







Eigen Vectors and Eigen Values

The eigen vector, \mathbf{x} , of a matrix A is a special vector, with the following property

Ax = Ix Where ? is called eigen value

To find eigen values of a matrix A first find the roots of:

 $\det(A - II) = 0$

Then solve the following linear system for each eigen value to find corresponding eigen vector

(A-II)x=0







Face Recognition

Collect all gray levels in a long vector *u*:

 $u = (I(1,1),...,I(1,N),I(2,1),...,I(2,N),...,I(M,1),...,I(M,N))^T$ Collect *n* samples (views) of each of *p* persons in matrix A (MN X pn):

 $A = \left[u_1^1, \dots, u_n^1, u_1^2, \dots, u_n^2, \dots, u_1^p, \dots, u_n^p \right]$

Form a correlation matrix L (MN X MN):

 $L = AA^T$

Compute eigen vectors, $f_1, f_2, f_3, \dots, f_{n_1}$, of L, which form a bases for whole face space

Face Recognition

Each face, u, can now be represented as a linear combination of eigen vectors

$$u = \sum_{i=1}^{n_1} a_i \mathbf{f}_i$$

Eigen vectors for a symmetric matrix are orthonormal:

$$\boldsymbol{f}_{i}^{T}\boldsymbol{f}_{j} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

Face Recognition

$$u_x^T \mathbf{f}_i = (\sum_{i=1}^n a_i \mathbf{f}_i)^T \mathbf{f}_i$$

$$= (\mathbf{a}_1 \mathbf{f}_1^T + \mathbf{a}_2 \mathbf{f}_2^T + \ldots + \mathbf{a}_i \mathbf{f}_i^T + \ldots + \mathbf{a}_n \mathbf{f}_n^T) \mathbf{f}_i$$

$$u_x^T \cdot \mathbf{f}_i = (\mathbf{a}_1 \mathbf{f}_1^T \cdot \mathbf{f}_i + \mathbf{a}_2 \mathbf{f}_2^T \cdot \mathbf{f}_i + \ldots + \mathbf{a}_i \mathbf{f}_i^T \cdot \mathbf{f}_i + \ldots + \mathbf{a}_n \mathbf{f}_n^T \cdot \mathbf{f}_i)$$

$$u_x^T \cdot \mathbf{f}_i = a_i$$
herefore:

$$a_i = u_x^T \cdot \mathbf{f}_i$$

Therefore:

Face Recognition

L is a large matrix, computing eigen vectors of a large matrix is time consuming. Therefore compute eigen vectors of a smaller matrix, C:

$$C = A^T A$$

Let \boldsymbol{a}_i be eigen vectors of C, then $A\boldsymbol{a}_i$ are the eigen vectors of A:

 $C \boldsymbol{a}_i = \boldsymbol{l}_i \boldsymbol{a}_i$ $A^T A \boldsymbol{a}_i = \boldsymbol{l}_i \boldsymbol{a}_i$ $AA^{T}(A\boldsymbol{a}_{i}) = \boldsymbol{l}_{i}(A\boldsymbol{a}_{i})$ $L(A\boldsymbol{a}_i) = \boldsymbol{l}_i(A\boldsymbol{a}_i)$

Training

- Create *A* matrix from training images
- Compute *C* matrix from *A*.
- Compute eigenvectors of *C*.
- Compute eigenvectors of *L* from eigenvectors of *C*.
- Select few most significant eigenvectors of *L* for face recognition.
- Compute coefficient vectors corresponding to each training image.
- For each person, coefficients will form a cluster, compute the mean of cluster.



```
load faces.mat
C=A'*A:
[vectorC,valueC]=eig(C);
ss=diag(valueC);
[ss,iii]=sort(-ss);
vectorC=vectorC(:,iii);
vectorL=A*vectorC(:,1:5);
Coeff=A'*vectorL;
for I=1:30
         model(i, :)=mean(coeff((5*(i-1)+1):5*I,:));
end
while (1)
         imagename=input('Enter the filename of the image to
         Recognize(0 stop):');
         if (imagename <1)
         break;
         end;
         imageco=A(:,imagename)'*vectorL;
         disp ('');
         disp ('The coefficients for this image are:');
```

```
mess1=sprintf('%.2f %.2f %.2f %.2f %.2f',
         imageco(1), imageco(2), imageco(3), imageco(4),
         imageco(5));
         disp(mess1);
         top=1;
         for I=2:30
                  if (norm(model(i,:)-imageco,1)<norm(model
                  (top, :)-imageco,1))
                  top=i
                  end
         end
         mess1=sprintf('The image input was a image of person
         number %d',top);
         disp(mess1);
         end
b=A(:,81);
b=reshape(b,34,51);
imshow(b,gray(255)):
```



Visual Lipreading





Feature Subspace Generation

- Generate a lower dimension subspace onto which image sequences are projected to produce a vector of coefficients.
- Components
 - Sample Matrix
 - Most Expressive Features

Generating the Sample Matrix

• Consider *e* letters, each of which has a training set of K sequences. Each sequence is compose of images:

$$I_1, I_2, \ldots, I_p$$

• Collect all gray-level pixels from all images in a sequence into a vector:

 $u = (I_1(1,1), \dots, I_1(M,N), I_2(1,1), \dots, I_2(M,N), \dots, I_p(1,1), \dots, I_p(M,N))$

. Generating the Sample Matrix

• For letter \boldsymbol{W} , collect vectors into matrix T $T_{\boldsymbol{w}} = \begin{bmatrix} u^1, u^2, \dots u^K \end{bmatrix}$

• Create sample matrix A:

$$A = \left[T_1, T_2, \dots T_e\right]$$

•The eigenvectors of a matrix $L = AA^T$ are defined as:

$$L \boldsymbol{f}_i = \boldsymbol{l}_i \boldsymbol{f}_i$$

The Most Expressive Features • f is an orthonormal basis of the sample matrix. • Any image sequence, u, can be represented as: $u = \sum_{n=1}^{Q} a_{nf_n} = fa$ • Use Q most significant eigenvectors. • The linear coefficients can be computed as: $a_n = u^T f_n$













Extracting letters from Connected Sequences

• Detect valleys in g.

• From valley locations in g, find locations where f crosses high threshold.

• Locate beginning and ending frames.













