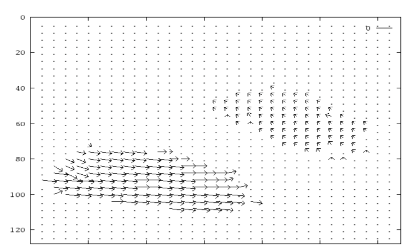
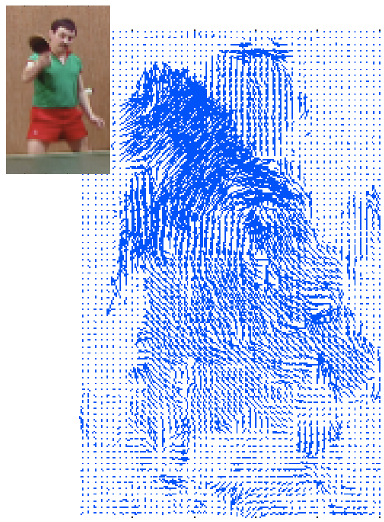
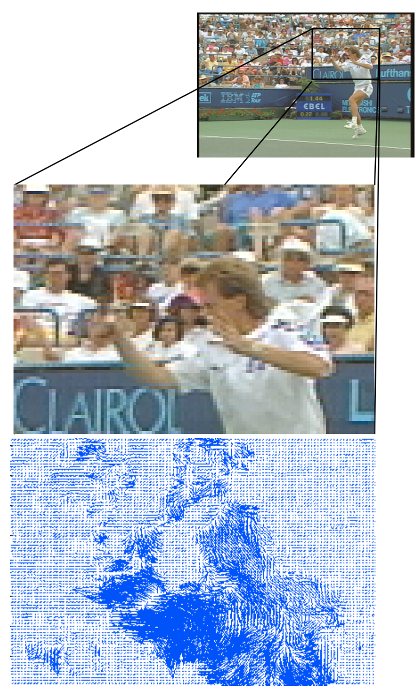
Lecture-3

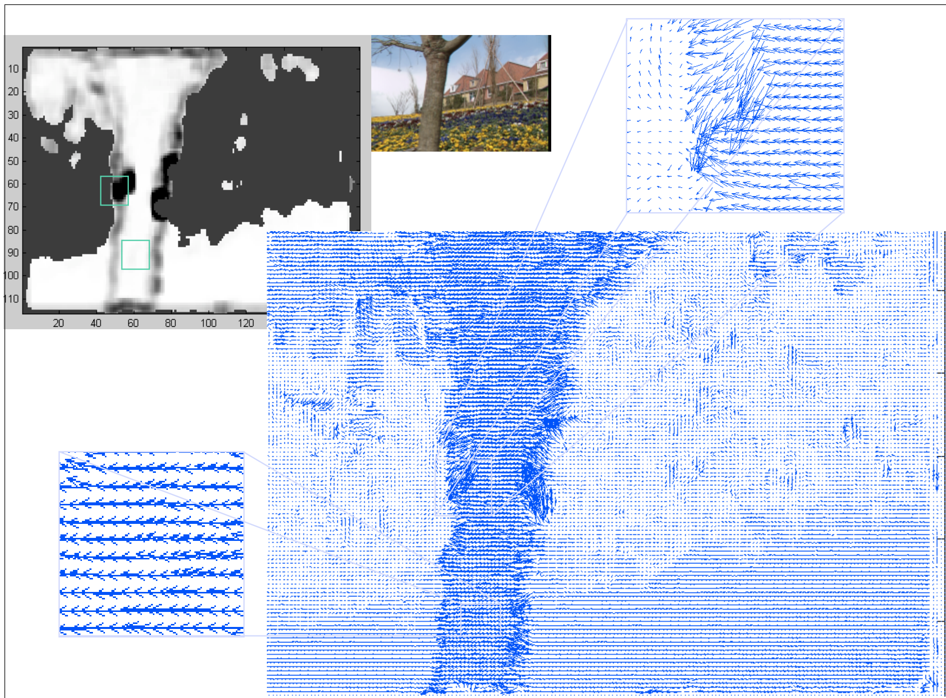
Computing Optical Flow

Hamburg Taxi seq






CLAIROL



Horn&Schunck Optical Flow

$f(x, y, t)$ Image Sequence

$$\frac{df(x,y,t)}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial t} = 0$$

brightness constancy eq

Horn&Schunck Optical Flow

$$f(x, y, t) = f(x + dx, y + dy, t + dt)$$

Taylor Series

$$f(x, y, t) = f(x, y, t) + \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial t}dt$$

$$f_x dx + f_y dy + f_t dt = 0$$

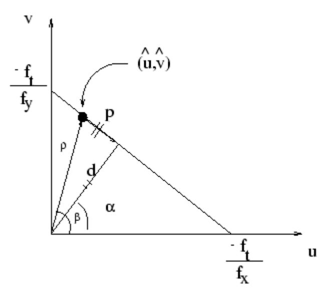
brightness constancy eq

Interpretation of optical flow eq

$$f_x u + f_y v + f_t = 0$$

$$v = -\frac{f_x}{f_y}u - \frac{f_t}{f_y}$$

Equation of st.line



d=normal flow

p=parallel flow

$$d = \frac{f_t}{\sqrt{f_x^2 + f_y^2}}$$

Horn&Schunck (contd)

$$\iint \{(f_x u + f_y v + f_t)^2 + \lambda(u_x^2 + u_y^2 + v_x^2 + v_y^2)\} dxdy$$

min

$$(f_x u + f_y v + f_t) f_x + \lambda(\nabla^2 u) = 0$$

$$(f_x u + f_y v + f_t) f_y + \lambda((\nabla^2 v) = 0$$

discrete version

$$(f_x u + f_y v + f_t) f_x + \lambda(u - u_{av}) = 0$$

$$(f_x u + f_y v + f_t) f_y + \lambda((v - v_{av}) = 0$$

variational calculus

$$u = u_{av} - f_x \frac{P}{D}$$

$$v = v_{av} - f_y \frac{P}{D}$$

$$P = f_x u_{av} + f_y v_{av} + f_t$$

$$D = \lambda + f_x^2 + f_y^2$$

$$\nabla^2 u = u_{xx} + u_{yy}$$

Algorithm-1

- k=0
- Initialize u^K v^K
- Repeat until some error measure is satisfied (converges)

$$u^K = u_{av}^{k-1} - f_x \frac{P}{D}$$

$$v = v_{av}^{K-1} - f_y \frac{P}{D}$$

$$P = f_x u_{av} + f_y v_{av} + f_t$$

$$D = \lambda + f_x^2 + f_y^2$$

Derivatives

- Derivative: Rate of change of some quantity
  - Speed is a rate of change of a distance
  - Acceleration is a rate of change of speed

Derivative

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x) - f(x - \Delta x)}{\Delta x} = f'(x) = f_x$$

$$v = \frac{ds}{dt} \quad \text{speed}$$

$$a = \frac{dv}{dt} \quad \text{acceleration}$$

Examples

$$y = x^2 + x^4 \qquad y = \sin x + e^{-x}$$

$$\frac{dy}{dx} = 2x + 4x^3 \qquad \frac{dy}{dx} = \cos x + (-1)e^{-x}$$

Second Derivative

$$\frac{df_x}{dx} = f''(x) = f_{xx}$$

$$y = x^2 + x^4$$

$$\frac{dy}{dx} = 2x + 4x^3$$

$$\frac{d^2 y}{dx^2} = 2 + 12x^2$$

Discrete Derivative

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x) - f(x - \Delta x)}{\Delta x} = f'(x)$$

$$\frac{df}{dx} = \frac{f(x) - f(x-1)}{1} = f'(x)$$

$$\frac{df}{dx} = f(x) - f(x-1) = f'(x)$$

(Finite Difference)

Discrete Derivative

$$\frac{df}{dx} = f(x) - f(x-1) = f'(x)$$ Left difference

$$\frac{df}{dx} = f(x) - f(x+1) = f'(x)$$ Right difference

$$\frac{df}{dx} = f(x+1) - f(x-1) = f'(x)$$ Center difference

Example

	F(x)=10	10	10	10	20	20	20
Left	F'(x)=0	0	0	0	10	0	0
difference	F''(x)=0	0	0	0	10	-10	0

-1	1		left difference
1	-1		right difference
-1	0	1	center difference

Derivatives in Two Dimensions

$$f(x,y)$$

(partial Derivatives)

$$\frac{\partial f}{\partial x} = f_x = \lim_{\Delta x \to 0} \frac{f(x,y) - f(x - \Delta x, y)}{\Delta x}$$

$$\frac{\partial f}{\partial y} = f_y = \lim_{\Delta y \to 0} \frac{f(x,y) - f(x, y - \Delta y)}{\Delta y}$$

(f_x, f_y) Gradient Vector

magnitude $= \sqrt{(f_x^2 + f_y^2)}$

direction $= \theta = \tan^{-1} \frac{f_y}{f_x}$

$$\nabla^2 f = f_{xx} + f_{yy} = \text{Laplacian}$$

Derivatives of an Image

Derivative & average

$$\begin{matrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{matrix}$$

f_x

$$\begin{matrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{matrix}$$

f_y

Prewit

$$I(x,y) = \begin{bmatrix} 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \end{bmatrix}$$

$$I_x = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 30 & 30 & 0 & 0 \\ 0 & 30 & 30 & 0 & 0 \\ 0 & 30 & 30 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Derivatives of an Image

$$I(x,y) = \begin{bmatrix} 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \end{bmatrix}$$

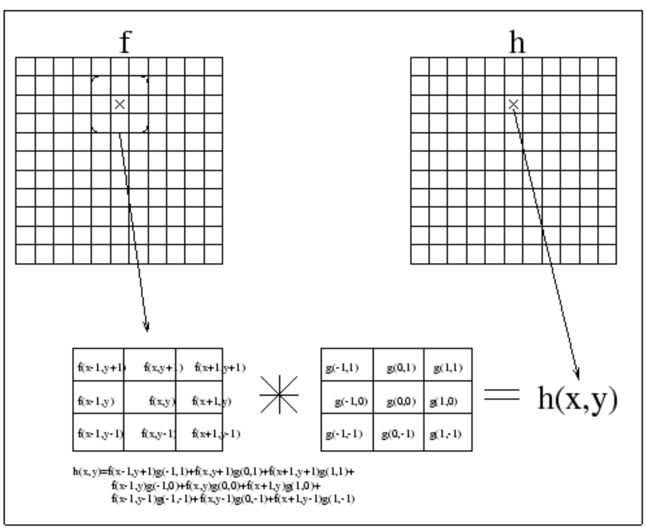
$$I_y = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Laplacian

$$\begin{matrix} 0 & \square\frac{1}{4} & 1 \\ \square\frac{1}{4} & 1 & \square\frac{1}{4} \\ 0 & \square\frac{1}{4} & 0 \end{matrix}$$

$$f_{xx} + f_{yy}$$

Convolution



Convolution (contd)

$$h(x,y) = \sum_{i=-1}^{1} \sum_{j=-1}^{1} f(x+i, y+j) g(i,j)$$

$$h(x,y) = f(x,y) * g(x,y)$$

$$\begin{aligned} h(x,y) = {} & f(x-1,y-1)g(-1,-1) + f(x,y-1)g(0,-1) + f(x+1,y-1)g(1,-1) \\ & + f(x-1,y)g(-1,0) + f(x,y)g(0,0) + f(x+1,y)g(1,0) \\ & + f(x-1,y+1)g(-1,1) + f(x,y+1)g(0,1) + f(x+1,y+1)g(1,1) \end{aligned}$$

Derivative Masks

f_x:

$$\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \text{first image} \qquad \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \text{second image}$$

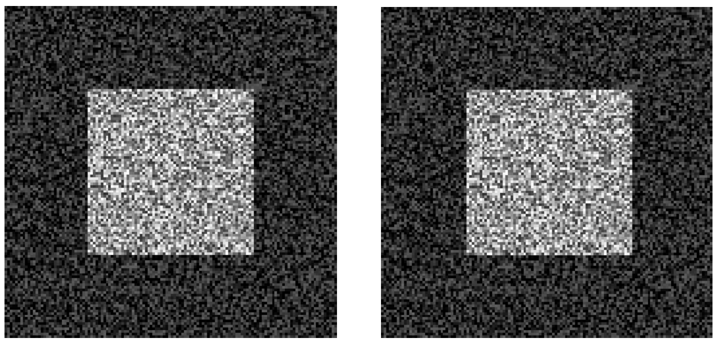
f_y:

$$\begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} \text{first image} \qquad \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} \text{second image}$$

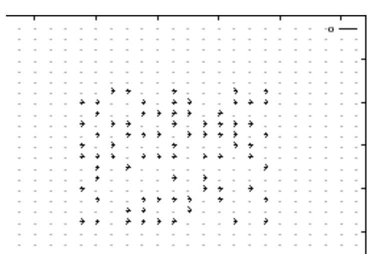
f_t:

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \text{first image} \qquad \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \text{second image}$$

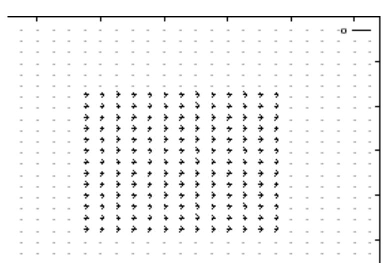
Synthetic Images



Results



One iteration



10 iterations

□ = 4

Homework Due 9/9/02

- Derive Euler Angles matrix from three rotations around x, y and Z. (Lecture-2, page 3, do not hand in).
- Derive bi-quadratic motion model from the projective motion model using Taylor series. (Lecture-2, page 11).
- Verify 3-D rigid motion using instantaneous motion model can be written as a cross product of rotational velocities and object location (X). Lecture-2, page 16.
- Verify that pseudo perspective motion model can be derived assuming planar scene and perspective projection. Lecture-2, page 18.