f(x-1) = f'(x)$$

(Finite Difference)

Discrete Derivative

$$\frac{df}{dx} = f(x) - f(x-1) = f'(x)$$ Left difference

$$\frac{df}{dx} = f(x) - f(x+1) = f'(x)$$ Right difference

$$\frac{df}{dx} = f(x+1) - f(x-1) = f'(x)$$ Center difference

Example

	F(x)=10	10	10	10	20	20	20
Left difference	F'(x)=0	0	0	0	10	0	0
	F''(x)=0	0	0	0	10	-10	0

-1	1		left difference
1	-1		right difference
-1	0	1	center difference

Derivatives in Two Dimensions

$f(x,y)$

(partial Derivatives)

$$\frac{\partial f}{\partial x} = f_x = \lim_{\Delta x \to 0} \frac{f(x,y) - f(x - \Delta x, y)}{\Delta x}$$

$$\frac{\partial f}{\partial y} = f_y = \lim_{\Delta y \to 0} \frac{f(x,y) - f(x, y - \Delta y)}{\Delta y}$$

(f_x, f_y) Gradient Vector

$$\text{magnitude} = \sqrt{(f_x^2 + f_y^2)}$$

$$\text{direction} = \theta = \tan^{-1} \frac{f_y}{f_x}$$

$$\Delta^2 f = f_{xx} + f_{yy} = \text{Laplacian}$$

Derivatives of an Image

Derivative & average

$$f_x = \begin{matrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{matrix} \qquad f_y = \begin{matrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{matrix}$$

Prewit

$$I(x,y) = \begin{bmatrix} 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \end{bmatrix} \qquad I_x = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 30 & 30 & 0 & 0 \\ 0 & 30 & 30 & 0 & 0 \\ 0 & 30 & 30 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Derivatives of an Image

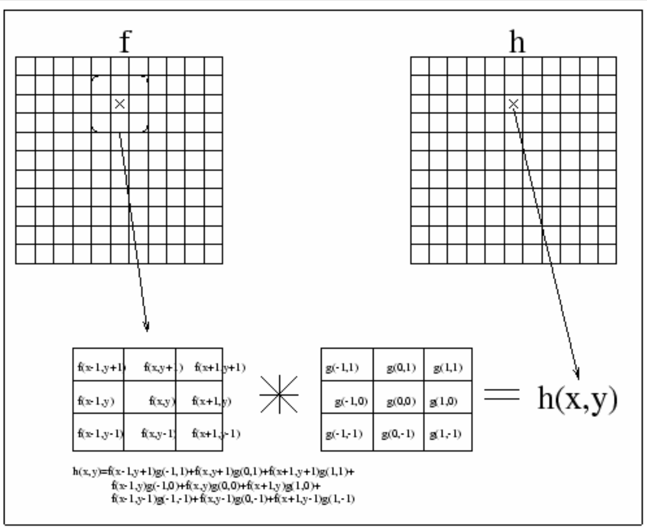
$$I(x,y) = \begin{bmatrix} 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \end{bmatrix} \qquad I_y = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Laplacian

$$\begin{matrix} 0 & -\frac{1}{4} & 0 \\ -\frac{1}{4} & 1 & -\frac{1}{4} \\ 0 & -\frac{1}{4} & 0 \end{matrix}$$

$$f_{xx} + f_{yy}$$

Convolution



Convolution (contd)

$$h(x,y)=\sum_{i=-1}^{1}\sum_{j=-1}^{1} f(x+i,y+j)g(i,j)$$

$$h(x,y)=f(x,y)*g(x,y)$$

$$\begin{aligned}h(x,y)=\ & f(x-1,y-1)g(-1,-1)+f(x,y-1)g(0,-1)+f(x+1,y-1)g(1,-1)\\ &+f(x-1,y)g(-1,0)+f(x,y)g(0,0)+f(x+1,y)g(1,0)\\ &+f(x-1,y+1)g(-1,1)+f(x,y+1)g(0,1)+f(x+1,y+1)g(1,1)\end{aligned}$$

Derivative Masks

$\begin{bmatrix}-1 & 1\\ -1 & 1\end{bmatrix}$ first image

$\begin{bmatrix}-1 & 1\\ -1 & 1\end{bmatrix}$ second image

f_x

$\begin{bmatrix}-1 & -1\\ 1 & 1\end{bmatrix}$ first image

$\begin{bmatrix}-1 & -1\\ 1 & 1\end{bmatrix}$ second image

f_y

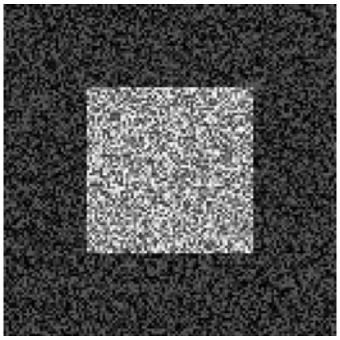
$\begin{bmatrix}-1 & -1\\ -1 & -1\end{bmatrix}$ first image

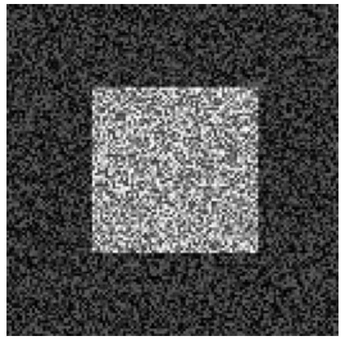
$\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$ second image

f_t

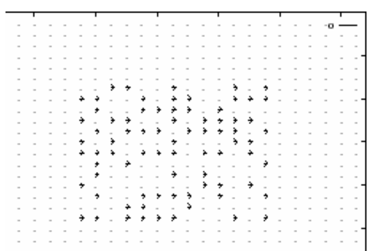
Apply first mask to 1st image
Second mask to 2nd image
Add the responses to get *f_x*, *f_y*, *f_t*

Synthetic Images

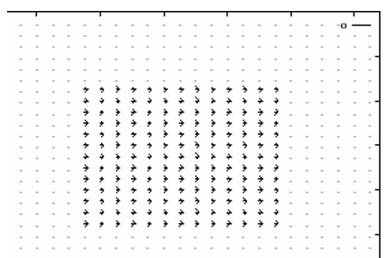




Horn & Schunck Results



One iteration



10 iterations

$$\lambda = 4$$

Lucas & Kanade (Least Squares)

- Optical flow eq

$$f_x u + f_y v = -f_t$$

  - Consider 3 by 3 window

$$f_{x1} u + f_{y1} v = -f_{t1}$$

$$\vdots$$

$$f_{x9} u + f_{y9} v = -f_{t9}$$

$$\begin{bmatrix} f_{x1} & f_{y1} \\ \vdots & \vdots \\ \\ f_{x9} & f_{y9} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} -f_{t1} \\ \vdots \\ \\ -f_{t9} \end{bmatrix}$$

$$\mathbf{Au} = \mathbf{f_t}$$

Lucas & Kanade

$$\mathbf{Au} = \mathbf{f_t}$$

$$\mathbf{A^T A u} = \mathbf{A^T f_t}$$

$$\mathbf{u} = (\mathbf{A^T A})^{-1} \mathbf{A^T f_t}$$

$$\min \sum (f_{xi} u + f_{yi} v + f_t)^2$$

Lucas & Kanade

$$\min \sum (f_{xi}u + f_{yi}v + f_t)^2$$

$$\Downarrow$$

$$\sum (f_{xi}u + f_{yi}v + f_{ti}) f_{xi} = 0$$

$$\sum (f_{xi}u + f_{yi}v + f_{ti}) f_{yi} = 0$$

$$\sum (f_{xi}u + f_{yi}v + f_{ti}) f_{xi} = 0$$

$$\sum (f_{xi}u + f_{yi}v + f_{ti}) f_{yi} = 0$$

$$\sum f_{xi}^2 u + \sum f_{xi} f_{yi} v = -\sum f_{xi} f_{ti}$$

$$\sum f_{xi} f_{yi} u + \sum f_{yi}^2 v = -\sum f_{yi} f_{ti}$$

$$\begin{bmatrix} \sum f_{xi}^2 & \sum f_{xi} f_{yi} \\ \sum f_{xi} f_{yi} & \sum f_{yi}^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} -\sum f_{xi} f_{ti} \\ -\sum f_{yi} f_{ti} \end{bmatrix}$$

Lucas & Kanade

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \sum f_{xi}^2 & \sum f_{xi} f_{yi} \\ \sum f_{xi} f_{yi} & \sum f_{yi}^2 \end{bmatrix}^{-1} \begin{bmatrix} -\sum f_{xi} f_{ti} \\ -\sum f_{yi} f_{ti} \end{bmatrix}$$

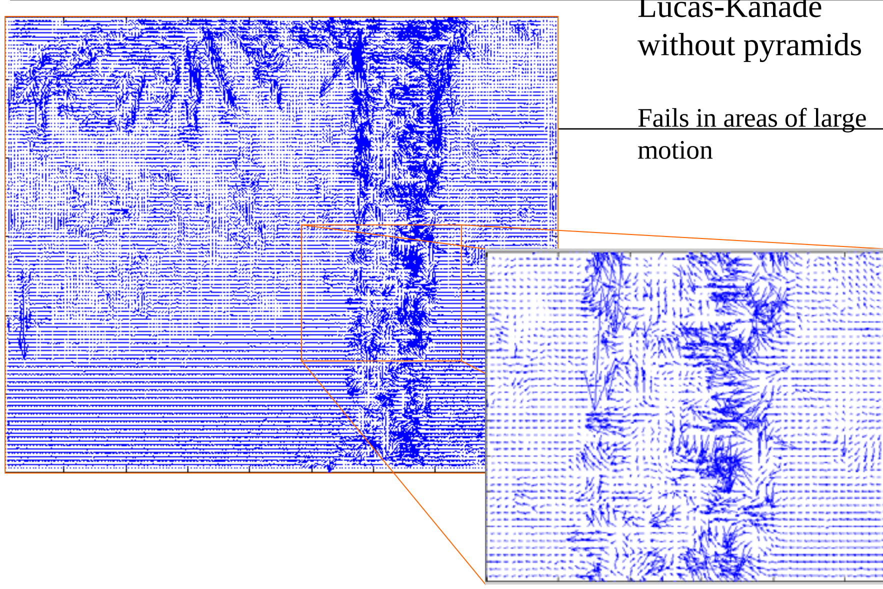
$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{1}{\sum f_{xi}^2 \sum f_{yi}^2 - (\sum f_{xi} f_{yi})^2} \begin{bmatrix} \sum f_{yi}^2 & -\sum f_{xi} f_{yi} \\ -\sum f_{xi} f_{yi} & \sum f_{xi}^2 \end{bmatrix} \begin{bmatrix} -\sum f_{xi} f_{ti} \\ -\sum f_{yi} f_{ti} \end{bmatrix}$$

$$u = \frac{-\sum f_{yi}^2 \sum f_{xi} f_{ti} + \sum f_{xi} f_{yi} \sum f_{yi} f_{ti}}{\sum f_{xi}^2 \sum f_{yi}^2 - (\sum f_{xi} f_{yi})^2}$$

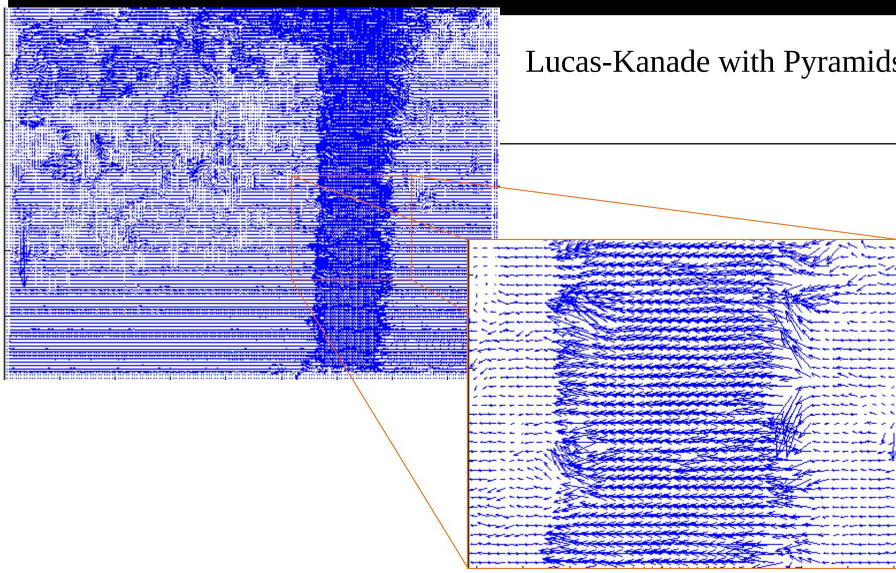
$$v = \frac{\sum f_{xi} f_{ti} \sum f_{xi} f_{yi} - \sum f_{xi}^2 \sum f_{yi} f_{ti}}{\sum f_{xi}^2 \sum f_{yi}^2 - (\sum f_{xi} f_{yi})^2}$$

Lucas-Kanade without pyramids

Fails in areas of large motion



Lucas-Kanade with Pyramids



Comments

- Horn-Schunck and Lucas-Kanade optical method works only for small motion.
- If object moves faster, the brightness changes rapidly, 2x2 or 3x3 masks fail to estimate spatiotemporal derivatives.
- Pyramids can be used to compute large optical flow vectors.