

Homework Due 1/16/01 • 2.1, 2.2, 2.3, 2.8, 2.13, 2.14

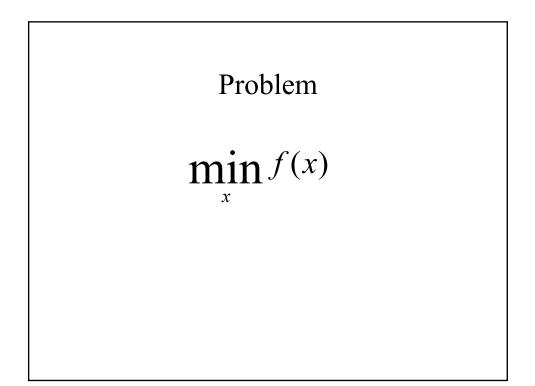
Rate of Convergence

Definition : Suppose $\{p_n\}_{n=0}^{\infty}$ is a sequence that converges to p and that $e_n = p_n - p$

$$\lim_{n \to \infty} \frac{|p_{n+1} - p|}{|p_n - p|^{\alpha}} = \lim_{n \to \infty} \frac{|e_{n+1}|}{|e_n|^{\alpha}} = \lambda$$

then the seq is said to converge to p of order α with asymptotic error constant λ .

 $\alpha = 1$, linear $\alpha = 2$, quadratic $\alpha = 1$, and $\lambda = 0$, superlinear



Definitions

A point x^* is a stationary point if $f'(x^*) = 0$ A point x^* is a global minimizer if $f(x^*) \le f(x) \ \forall x$ A point x^* is a local minimizer if there is a neighborhood N s.t. $f(x^*) \le f(x) \ \forall x \in N$ A point x^* is a strict local minimizer if there is a neighborhood N s.t. $f(x^*) < f(x) \ \forall x \in N, x \neq x^*$ if $\nabla f(x^*) = 0$, but x^* is neither a minimum nor a maxima, it is called a saddle point.

First Order necessary conditions If x^* is a local minimizer and fis continuously differentiable in an open neighborhood of x^* , then $\nabla f(x^*) = 0$.

Second order necessary conditions

If x^* is a local minimizer and $\nabla^2 f$ is continuous in an open neighborhood of x^* , then $\nabla f(x^*) = 0$ and $\nabla^2 f(x^*)$ is positive semidefinite.

Second order sufficient conditions

Suppose that $\nabla^2 f$ is continuous in an open neighborhood of x^* and that $\nabla f(x^*) = 0$ and $\nabla f(x^*)$ is positive definite. Then x^* is a strict local minimizer of f.

Convex Function

f is a convex function if for any two points *x* and *y* in its domain, the graph of *f* lies below straight line connecting(x, f(x)) to (y, f(y))

 $f(\alpha x + (1 - \alpha)y) \le \alpha f(x) + (1 - \alpha)f(y) \quad \forall \alpha \in [0, 1]$

Convex Function

When f is convex, any local minimizer x^* is a global minimizer of f. If in addition f is differentiable, then any stationary point x^* is a gobal minimizer of f.

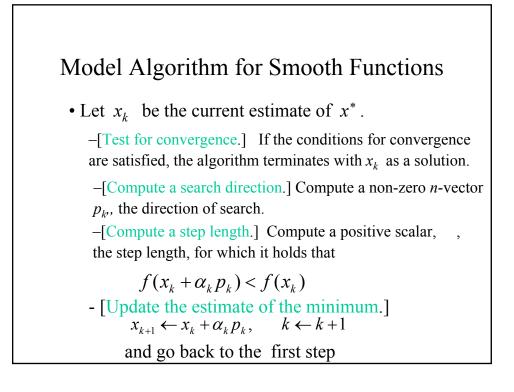
Line Search

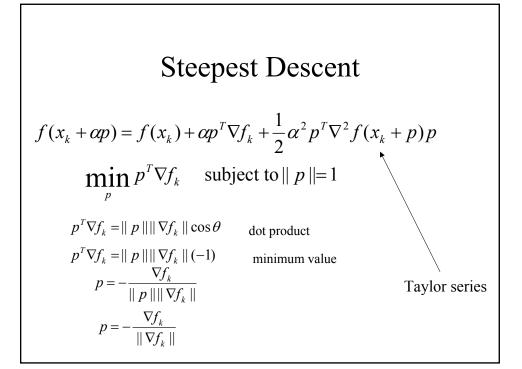
 $\min f(x_k + \alpha p_k)$

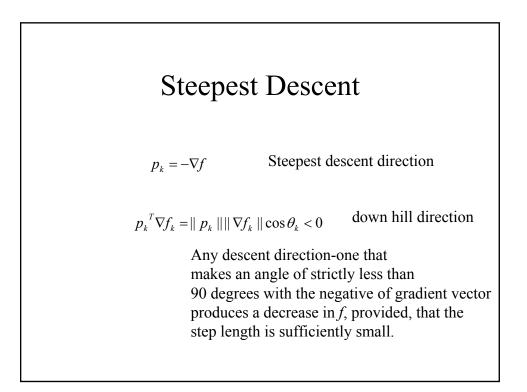
 x_k current iterate

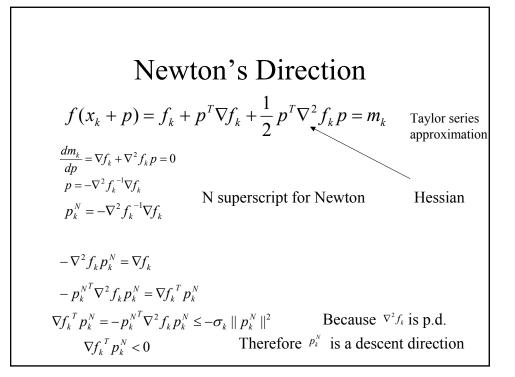
 p_k direction of a search

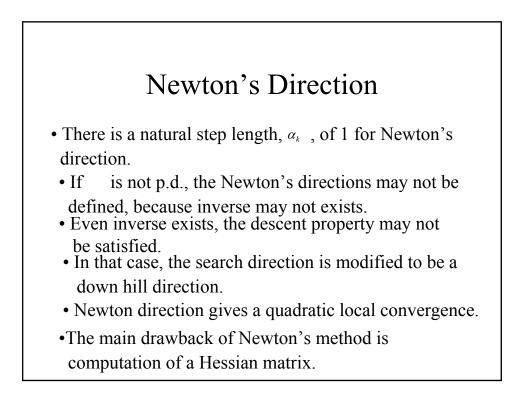
 α distance to move along

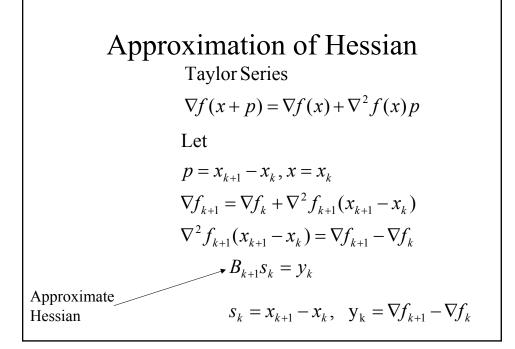


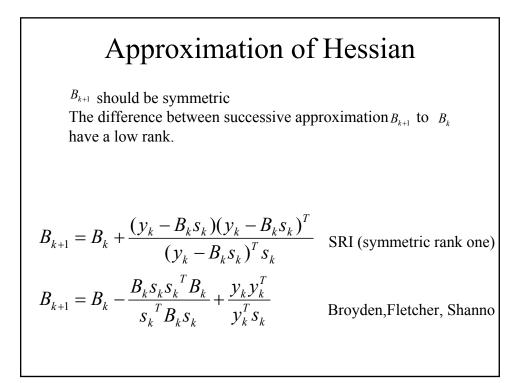












Quasi-Newton

$$p_k = -B_k^{-1} \nabla f_k$$

Inverse Hessian Instead of inverting approximation of Hessian, we can directly compute the approximation of inverse of Hessian: $H_{k+1} = (I - \rho_k s_k y_k^T) H_k (I - \rho_k s_k y_k^T) + \rho_k s_k s_k^T,$ $\rho_k = \frac{1}{y_k^T s_k} \qquad H_k = B_k^{-1}$ $\rho_k = -H_k \nabla f_k$

Conjugate Gradient

$$p_k = -\nabla f(x_k) + \beta_k p_{k-1}$$

 β_k is scalar that p_{k-1} and p_k are conjugate

Two vectors are conjugate with respect to a matrix G if

$$p_k^T G p_{k-1} = 0$$

Where G is PD