Recap

Typical Visual Features

- Color
  - RGB,
  - HSV,
  - Binary, IR
- Edges
  - Sobel, Canny
## Recap

### Typical Visual Features

- **Optical flow**
  - Lucas Kanade

- **Texture**
  - GLCM,
  - Law’s texture
  - Wavelets, Steerable pyramids
  - Energy, entropy, kurtosis…

\[ uI_x + vI_y + I_t = 0 \]

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## Object Detection
Object Detection

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Point Detection
Where can we use it?

- Automate object tracking
- Point matching for computing disparity
- Stereo calibration
  - Estimation of fundamental matrix
- Motion based segmentation
- Recognition
- 3D object reconstruction
- Robot navigation
- Image retrieval and indexing

What is an interest point

- Expressive texture
  - The point at which the direction of the boundary of object changes abruptly
  - Intersection point between two or more edge segments
Synthetic & Real Interest Points

Corners are indicated in red

Properties of Interest Point Detectors

- Detect all (or most) true interest points
- No false interest points
- Well localized.
- Robust with respect to noise.
- Efficient detection
Possible Approaches to Corner Detection

- Based on brightness of images
  - Usually image derivatives
- Based on boundary extraction
  - First step edge detection
  - Curvature analysis of edges

Harris Corner Detector

- Corner point can be recognized in a window
- Shifting a window in any direction should give a large change in intensity

**Basic Idea**

- **“flat” region:** no change in all directions
- **“edge”:** no change along the edge direction
- **“corner”:** significant change in all directions

**Mathematics of Harris Detector**

- **Change of intensity for the shift** \((u,v)\)

\[
E(u,v) = \sum_{x,y} w(x,y) \left[ I(x+u, y+v) - I(x,y) \right]^2
\]

Window functions →
Mathematics of Harris Detector

$$E(u,v) = \sum_{x,y} w(x,y) [u(I(x+u,y) - I(x,y)) + v(I(x,y+v) - I(x,y))]^2$$

$$E(u,v) = \sum_{x,y} w(x,y) [uI_x + vI_y]^2$$

$$E(u,v) = \sum_{x,y} w(x,y) \left(u \begin{pmatrix} I_x \\ I_y \end{pmatrix}ight)$$

$$E(u,v) = \sum_{x,y} w(x,y) \left(u \begin{pmatrix} I_x & I_y \\ I_x & I_y \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} \right)$$

$$E(u,v) = \left(u \begin{pmatrix} I_x \\ I_y \end{pmatrix} \begin{pmatrix} I_x & I_y \\ I_x & I_y \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} \right)$$

Mathematics of Harris Detector

$$E(u,v) = (u \begin{pmatrix} I_x & I_y \\ I_x & I_y \end{pmatrix} \begin{pmatrix} I_x & I_y \\ I_x & I_y \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix})$$

$$M = \sum_{x,y} w(x,y) \begin{pmatrix} I_xI_x & I_xI_y \\ I_xI_y & I_yI_y \end{pmatrix}$$

- E(u,v) is an equation of an ellipse, where M is the covariance
- Let $\lambda_1$ and $\lambda_2$ be eigenvalues of M

$$(\lambda_{\text{max}})^{-1/2} \quad (\lambda_{\text{min}})^{-1/2}$$
Mathematics of Harris Detector

Classification of image points using eigenvalues of $M$:

- $\lambda_1$ and $\lambda_2$ are small; $E$ is almost constant in all directions
- $\lambda_2 \gg \lambda_1$, "Corner" $\lambda_1$ and $\lambda_2$ are large, $\lambda_1 \sim \lambda_2$; $E$ increases in all directions
- $\lambda_2 >> \lambda_1$, "Edge" $\lambda_1 >> \lambda_2$, "Flat" region

Mathematics of Harris Detector

Measure of cornerness in terms of $\lambda_1, \lambda_2$

$$\det(M - I\lambda) = 0$$

$$\ldots$$

$$R = \det M - k(\text{trace}M)^2$$
Mathematics of Harris Detector

- $R$ depends only on eigenvalues of $M$
- $R$ is large for a corner
- $R$ is negative with large magnitude for an edge
- $|R|$ is small for a flat region
Properties of Harris Detector

- Rotation invariance (Homework: Algebraically prove this)

- Ellipse rotates but its shape remains same
  - Hence, Harris is rotation invariant

Properties of Harris Detector

- Invariance to conformal changes in intensities
  - Translation change: I=I+b
  - Scale change: I=al
Properties of Harris Detector

- Varies with image scaling

All points will be classified as edges

Corner!

Scale Invariant Harris Corners

- Build Gaussian Pyramids
- Start from the biggest scale and apply Harris
- Keep points that are also detected in smaller scales.
- Hence Harris becomes rotation and isotropic scale invariant
KLT Corner Detector

- Same as the Harris only difference is use of eigenvalues directly
- Threshold is applied to lowest eigenvalue
  - The threshold $\lambda_{thr}$ can be estimated from the histogram of $\lambda_2$
- Window size effects closeness of features

**SUSAN Detector**

- Proposed by Smith and Brady in 1995
- SUSAN stands for Smallest “Univalue Segment Assimilating Nucleus (USAN)”
- It doesn’t use any derivatives
- It is based on the fact that each point within an image is associated with it a local area of comparable brightness


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**Principle**

- Considered in a circular mask around a pixel
- Comparison of intensity in a neighborhood
  - Area with similar intensities is called USAN
- Repeat the procedure for each pixel
The SUSAN Detector

- USAN area varies with respect to features of the image
- USAN area
  - is maximum within the rectangular area
  - falls to a minimum at an edge
  - smaller value corresponding to a local minimum at a corner
- This is the property upon which the corner finder algorithm is based

Algorithm

1. Determine a circular mask
   - Typically 37 pixels around each pixel (nucleus)
2. Calculate the brightness difference between each pixel in the mask with its nucleus
   \[
   c(r, r_0) = \begin{cases} 
   1 & \text{if } |I(r) - I(r_0)| \leq t \\
   0 & \text{otherwise}
   \end{cases}
   \]
3. Sum the number of pixels with similar intensity levels to that of the nucleus
   \[
   n(r_0) = \sum_r c(r, r_0)
   \]
Algorithm

4. Compare $n$ with $g$, the geometric threshold which is set to half of the maximum value that $n$ can be $(n_{\text{max}}/2)$

5. At a perfect corner the USAN area will always be less than half the size of the mask area, and will be a local minimum

$$R(r_0) = \begin{cases} g - n(r_0) & \text{if } n(r_0) < g \\ 0 & \text{otherwise} \end{cases}$$

Problems

- Strong edges and noise results in false detection
- (Left figure) The USAN is not continuous. Obviously nucleus is not a corner, even though the function shows it is the local maxima.
- (Right figure) Nucleus lies in a long thin area, which depicts USAN is also very small. However, the value is high, which contradicts the fact that the point in question is not a corner.
Improving SUSAN Detector

- Two rules
  - Find centroid of USAN area and distance from nucleus. A point cannot become a corner if the distance is small.
  - One of the pixels on the line connecting centroid to center of circular region can be a corner.

Benefits

- Accuracy, speed and localization

Output of the SUSAN corner detector ($t=10$) given the test image. (0.3 sec)

Output of the Plessey corner detector ($o=2.0$) given the test image. (3.5 sec)
But we want more!!

- Affine invariance
  - Not only rotation, isotropic scaling but also shearing, and anisotropic scaling