Lecture-7

Feature-based Registration

Steps in Feature-based Registration

- Find features
- Establish correspondence (correlation, point correspondence)
- Fit transformation
- Apply transformation (warp)

Features

- Any pixels
- Corner points
- Interest points
- Features obtained using Gabor/Wavelet filters
- Straight lines
- Line intersections

Transformations

- Affine
- Projective
- Psuedo-perspective
- Rational polynomial

Good Features to Track

- Corner like features
- Moravec's Interest Operator

Corner like features

$$C = \begin{bmatrix} \sum_{Q} f_x^2 & \sum_{Q} f_x f_y \\ \sum_{Q} f_x f_y & \sum_{Q} f_y^2 \end{bmatrix}$$

$$\Downarrow$$

$$C = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

Corners

- For perfectly uniform region $\lambda_1 = \lambda_2 = 0$
- If Q contains an ideal step edge, then

$$\lambda_2 = 0, \lambda_1 > 0$$

- if Q contains a corner of black square on white background

$$\lambda_1 \geq \lambda_2 \succ 0$$

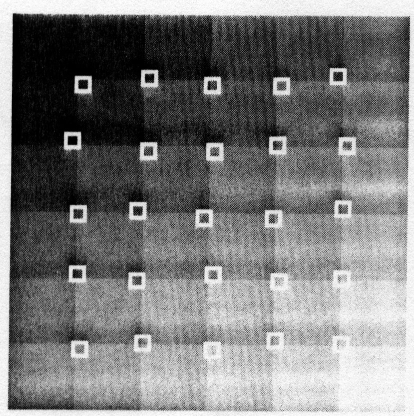
Algorithm Corners

- Compute the image gradient over entire image f.
- For each image point p:
  - form the matrix C over (2N+1)X(2N+1) neighborhood Q of p;
  - compute the smallest eigenvalue of C;
  - if eigenvalue is above some threshold, save the coordinates of p into a list L.

Algorithm Corners

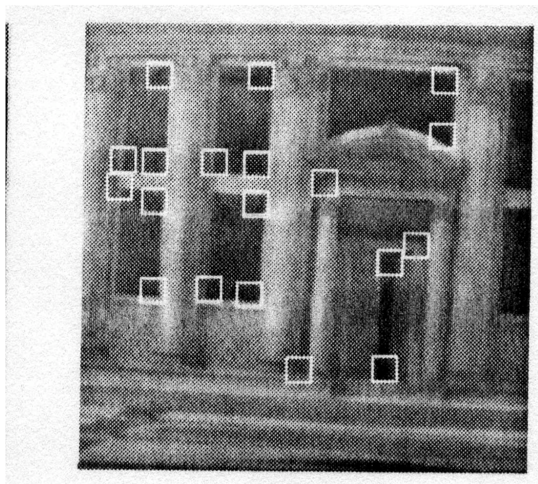
- Sort L in decreasing order of eigenvalues.
- Scanning the sorted list top to bottom: for each current point, p, delete all other points on the list which belong to the neighborhood of p.

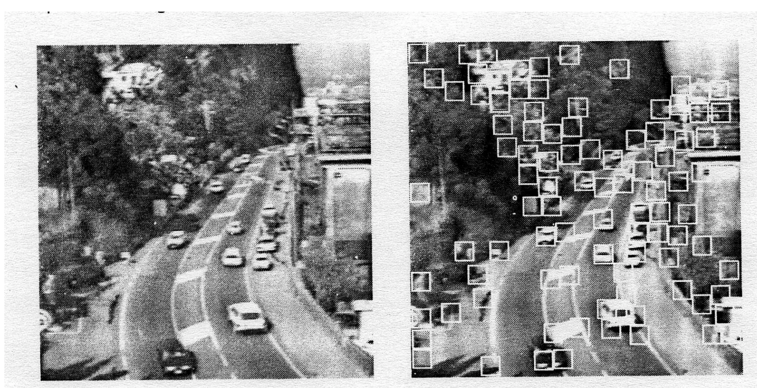
Results



(a)

Results



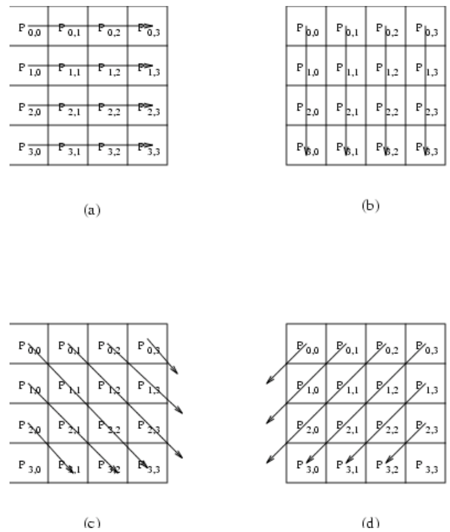


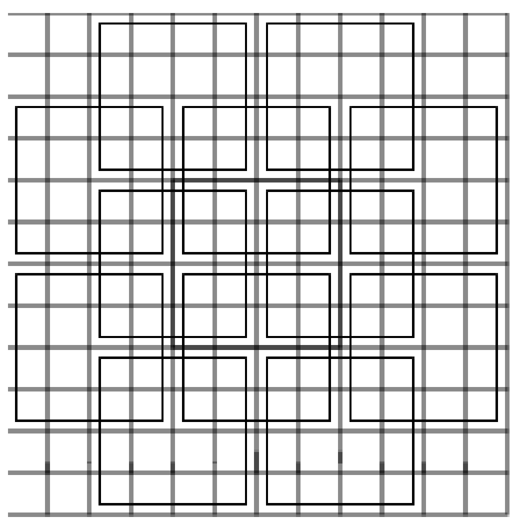
(a) (b)

Moravec's Interest Operator

Algorithm

- Compute four directional variances in horizontal, vertical, diagonal and anti-diagonal directions for each 4 by 4 window.
- If the minimum of four directional variances is a local maximum in a 12 by 12 overlapping neighborhood, then that widow (point) is interesting.


(a)
(b)
(c)
(d)



$$V_h = \sum_{j=0}^{3} \sum_{i=0}^{2} (P(x+i, y+j) - P(x+i+1, y+j))^2$$

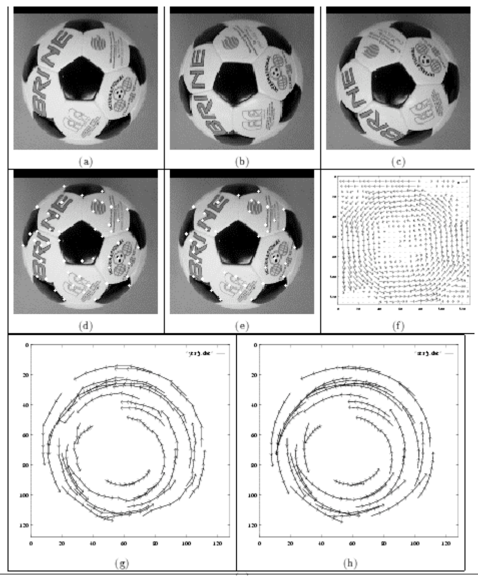
$$V_v = \sum_{j=0}^{2} \sum_{i=0}^{3} (P(x+i, y+j) - P(x+i, y+j+1))^2$$

$$V_d = \sum_{j=0}^{2} \sum_{i=0}^{2} (P(x+i, y+j) - P(x+i+1, y+j+1))^2$$

$$V_a = \sum_{j=0}^{2} \sum_{i=1}^{3} (P(x+i, y+j) - P(x+i-1, y+j+1))^2$$

$$V(x,y) = \min(V_h(x,y), V_v(x,y), V_d(x,y), V_a(x,y))$$

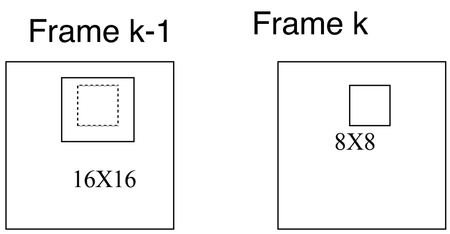
$$I(x,y) = \begin{cases} 1 & if V(x,y) local \max \\ 0 & 0therwise \end{cases}$$



Correlation

- Similarity/Dissimilarity Measures
  - Sum of Squares Difference (SSD)
  - Normalized Correlation
  - Mutual Correlation
  - Mutual information $H(X;Y)=H(X)-H(X|Y)$
- Use
  - Gray levels
  - Laplacian of Gaussian
  - Gradient magnitude

Block Matching



Block Matching

- For each 8X8 block, centered around pixel (x,y) in frame k, B_k
  - Obtain 16X16 block in frame k-1, centered around (x,y), B_{k-1}
  - Compute Sum of Squares Differences (SSD) between 8X8 block, B_k, and all possible 8X8 blocks in B_{k-1}
  - The 8X8 block in B_{k-1} centered around (x',y'), which gives the least SSD is the match
  - The displacement vector (optical flow) is given by u=x-x'; v=y-y'

Sum of Squares Differences (SSD)

$$(u(x,y),v(x,y)) = \arg\min_{u,v=-3\ldots3} \sum_{i=0}^{7} \sum_{j=0}^{7} \left(f_k(x+i,y+j) - f_{k-1}(x+i+u,y+j+v)\right)^2$$

Minimum Absolute Difference (MAD)

$$(u(x,y),v(x,y)) = \arg\min_{u,v=-3\ldots3} \sum_{i=0}^{7} \sum_{j=0}^{7} \left|\left(f_k(x+i,y+j) - f_{k-1}(x+i+u,y+j+v)\right)\right|$$

Maximum Matching Pixel Count (MPC)

$$T(x,y;u,v) = \begin{cases} 1 & \text{if } |f_k(x,y) - f_{k-1}(x+u,y+v)| \le t \\ 0 & \text{Otherwise} \end{cases}$$

$$(u(x,y),v(x,y)) = \arg\max_{u,v=-3\ldots3} \sum_{i=0}^{7} \sum_{j=0}^{7} T(x+i,y+j;u,v)$$

Cross Correlation

$$(u,v) = \arg\max_{u,v=-3\ldots3} \sum_{i=0}^{7} \sum_{j=0}^{7} \left(f_k(x+i,y+j)).(f_{k-1}(x+i+u,y+j+v) \right)$$

Normalized Correlation

$$(u,v) = \arg\max_{u,v=-3\ldots3} \frac{\sum_{i=0}^{7}\sum_{j=0}^{7}\left(f_k(x+i,y+j)).(f_{k-1}(x+i+u,y+j+v)\right)}{\sqrt{\sum_{i=0}^{7}\sum_{j=0}^{7} f_{k-1}(x+i+u,y+j+v).f_{k-1}(x+i+u,y+j+v)}}$$

Mutual Correlation

$$(u,v) = \arg\max_{u,v=-3\ldots3} \frac{1}{64\sigma_1\sigma_2}\sum_{i=0}^{7}\sum_{j=0}^{7}\left(f_k(x+i,y+j)-\mu_1).(f_{k-1}(x+i+u,y+j+v)-\mu_2\right)$$

Sigma and mu are standard deviation and mean of patch-1 and patch-2 respectively

Issues with Correlation

- Patch Size
- Search Area
- How many peaks

Spatiotemporal Models

- First order Taylor series

$$f(x, y, t) = f(x + dx, y + dy, t + dt)$$

$$f(x, y, t) = f(x, y, t) + \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial t} dt$$

$$\left[\sum \nabla X^T \mathbf{f_X} \mathbf{f_X^T} \nabla X\right] \partial a = -\sum \nabla X^T \mathbf{f_X} f_t$$

$$\left(u(x,y), v(x,y)\right) = \arg\min_{u,v=-3\ldots3} \sum_{i=0}^{\square 7} \sum_{j=0}^{\square 7} \left|\left(f_k(x+i, y+j) - f_{k-1}(x+i+u, y+j+v)\right)\right|$$

Correlation

Bilinear and Pseudo-Perspective

$$(\Box\ \Box\Box^{T})\mathbf{q} = \Box\Box\ f_t\Box$$

$$\Box^{T} = \left[f_x(xy,x,y,1),\quad f_y\left(xy,x,y,1\right)\right]$$ **bilinear**

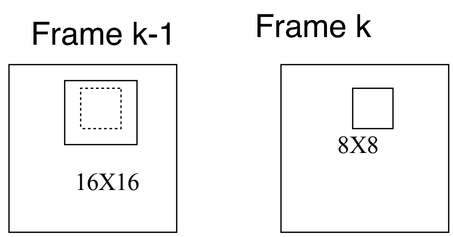
$$\Box^{T} = \left[f_x(x,y,1)\quad f_y(x,y,1)\quad c_1\quad c_2\right]$$

$$c_1 = x^2 f_x + xyf_x$$

$$c_2 = xyf_x + y^2 f_y$$

Pseudo perspective

Correlation Vs Spatiotemporal



Correlation Complexity

- m*m multiplications and additions
- 2*m*m additions and 2 divisions for two means
- 2*m*m multiplications and additions for variances

$$(u,v) = \arg\max_{u,v=-3\ldots3} \frac{1}{64\sigma_1\sigma_2} \sum_{i=0}^{7} \sum_{j=0}^{7} \left(f_k(x+i, y+j) - \mu_1 \right).\left(f_{k-1}(x+i+u, y+j+v) - \mu_2 \right)$$

Spatiotemporal Complexity

- 3* m*m subtractions for sptiotemporal derivatives
- (36+6)*m*m additions for generating linear system
- 6*6*6 multiplications and additions for solving 6 by 6 linear system

$$\left[\sum X^T \mathbf{f_X} \mathbf{f_X^T} X\right] a = -\sum X^T \mathbf{f_X} f_t$$

Feature-based Matching

Feature-based Matching

- The input is formed by f1 and f2, two frames of an image sequence.
- Let Q1, Q2 and Q' be three NXN image regions.
- Let "d" be the unknown displacement vector between f1 and f2 of a feature point "p", on which Q1 is centered.

Algorithm

- Set d=0, center Q1 on p1.
- Estimate the displacement "d0" of "p", center of "Q1", using Lucas and Kanade method. Let d=d+d0.
- Let Q' bet the patch obtained by warping Q1 according to "d0". Compute Sum of Square (SSD) difference between new patch Q' and corresponding patch Q2 in frame f2.
- If SSD more than threshold, set Q1=Q' and go to step 1, otherwise exit.

Lucas & Kanade (Least Squares)

- Optical flow eq

$$f_x u + f_y v = -f_t$$

- Consider 3 by 3 window

$$f_{x1} u + f_{y1} v = -f_{t1}$$

$$\vdots$$

$$f_{x9} u + f_{y9} v = -f_{t9}$$

$$\begin{bmatrix} f_{x1} & f_{y1} \\ \vdots & \vdots \\ f_{x9} & f_{y9} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} -f_{t1} \\ \vdots \\ -f_{t9} \end{bmatrix}$$

$$\mathbf{Au} = \mathbf{f_t}$$

Shi-Tomasi-Kanade(STK) Tracker

$$I(\delta(\mathrm{x}), t+\tau) = I(\mathrm{x}, t)$$

$$\delta(\mathrm{x}) = A\mathrm{x} + d$$

$$\varepsilon = \sum_W \left[I(A\mathrm{x} + d, t+\tau) - I(\mathrm{x}, t) \right]^2$$

After the first order Taylor expansion at A=I and d=0, we can get a linear 6_6 system:

$$Tz = f$$

$$z = \begin{bmatrix} A_{11} & A_{12} & A_{21} & A_{22} & d_1 & d_2 \end{bmatrix}$$

$$f = I_t \sum_W \begin{bmatrix} xI_x & yI_x & xI_y & yI_y & I_x & I_y \end{bmatrix}$$

Shi-Tomasi-Kanade(STK) Tracker

- Advantages:
  - Easy to implement .
  - Works very well with a small transformation.
- Drawbacks:
  - Fail to track in presence of a large rotation.
- Reason:
  - The rotation component implied in affine model is non-linear.

Useful Links

http://twtelecom.dl.sourceforge.net/sourceforge/opencvlibrary/OpenCVReferenceManual.pdf

http://vision.stanford.edu/~birch/klt/.